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T H È S E

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Présentée par
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Modélisation des marchés du gaz naturel en Europe en concurrence oligopolistique. Le modèle GaMMES et quelques applications

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Un jour, tu creuses. Si tu as un peu de chance, tu trouves du pétrole. Si tu n'as pas de chance, tu ne trouves rien. Mais si tu n'as vraiment pas de chance, tu trouves du gaz naturel.

Anonyme.

Normal completion... Other error.

Gams.

Abstract

This thesis studies the evolution of the natural gas markets in Europe, until 2035, using optimization theory tools. The model we develop, named GaMMES, is based on an oligopolistic description of the markets. Its main advantages are the following : we consider an important level of detail in the economic structure of the gas chain and we endogenously take into account long-term contracts in the upstream as well as energy substitution between gas, oil, and coal in the demand.

In the first part of this thesis, we study the issue of security of supply in Europe and the conditions under which it is necessary to regulate the gas markets that are strongly dependent on foreign imports. Three case studies are then presented, regarding the level of dependence and the markets' specificities : the German gas trade of the 1980s and the current Spanish and Bulgarian markets. We study in particular the evolution of the markets' outcome as a function of the supply disruption probability and the kind of regulation to implement in order to maximize the social welfare.

In the second part, we develop a system dynamics model in order to capture fuel substitution between oil, coal, and natural gas. Our approach allows one to calculate a new functional form of the demand function for natural gas that contains energy substitution and consumption inertia effects due to end-users' investments.

In the third part, we take advantage of our demand function and use it in a partial equilibrium model of natural gas markets in Europe. The GaMMES model, when written as a complementarity problem, describes the principal gas chain actors as well as their strategic interactions and market power. It was applied to the northwestern European gas trade to analyze the evolution of consumption, spot and long-term contract prices and volumes, production, and natural gas dependence, until 2035.

In the last part, we present a stochastic extension of the GaMMES model in order to study the impact of the strong Brent price fluctuation on the gas markets. An econometric analysis allowed us to calculate the probability law of the oil price, when taken as a random variable, in order to construct the scenario tree and estimate its weights. Our results show how uncertainty changes the strategic behavior, in particular for the long-term contracting activity. Finally, the value of the stochastic solution is calculated to quantify the importance of taking into account randomness in the optimization programs of the gas chain actors.

Résumé

Cette thèse étudie l'évolution des marchés du gaz naturel en Europe jusqu'en 2035 en utilisant les outils de la modélisation. Le modèle proposé, intitulé GaMMES, repose sur une description oligopolistique des marchés. Ses principaux avantages sont les suivants : un niveau de détail important de la structure économique de la chaîne gazière et une prise en compte endogène des contrats de long-terme en amont ainsi que de la substitution avec les produits pétroliers et le charbon, au niveau de la demande.

Dans un premier temps, nous étudions la question de la sécurité d'approvisionnement en gaz en Europe et les conditions favorables à la régulation des marchés vulnérables au risque de rupture d'approvisionnement, notamment de la part de la Russie. Trois études de cas sont proposées selon le degré de dépendance et la nature de la régulation en place : le marché allemand des années 1980 et les marchés actuels de la Bulgarie et de l'Espagne. Nous étudions en particulier l'évolution des caractéristiques des marchés en fonction du risque de rupture et le type de régulation à mettre en place afin d'assurer l'optimalité du bien-être social.

Ensuite, nous proposons un modèle de type systèmes dynamiques afin de prendre en compte la substitution énergétique entre le charbon, le pétrole et le gaz naturel. Notre approche permet d'estimer une nouvelle forme fonctionnelle de la fonction de demande pour le gaz naturel, qui englobe à la fois la substitution énergétique et les inerties de consommation dues aux investissements des usagers finaux.

Dans un troisième temps, nous utilisons cette fonction de demande dans un modèle d'équilibre partiel des marchés du gaz naturel en Europe. Le modèle GaMMES, écrit sous forme de problème de complémentarité, représente les principaux acteurs de l'industrie du gaz naturel en considérant leurs interactions stratégiques et les pouvoirs de marchés. Il a été appliqué au marché du gaz naturel dans la zone nord-ouest de l'Europe afin d'étudier l'évolution, jusqu'en 2035, de la consommation, des prix spot, des prix et volumes long-terme, de la production et de la dépendance par rapport aux imports étrangers.

Finalement, nous proposons une extension stochastique du modèle GaMMES afin d'analyser l'impact de la forte fluctuation du prix du Brent sur les marchés gaziers. Une étude économétrique a été menée afin de calculer la loi de probabilité du prix du pétrole, lorsqu'il est modélisé en tant que variable aléatoire, dans le but de construire et pondérer l'arbre des scénarii. Les résultats permettent de comprendre comment l'aléa modifie les comportements stratégiques des acteurs, notamment au niveau des contrats de long-terme. Enfin, la valeur de la solution stochastique est calculée afin de quantifier l'importance de la prise en compte des fluctuations du prix du pétrole pour chaque acteur de la chaîne.

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Notation and abreviations

Table 1 – *Abreviations*

APG	Average percentage growth
BCM	Billion cubic meter
CM	Cubic meter
GaMMES	Gas Markets Modeling with Energy Substitution
GNC or GNCG	Generalized Nash-Cournot problem
GSS	Gain of the stochastic solution
KKT	Karush-Khun and Tucker conditions
LNG	Liquefied natural gas
LP	Linear programming
LTC	Long-term contract
LSS	Loss of the stochastic solution
MAPE	Mean absolute percent error
MCP	Mixed complementarity problem
MVM	Mean value model
NCE	Nash-Cournot equilibrium
NG	Natural gas
QVI	Quasi-Variational inequality
RMSE	Root mean square error
SD	System dynamics
S-GaMMES	Stochastic Gas Markets Modeling with Energy Substitution
SM	Stochastic model
SNC or SNCG	Standard Nash-Cournot problem
SSP	Security of supply parameters
TOP	Take or pay clauses
TOE	Ton of oil equivalent
VI	Variational inequation
VSS	Value of the stochastic solution
$	US dollar

Première partie

Introduction

Introduction

§ 0.1 La consommation énergétique

Il est aisé de constater que notre société devient de plus en plus dépendante de l'énergie. Pour s'en convaincre, il suffit d'énumérer quelques secteurs et activités où elle est devenue indispensable : que ce soit pour le transport, le chauffage, l'éclairage ou les communications, l'énergie est devenue nécessaire à nos modes de vie contemporains. Plus particulièrement, depuis le début de l'ère industrielle, les énergies fossiles, extraites du sol terrestre, prennent une place très importante dans notre consommation puisqu'elle ont représenté, durant plusieurs décennies, la majorité de l'énergie que nous utilisons [1]. Cette énergie fossile est issue de la matière organique que la terre met quelques milliards d'années à produire à des profondeurs assurant des conditions de température et de pression favorables à sa formation. Cette énergie se trouve principalement sous trois formes qui coïncident avec les trois états de la matière : le pétrole (liquide), le gaz naturel (gazeux) et le charbon (solide). La matière fossile a très rapidement été exploitée à grande échelle car son utilisation est simple, pratique et, à certains égards, peu coûteuse par rapport aux autres sources d'énergie.

Etant donné la durée de fabrication naturelle de la matière fossile, force est de constater que son utilisation a été d'une rapidité sidérante. En 2005, entre 45 et 70% des réserves mondiales de pétrole ont été consommées (BP Statistical Review 2010). Ainsi, dans un monde qui doit faire face à une situation alarmante de ressource épuisable, se tourne-t-on vers des énergies renouvelables telles que l'énergie solaire et l'énergie éolienne. En outre, les problèmes environnementaux, dont on commence à appréhender l'importance, nous poussent à limiter l'impact de nos activités sur la planète, notamment en réduisant nos emissions de gaz à effet de serre. A cet effet, le gaz naturel peut apporter une solution transitoire entre notre mode de consommation actuel, fondé sur la matière fossile et un mode d'usage "décarboné". En effet, des trois énergies fossiles, le gaz naturel est celle qui libère le moins de dioxyde de carbone et de polluants (tels que les NO_X et les SO_X) et est par conséquent souvent considérée comme comparativement moins polluante. [2] En outre, les réserves prouvées de gaz naturel sont plus importantes que celles du pétrole. Cependant, il paraît

1. Cette remarque s'applique surtout pour les pays développés puisque pour ceux en voie de développement, la biomasse continue de prendre une part majoritaire de la consommation énergétique.
2. Toutefois, le gaz naturel emet de fortes quantités de méthane, un puissant gaz à effet de serre.

important de ne pas reporter notre dépendance énergétique au gaz naturel qui, ne l'oublions pas, est une ressource épuisable, mais plutôt de l'utiliser comme un appui provisoire des approvisionnements renouvelables. Ainsi, selon un rapport de l'AIE (WEO 2011), un usage important du gaz naturel en remplacement des autres sources ne permettrait pas réduire significativement la concentration de dioxyde de carbone dans l'atmosphère.

Les scénarii d'évolution de la croissance économique et de notre consommation énergétique prévoient une hausse continue dans les décennies à venir. Cela est principalement dû à une sorte d'inertie de consommation, inhérente aux usages actuels de l'énergie. En outre, l'augmentation démographique de la planète induira indubitablement une augmentation de la consommation de l'énergie. [3] Ainsi, la croissance économique et démographique des pays en voie de développement (Chine, Inde, etc.) constitue un moteur important de la demande énergétique mondiale. Selon l'Agence Internationale de l'Energie (rapport IEA 2008), la consommation d'énergie primaire augmentera de plus de 40% entre 2008 et 2030. Ce phénomène s'accompagnera donc d'une augmentation de l'exploitation et de la consommation du gaz naturel dans le monde. La figure 1 donne l'évolution de la consommation mondiale de gaz naturel entre 1990 et 2011, ainsi qu'une prévision d'évolution de la consommation jusqu'en 2030 (BP Statistical Review 2011).

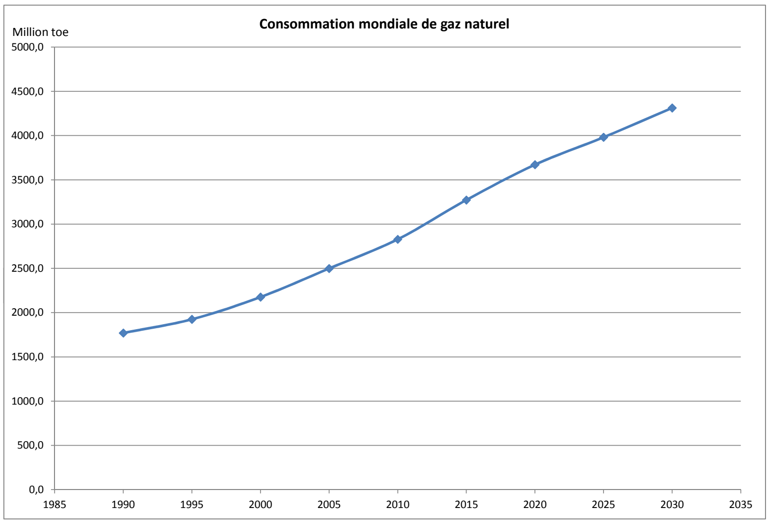

Figure 1 – *Evolution de la consommation mondiale de gaz naturel.*

3. Cet argument est valable même si l'on améliore l'efficacité énergétique au niveau mondiale, ce qui aurait pour effet de réduire la consommation par tête.

Cette courbe indique ainsi que le gaz naturel jouera un rôle de plus en plus important dans notre consommation, avec un taux de croissance estimé de 52% entre 2010 et 2030. Cependant, il serait plus judicieux de comparer les tendances d'évolution de l'utilisation des trois matières fossiles, afin de quantifier l'essor à venir du gaz naturel dans le mix énergétique mondial. La figure 2 montre les tendances de croissance des énergies fossiles entre 1990 et 2030 (BP Statistical Review 2011).

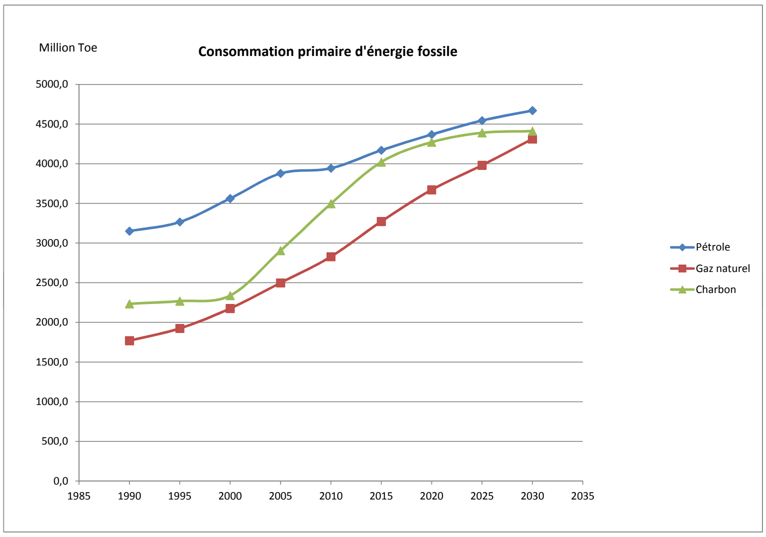

FIGURE 2 – *Evolution de la consommation primaire mondiale d'énergie fossile.*

Le tableau suivant fournit les taux d'évolution par an de la consommation des trois énergies :

Combustible	taux annuel de croissance de la consommation primaire (en %)
Pétrole	$0,85$
Gaz naturel	$2,13$
Charbon	$1,17$

On constate donc que les prévisions estiment que le gaz naturel aura le plus fort taux de croissance entre aujourd'hui et 2030 et sa consommation atteindra celle du charbon à partir de cette année. Par ailleurs, la catastrophe nucléaire qui a touché le Japon en mars 2011 et qui a conduit certains pays européens à annoncer l'abandon progressif de leur programme d'utilisation de l'énergie nucléaire devrait exacerber encore plus l'attractivité du gaz naturel. [4] Par conséquent, étant donné la forte croissance à venir de l'exploitation du gaz naturel dans le monde, son moindre

4. A cet effet, l'Allemagne a commencé à renégocier ses contrats d'importations de gaz russe.

impact sur l'environnement et la richesse et complexité de l'économie gazière, il nous est apparu intéressant d'étudier la problématique d'évolution des marchés gaziers dans les prochaines décennies.

§ 0.2 Economie et géopolitique du gaz naturel

0.2.1 L'essor des marchés gaziers

La libéralisation des marchés du gaz naturel en Europe a débuté en 1999, en application des directives européennes de 1996 et 1998. Initialement, cette ouverture ne concernait que les consommateurs professionnels (ou les gros consommateurs). La directive européenne de juin 2003 l'a étendue à l'ensemble des acteurs du marché, en incluant les consommateurs particuliers. Cette dernière directive est entrée en application le premier juillet 2007 et marque ainsi, la date symbolique de la libéralisation complète du marché européen du gaz naturel.

Depuis la libéralisation des marchés de l'énergie en Europe, les échanges de gaz naturel n'ont cessé de croître, favorisant ainsi l'emergence d'un marché européen connexe. L'apparition de nouveaux acteurs a permis un développement important des échanges de gaz naturel au niveau des principaux hubs européens. Aussi, la réduction des zones d'équilibrage a favorisé les échanges entre zones et la régulation de l'accès aux capacités de transport. A l'instar de ce qui s'est produit dans le monde, un marché européen du gaz naturel s'est développé depuis quelques années, entraînant une diversification des sources d'approvisionnement et des zones de consommation, ainsi qu'un développement d'une infrastructure de transport et de stockage.

Plus généralement, le facteur principal qui a conduit à la création et au développement des échanges mondiaux de gaz naturel est l'essor du Gaz Naturel Liquéfié (GNL). Ainsi, jusqu'au début des années 1990, l'économie gazière était dominée par des échanges entre zones de production/consommation dont le fonctionnement était fondé sur des relations bilatérales entre états. L'infrastructure de transport était principalement constituée de gazoducs (ou pipelines), tuyaux assurant le déplacement du gaz grâce à un différentiel de pressions. A titre d'exemple, la Russie exerçait (et continue d'exercer) sur l'Europe un pouvoir de marché conséquent, mû par une abondance des réserves russes et une proximité géographique. Les terminaux de liquéfaction et de regazification ont commencé à être exploités commercialement depuis 1964, le transport étant assuré par des bateaux que l'on nomme méthaniers. Les premiers échanges commerciaux ont été réalisés entre l'Algérie (le premier terminal de liquéfaction à des fins commerciales a été construit à Arzew, Algérie en 1964) et l'Europe (Grande-Bretagne). A une échelle plus large, les échanges de GNL sont restés, durant plusieurs décennies, confinés aux régions de consommation n'ayant pas d'approvisionnements alternatifs par gazoducs (Japon, Corée du Sud, etc.). Cela est dû au coût d'exploitation du GNL qui est relativement élevé par rapport à un transport par pipeline (surtout au niveau de l'investissement puisqu'il nécessite la construction de terminaux de liquéfaction et de regazification). Cependant, la déplétion des réserves dans les régions traditionnelles de production et l'apparition des problèmes de sécurité d'approvisionnement ont créé les conditions de l'emergence d'un marché mondial du gaz naturel, à partir des années 2000.

La figure 3 donne des prévisions de l'évolution des échanges gaziers dans le monde (IEA 2008). L'unité de volume est le Bcm (milliard de mètres cubes).

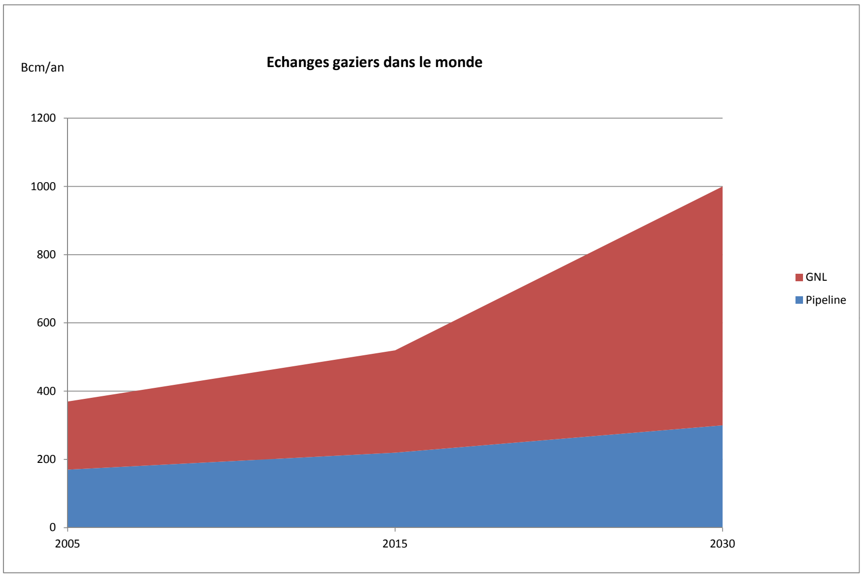

FIGURE 3 – *Evolution des échanges gaziers dans le monde.*

Ainsi, selon l'AIE, les échanges en gaz naturel vont croître de 110% entre 2010 et 2030. [5] Bien entendu, ce taux doit être comparé à celui relatif à l'évolution de la consommation sur la même période : 52%. Ce résultat suggère par conséquent que le marché mondial du gaz naturel va progressivement gagner en importance, mû par une demande croissante. Toutefois, puisque la ressource est épuisable, ce développement ne se fera pas sans dangers. Le plus important concerne la sécurité d'approvisionnement, surtout au sein d'une Europe fortement dépendante des approvisionnements étrangers à cause de la baisse de sa propre production. L'émergence de nouveaux producteurs, en particulier ceux exploitant le GNL et les gaz non-conventionnels, changera probablement la donne énergétique et les pouvoirs de marchés des producteurs usuels. Ainsi, l'exploitation récente du gaz de schiste aux Etats-Unis a modifié la tendance des prix de consommation et a rendu ce pays exportateur net de gaz naturel.

5. La croissance des échanges par GNL sera de 180% et celle des échanges par pipeline de 50%.

§ 0.3 Pouvoirs de marché et sécurité d'approvisionnement

Depuis l'exploitation du gaz naturel à des fins commerciales en Europe, son économie a souvent été caractérisée par l'exercice de pouvoirs de marché au niveau de l'approvisionnement. Cela est principalement dû à deux raisons : la concentration de la production et les monopoles historiques.

La répartition des réserves

Les réserves prouvées de gaz naturel sont réparties de manière très hétérogène dans le monde, comme le montre la figure 4 (BP Statistical Review 2011).

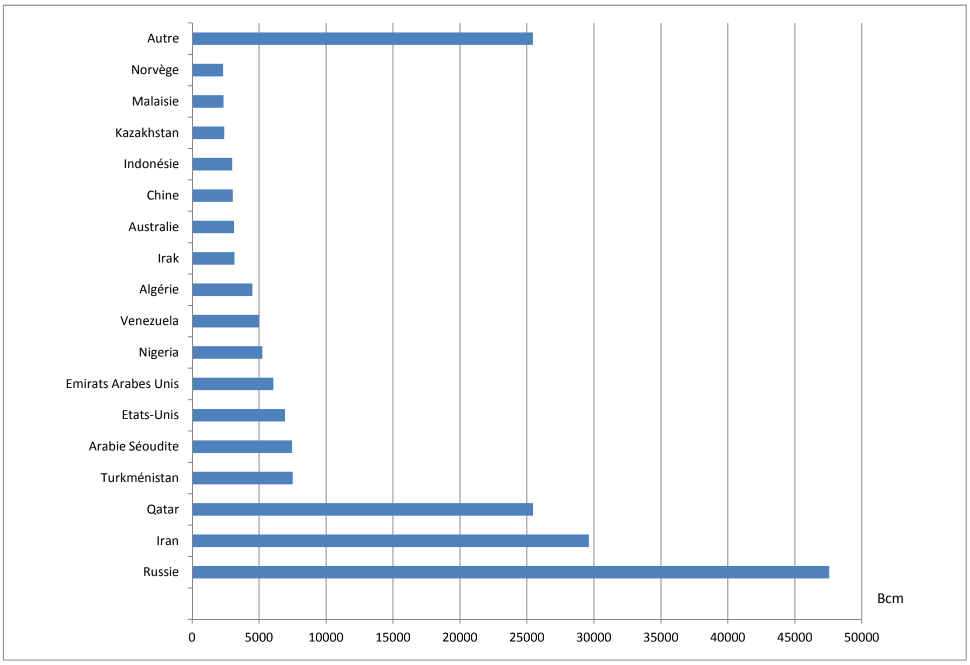

Figure 4 – *Répartition des réserves prouvées de gaz naturel dans le monde.*

La figure 4 montre que dix pays contiennent 75% des réserves mondiales de gaz naturel : la Russie, l'Iran, le Qatar, le Turkménistan, l'Arabie Séoudite, les Etats-Unis, les Emirats Arabes Unis, le Nigeria, le Venezuela et l'Algérie. En Europe, la situation est plus critique : les réserves atteignent seulement 2250 Bcm, soit seulement cinq fois la consommation de l'année 2010. Ainsi, l'amont de la chaîne gazière est caractérisé par le faible nombre de producteurs. Par conséquent, ces producteurs ont intérêt à exercer un pouvoir de marché. En économie, l'exercice d'un pouvoir de marché signifie que certains acteurs, ici les producteurs, connaissent *a priori* la fonction de réaction des consommateurs à leurs décisions. Plus simplement, un producteur possède un pouvoir de marché s'il connaît la fonction de demande des consommateurs et l'intègre dans son programme d'optimisation afin de gérer sa production (entre autres paramètres). Ainsi, un producteur aurait un certain intérêt à réduire sa production afin de faire monter les prix, ce qui serait dommageable pour le consommateur.

Plusieurs études économiques (le plus souvent fondées sur la modélisation) ont confirmé ce constat : il existe effectivement un exercice de pouvoir de marché en Europe, en amont de la chaîne gazière (Holz et al., 2008).

Fonctionnement des marchés gaziers et monopoles historiques européens

La gestion de l'approvisionnement en gaz naturel en Europe a longtemps été assurée, au niveau de plusieurs pays, par des compagnies publiques régulées. A l'instar des activités de GDF (actuellement GDF SUEZ) ou Total en France et Ruhrgas (actuellement E.ON Ruhrgas) en Allemagne, la production locale, l'approvisionnement auprès des producteurs, le transport ainsi que la distribution du gaz naturel ont longtemps été assurés par des sociétés publiques, souvent en situation monopolistique dans leur pays. Depuis la libéralisation des marchés de l'énergie (qui a commencé en Grande-Bretagne dans les années 1990), les pouvoirs publics ont voulu introduire plus de concurrence dans les marchés, en particulier ceux du gaz naturel, afin d'améliorer le bien-être du consommateur. On s'attendait ainsi à converger progressivement vers une situation de fluidité des échanges et de concurrence accrue en aval de la chaîne, niveau reliant les firmes locales aux consommateurs. Ainsi, depuis l'ouverture des marchés à la concurrence (directive de 2003), n'importe quel fournisseur de gaz naturel est potentiellement susceptible de pénétrer n'importe quel marché de consommation, sans barrières à l'entrée. Toutefois, ce genre de pratiques se sont peu développées en Europe pour des raisons qui tiennent au fonctionnement des marchés du gaz naturel.

Actuellement, l'approvisionnement européen en gaz se fait principalement hors d'Europe. Les plus grands fournisseurs de gaz naturel sont (par ordre décroissant de quantité fournie) : la Russie, la Norvège, l'Algérie, les Pays-Bas et la Grande-Bretagne. Le mode historique, antérieur à la libéralisation, de fourniture est le suivant : une firme (ou pays) productrice de gaz naturel contracte sur le long-terme avec la firme locale (que nous appelerons "trader") s'occupant d'assurer l'approvisionnement dans son pays de consommation. Les traders, tels que GDF SUEZ ou Total, assurent aussi la production locale en gaz naturel (qui est parfois négligeable par rapport aux importations). Les contrats long-terme fonctionnent de la manière suivante : le producteur et le trader contractent sur un volume, un prix de vente/achat et une durée de validité (généralement une dizaine d'années). Le volume doit être échangé dans la mesure où si le trader ne désire pas acquérir le gaz naturel, il devra tout de même le payer. Cette particularité du contrat se nomme le Take-Or-Pay (TOP). Elle permet au producteur de couvrir ses risques d'investissements en production et en infrastructure de transport en lui assurant une vente minimale pour son gaz. Parallèlement, le contrat long-terme (qui sera appelé LTC dans cette thèse, pour Long-

Term Contract), assure au trader un approvisionnement sûr, ce qui lui permettra de répondre à la demande des consommateurs. Bien entendu, ces contrats contiennent aussi des clauses de flexibilité qui permettent au trader de prendre une part légèrement inférieure du volume total contracté. De manière générale, nous dirons que les LTC permettent un partage des risques entre l'importateur (risque de volume) et le producteur (risque de prix). Le mode de fonctionnement des LTC précédemment décrit est le plus courant. Toutefois, Il existe d'autres types de contrats en amont de la chane, tels que les *depletion contracts*, les *interruptible contracts*, les *peak shaving contracts*, etc. Ces différentes sortes de contractualisations sont détaillées dans von Hirschhausen et al., 2008 et Hubbard et Weiner, 1986.

Aujourd'hui, malgré la libéralisation des marchés, on constate que l'amont de la chaîne fonctionne toujours selon la base de contrat long-terme. Ainsi, en 2010, la part des échanges LTC a atteint 70% des échanges totaux en Europe (International Gas Union, 2011). Cela est principalement dû à une volonté de couverture de risque, comme on l'a vu. Toutefois, les producteurs à même de créer des parts de marché au niveau aval peuvent directement vendre leur gaz aux consommateurs finaux, sans faire intervenir le trader local. Par exemple, on peut évoquer le cas de certains producteurs importants, tels que Gazprom qui ont contracté avec de grands électriciens européens, tels que E.ON. Ceci est une conséquence directe de la libéralisation. Cependant, les coûts d'investissement dans le transport et la distribution sont si importants qu'ils constituent, *a priori*, une barrière à l'entrée assez forte pour dissuader les nouveaux entrants. De ce fait, les différents marchés de consommation sont encore caractérisés aujourd'hui par une concentration de l'offre, qu'elle soit de la part des producteurs (au niveau amont) ou des traders. Cette situation est donc elle aussi favorable à l'exercice d'un pouvoir de marché : en effet, les monopoles historiques ont progressivement évolué en oligopoles.

(In)sécurité d'approvisionnement

La question de la sécurité d'approvisionnement en Europe est cruciale : elle se pose dès que l'on constate que le continent est à plus de 50% dépendant des importations étrangères, en gaz naturel. Une dépendance énergétique implique directement une fragilité géopolitique, que la Russie a exploité à deux reprises depuis l'année 2000. Deux crises politiques majeures ont opposé ces dernières années la Russie à l'Ukraine, et *a fortiori* l'Europe, portant sur le transit du gaz naturel. Elles ont abouti à une rupture d'approvisionnement laissant certains pays vulnérables face aux difficultés d'un hiver particulièrement froid. En effet, la majorité du gaz russe à destination de l'Europe passe par l'Ukraine qui bénéficie d'un tarif de transit. En effet, l'Ukraine impose un tarif de transit de l'ordre de 1,09$/cm/100km de pipeline, ce qui réduit la marge des producteurs de l'est. Or la Russie a à plusieurs reprises accusé l'Ukraine de récupérer, sans compensation, une partie de l'approvisionnement européen et d'exercer un fort tarif de transit. Face au refus des Ukrainiens de répondre aux exigences russes pour régler la question, la Russie a décidé d'interrompre, en janvier 2006 et janvier 2009, tous ses approvisionnements transitant par l'Ukraine, causant ainsi une forte pénurie de chauffage en Europe, pendant plusieurs semaines. En conséquence, certains pays d'Europe orientale tels que la Bulgarie, particulièrement dépendants des importations russes et subissant un hiver rigoureux, se sont retrouvés dans une situation critique et ont dû fournir en urgence du bois de chauffage aux particuliers. Outre la volonté politique de la Russie d'asseoir son hégémonie énergétique sur le continent, ses ambitions officielles étaient de faire réagir l'Europe afin qu'elle la soutienne dans le règlement du conflit avec l'Ukraine.

L'une des solutions proposées conjointement par la Russie et l'Allemagne au problème ukrainien consistait à financer la construction d'un gazoduc reliant directement les deux pays en traversant la mer Baltique : il s'agit du pipeline Nord Stream dont la capacité de transport est de 55 Bcm/an et qui sera opérationnel à partir de l'année 2015.

L'avantage de l'exploitation de ce pipeline est qu'il permet de contourner l'Ukraine lors du transit du gaz russe.

La principale autre mesure adoptée par l'Europe pour pallier cette insécurité d'approvisionnement consiste en une diversification des sources d'importation. Ainsi, pour atténuer sa dépendance par rapport au gaz algérien, le gouvernement espagnol a voté en octobre 1998 la "Loi Hydrocarbures" qui interdit aux traders de se fournir en gaz naturel à plus de 60% auprès d'un producteur. Ainsi, cette loi contraint les fournisseurs locaux à diversifier leurs origines d'importations. D'autres pays, tels que la France, ont également adopté cette stratégie de diversification. Sur une échelle plus large, plusieurs pays européens ont soutenu le développement de nouvelles routes d'approvisionnement, notamment venant de l'Asie, telles que le nouveau pipeline NABUCCO qui reliera directement les nouveaux producteurs de la mer Caspienne (comme l'Iran) à l'Europe et dont la capacité de transport sera de 30 Bcm/an.

La substitution énergétique et le prix du pétrole

Jusqu'à présent, notre discussion a principalement porté sur les volumes de gaz naturel échangés. Nous abordons à présent la question de la formation des prix. Une lecture rapide de l'évolution du prix spot du gaz naturel nous pousserait à affirmer que celui-ci est fixé par le prix du pétrole. La figure 5 montre l'évolution du prix du gaz naturel (prix NBP en \$/Mbtu) et celui du Brent (en $\times$ 10 \$/baril) (BP Statistical Review 2011).

Ainsi, il apparaît clairement que les deux prix sont fortement corrélés. Toutefois, il serait hâtif de conclure que le prix du pétrole définit celui du gaz car ceci ignorerait le fonctionnement complexe du marché du gaz naturel. Cette corrélation s'explique par deux phénomènes : l'établissement des contrats long-terme, au niveau de l'offre, et la substitution énergétique, au niveau de la demande, notamment pour la production électrique.

- Avant la libéralisation, la détermination du prix du gaz naturel pour les consommateurs finaux et au niveau des contrats long-terme se faisait grâce à la valeur Netback, pour certains marchés. [6] Pour chaque secteur de consommation, la valeur Netback se calcule par l'estimation du prix de la source d'énergie alternative au gaz naturel la moins chère. Les prix des contrats en amont de la chaîne sont calculés une fois les coûts de transport, de distribution

6. L'utilisation de la valeur Netback a été introduite pour l'exploitation du gisement de Groningue.

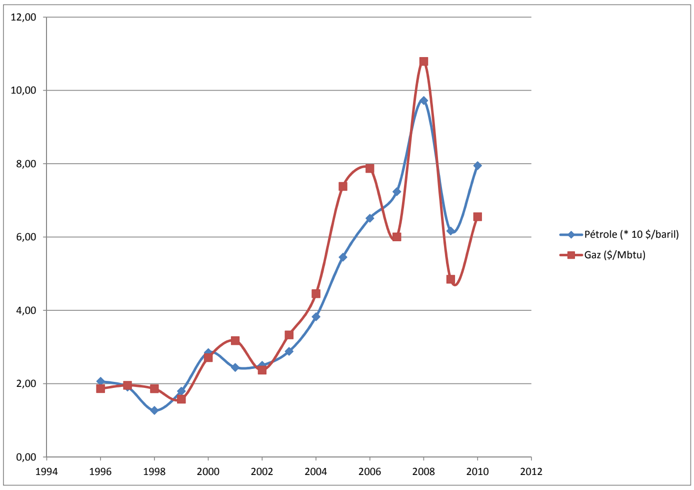

FIGURE 5 – *L'évolution des prix du gaz et du pétrole en Europe.*

et de stockage déduits.

Actuellement, la fixation du prix des contrats long-terme entre producteurs et traders est réalisée sur la base d'une formule d'indexation sur le prix du pétrole. En effet, afin de permettre au gaz naturel de rester compétitif par rapport aux énergies concurrentes, les producteurs et traders ont préféré lier leur prix de contrat au prix du pétrole. Cela se fait grâce à une formule de fixation du prix du contrat long-terme faisant intervenir le prix du pétrole. De manière schématique, on peut considérer que le prix du contrat long-terme représente un coût marginal d'approvisionnement pour les traders. Puisque ces derniers revendent le gaz naturel aux consommateurs finaux, il devient facile de comprendre la corrélation gaz/pétrole au niveau des prix : le prix spot du gaz naturel est, pour des raisons économiques, lié au coût marginal d'approvisionnement au niveau des traders locaux. Ce dernier est lui même lié au prix du contrat long-terme au niveau amont qui est indexé sur le prix des produits pétroliers. Bien entendu, l'utilisation de la valeur Netback dans la détermination du prix du gaz naturel permet également de rendre compte de la corrélation constatée.

Aujourd'hui, on commence à réfléchir à un autre mode d'indexation fondé sur le prix du charbon. Toutefois, on constate que les échanges de court-terme, sans contrats LTC, gagnent en importance. En outre, certaines analyses économiques, telles que celle de von Hirschhausen et al., 2008, tendent à montrer que la durée des contrats long-terme serait en diminution, pour les contrats récemment renouvelés. Ainsi, ces nouveaux contrats dureraient en moyenne quatre années de moins. Par conséquent, il existe une autre explication économique inhérente à la corrélation apparente.

– Certains secteurs de consommation sont caractérisés par une forte substitution énergétique. Cette dernière intervient lorsque les consommateurs ont la possibilité de varier, modulo certaines inerties d'usage, la source d'énergie qu'ils consomment, en fonction des prix de marché des différent combustibles auxquels ils ont accès. Le secteur industriel présente par exemple de fortes substitutions énergétiques. Si de telles substitutions existent, elles sont susceptibles de créer une concurrence entre les énergies, au sein même de la consommation. Par exemple, une énergie chère deviendrait ainsi relativement moins utilisée. Au contraire, une énergie peu coûteuse verrait sa consommation augmenter dans le temps. Ainsi, dans un état d'équilibre, les prix des énergies concurrentes sont corrélés. Du point de vue purement économique, cette explication invoque les élasticités croisées de court et de long-terme entre les prix des différentes énergies.

  Bien entendu, ces considérations sont à prendre avec précaution : en effet, certains secteurs de consommation finale, tels que les transports, présentent très peu de substitution possible. Ce secteur représente actuellement environ 20% (IEA, 2008) de la consommation totale en France, ce qui laisse 80% de la consommation susceptible de supporter une substitution plus ou moins forte. Aussi, l'estimation du coût de chaque énergie est complexe. Celui-ci doit faire intervenir par exemple le prix de marché, les coûts relatifs d'investissement en capital et le coût du CO_2 qui permet de prendre en compte l'impact des politiques environnementales. Cependant, la substitution énergétique permet également de comprendre la corrélation entre le prix du gaz et celui des autres énergies concurrentes. Une augmentation du prix du pétrole par exemple induirait une hausse de celui du gaz naturel (puisque la concurrence au niveau de la consommation ferait que le substitut deviendrait plus cher).

§ 0.4 Motivations de ce travail de recherche

Résumons ce que nous avons déjà développé :

– La consommation énergétique est en constante croissance et la plupart des prévisions indiquent que cette tendence va se poursuivre dans les décennies à venir : +52% entre 2010 et 2030.

– Les études et analyses économiques prévoient que le gaz naturel jouera un rôle de plus en plus important dans notre mix énergétique. Cela est dû à sa relative abondance (par rapport au pétrole) et son faible taux d'emission de gaz polluants.

– Les échanges de gaz naturel sont en constante augmentation. Les prévisions estiment que ces échanges croîtront de 110% entre 2010 et 2030. Cette situation est favorable à l'émergence d'un marché mondial du gaz naturel.

– La production de gaz naturel est très concentrée : les dix plus grands producteurs de gaz possèdent plus de 75% des réserves mondiales. ce qui facilite l'exercice d'un pouvoir de marché en amont.

- La gestion de l'approvisionnement, du transport et de la distribution au niveau des pays consommateurs a longtemps été effectuée par des firmes souvent en situation de monopoles régulés. Aujourd'hui, avec la libéralisation des marchés, on constate que l'on est passé à une situation oligopolistique.

- Les contrats long-terme entre producteurs et traders continuent de subsister dans un marché libéralisé. Actuellement, la part LTC dans les échanges en Europe est de 70%.

- La libéralisation des marchés a eu pour effet de faire évoluer les monopoles historiques en oligopoles. Cela est dû aux barrières d'entrée indirectement causées par les coûts de déploiement. Ainsi, les marchés européens du gaz naturel sont dominés par les oligopoles, ce qui est susceptibles d'induire des exercices de pouvoir de marché.

- La libéralisation des marchés a aussi complexifié la structure économique des marchés en Europe. Ainsi, outre le phénomène structurel à double niveau représenté par la chaîne producteur-trader-consommateur, il est possible aux producteurs de contourner leurs contrats long-terme et de vendre leur gaz directement aux marchés spot où ils se retrouvent en concurrence avec les traders.

- L'Europe est particulièrement vulnérable face aux risques d'interruption d'approvisionnements étrangers. La question de la sécurité d'approvisionnement est donc au cœur des problématiques de géoplitique gazière.

- La corrélation entre prix du gaz et prix du pétrole est un aspect important du marché gazier. La substitution énergétique ainsi que les contrats long-terme sont donc des caractéristiques importantes qu'il faut prendre en compte lorsqu'on tente d'analyser les marchés du gaz naturel.

Par conséquent, notre travail de recherche s'efforce de répondre aux questions suivantes :

1. Comment les problématiques de sécurité d'approvisionnement conditionnent-elles le comportement des traders dont le rôle est de choisir scrupuleusement les origines de leurs importations ? Notre analyse doit prendre en compte la structure économique actuelle des marchés : double pouvoir de marché, oligopoles et contrats long-terme par exemple. Plus particulièrement, nous nous sommes intéressés aux impacts que peut avoir un risque de rupture d'approvisionnement sur les caractéristiques du marché gazier en Europe : prix, consommations et bien-être social notamment. Ainsi, nous nous sommes interrogés sur la nécessité (ou non) de réguler le marché, en fonction du risque de rupture, à des fins d'amélioration du bien-être. Concernant les régulations existantes, notamment celle de l'Espagne, nous avons tenté de comprendre si elles assuraient l'optimalité du bien-être social et dans

le cas contraire, nous avons cherché d'autres régulations à même de contrebalancer le risque de rupture plus efficacement.

2. Dans un contexte de libéralisation, de suppression des clauses de destination et d'augmentation de la concurrence, quelles sont les évolutions possibles du marché du gaz naturel en Europe ? Comment les producteurs vont-ils faire face à l'épuisement de la ressource dans les décennies à venir ? Comment vont donc évoluer les stratégies de production et comment celles-ci vont-elles influencer les comportements de consommation et les prix ? Plus simplement, nous nous sommes intéressés à la réalisation et à l'étude de scénarii d'évolution des marchés du gaz naturel en Europe, afin d'en déduire une trajectoire temporelle de prix, de consommations et de productions entre autres.

3. Comment risque d'évoluer la dépendance énergétique de l'Europe dans le futur et comment va-t-elle conditionner la stratégie d'approvisionnement ? Dans le cas d'une augmentation de la dépendance, comment cette dernière va-t-elle influencer les prix sur le marché et la consommation ?

4. Comment peut-on prendre en compte la substitution énergétique dans une représentation formelle des marchés du gaz naturel ? Plus spécifiquement, puisque la substitution énergétique est due à une strategie de consommation, nous avons voulu la prendre en compte au sein de la fonction de demande en gaz naturel et l'inclure dans un cadre plus général de modélisation des marchés. Ainsi, nous avons cherché à rendre compte de la corrélation entre prix du gaz/prix des énergies concurrentes en analysant et modélisant directement le fonctionnement du marché tout en incluant la substitution dans la fonction de demande.

5. Les caractéristiques des contrats long-terme sont en général difficiles à obtenir via des données publiques, en particulier les prix, car ces contrats sont le fruit d'une négociation directe entre producteurs et traders dont les modalités sont souvent tenues secretes. Ainsi, nous avons voulu savoir si une approche s'appuyant sur la modélisation (en rendant les contrats endogènes au modèle) pouvait conduire à une estimation des volumes et prix des contrats long-terme entre chaque paire de producteur/trader. Plus généralement, nous avons voulu comprendre comment ces contrats long-terme pouvaient évoluer dans un marché de plus en plus libéralisé et dans quelle mesure ils permettaient de rentabiliser les différents investissements en infrastructure de production et de transport.

6. Comment la double application du pouvoir de marché, de la part des producteurs et des traders, au niveau aval influence-t-elle le marché ? Il est évident que les producteurs possèdent un degré de liberté en plus, concernant la destination de leur gaz, par rapport aux traders, puisqu'ils ont la possibilité de contracter sur le long-terme, mais également de cibler directement le marché spot (des échanges court-terme). En quoi cette dissymétrie de pouvoir de marché a-t-elle un impact sur l'économie gazière ?

7. Quelle est la relation entre les prix des contrats long-terme et les prix des marchés spots ?

8. Puisque le prix et la consommation du gaz naturel sont corrélés au prix du pétrole (entre autres), comment la fluctuation temporelle du prix du pétrole influence-t-elle les caractéristiques du marché, telles que les contrats long-terme, la consommation et les prix ?

Nous avons tenté de traiter ces problématiques à l'aide d'analyses économiques s'appuyant sur la modélisation afin de pouvoir quantifier les phénomènes étudiés et prévoir leur évolution dans le temps. Puisque les différents acteurs de la chaîne sont souvent confrontés à des contraintes de gestion des quantités (capacités de production, de transport et de stockage, contraintes d'offre et de demande), nous avons décidé de représenter la concurrence sur les marchés comme une concurrence de type Cournot.

Les principaux outils mathématiques que nous avons exploités sont les suivants : la théorie de l'optimisation, la théorie des jeux non-coopératifs, les problèmes de Nash-Cournot généralisés et les problèmes de complémentarité (Mixed Complementarity Problems ou MCP). En effet, nous avons modélisé chaque protagoniste de la chaîne gazière comme un acteur rationnel dont le but est d'optimiser son utilité (s'il est stratégique) ou le bien-être social (s'il est régulé ou compétitif), d'où le recours à l'optimisation. Par ailleurs, les différentes interactions stratégiques (ou les jeux d'acteurs) sont prises en compte grâce à une concurrence à la Cournot (les joueurs décident des quantités et le prix en résulte). C'est à ce stade qu'intervient la théorie des jeux non-coopératifs, puisque chaque acteur stratégique (exerçant un pouvoir de marché) tente d'optimiser son utilité en essayant d'anticiper la réaction des concurrents à ses décisions. Ainsi, la formulation mathématique de notre modèle gazier aboutit à un problème de complémentarité. Enfin, la prise en compte de manière endogène des contrats long-terme nous a poussés à étudier les problèmes de type Nash-Cournot généralisés.

§ 0.5 Etat de l'art et apports de la thèse

La question de la sécurité d'approvisionnement en Europe a largement été traitée en économie de l'énergie. La plupart des travaux analysant cette problématique peuvent être divisés en deux catégories : la première, qui est la plus abondante regroupe les études reposant sur des analyses purement géopolitiques : Percebois, 2006, Lefèvre, 2010 et Kruyt et al., 2010. La deuxième catégorie englobe les travaux s'appuyant sur une analyse microéconomique afin d'étudier et mesurer la sécurité d'approvisionnement. Ce genre d'analyses appliquées au cas du gaz naturel en Europe ne sont pas très nombreuses, les articles les plus importants étant Manne et al., 1986, Hoel et al., 1987 et Markandya et al., 2010. Par ailleurs, la plupart de ces contributions se fondent sur une description des marchés du gaz naturel en Europe qui ne prend pas en compte son évolution récente. En effet, avant la libéralisation, les marchés gaziers européens étaient régulés pas les gouvernements. Dans beaucoup de pays, la gestion de l'approvisionnement, du transport et de la distribution du gaz étaient assurée par des firmes publiques, ou des monopoles régulés (tel qu'en France). Ainsi, les modèles proposés par Manne et al., 1986 ou Hoel et al., 1987 étaient particulièrement adaptés à cette situation. A titre d'exemple, ces deux modèles supposent que les firmes ont pour objectif d'optimiser le bien-être social sous des contraintes d'insécurité des importations.

Notre travail s'efforce de prendre en compte la situation actuelle des marchés gaziers. Ainsi, nous avons voulu prendre en considération la libéralisation des marchés qui a conduit à une situation de concurrence imparfaite (tel qu'on l'a expliqué plus haut) et la contractualisation de long-terme entre producteurs et traders. Par conséquent, nous avons développé un modèle statique de concurrence oligopolistique où chaque acteur (trader) cherche à optimiser son profit en gérant les origines de ses importations et en sachant qu'il existe des producteurs en amont capables d'interrompre leurs exportations à tout moment, pour des raisons qui échappent à son contrôle (terrorisme, crises politiques, problèmes techniques, etc.). Dans le modèle proposé, nous tentons d'estimer la perte de bien-être des consommateurs, en cas de crise (interruption), grâce à une distinction entre fonctions de demande de court et de long-terme. L'objectif de notre étude est de comprendre et mesurer l'impact de l'insécurité d'approvisionnement sur les paramètres du marché : prix, consommation, production ainsi que le bien-être etc. Nous analysons les stratégies de couverture de risque de la part des traders et nous cherchons des régulations efficaces des marchés dans le cas où le risque d'insécurité est élevé.

La littérature contient pléthore de modèles mathématiques des marchés du gaz naturel ayant des objectifs divers. Le tableau suivant fournit quelques caractéristiques de certains modèles existants. Nous y avons ajouté le modèle GaMMES que nous avons développé dans le cadre de la thèse afin de pouvoir le comparer aux autres :

Table 2 – *Description de quelques modèles gaziers*

Nom	Institut	Pouvoir de marché	Echelle	LTC	Substitution
MAGELAN	Université de Cologne	non	2050, Monde	non	non
World Trade Gas Model	Baker institute	non	2040, Monde	non	non
NATGAS	CPB Netherlands Bureau of Economic Policy Analysis	oui	2030, Europe	non	non
GASTALE	ECN	oui	2030, Europe	non	non
GASMOD	DIW Berlin	oui	Statique, Europe	non	non
World Gas Model	Université du Maryland	oui	2030, Monde	oui (exogènes)	non
GaMMES	EDF, IFP energies nouvelles, Université de Nanterre-La Défense	oui	2035, Europe	oui (endogènes)	oui

La liste des modèles que nous avons indiquée n'est pas exhaustive. Toutefois, elle permet de mettre en exergue les principales propriétés de la modélisation des marchés gaziers en général et de comparer notre modèle aux précédents.

Ces modèles peuvent être séparés en deux catégories : ceux qui supposent qu'il n'y a pas d'exercice de pouvoirs de marchés [7] et ceux qui justement les prennent en compte. Une situation de non exercice de pouvoir de marché s'apparente à une concurrence pure et parfaite, configuration de marché où les acteurs sont *price-takers*. Cette hypothèse est valide s'il existe un fort degré d'atomicité au sein des acteurs qui les empêcherait d'influencer volontairement le prix du bien, indépendamment des autres protagonistes du marché. Cette hypothèse est relativement valide si l'on s'intéresse au marché américain du gaz naturel, par exemple. Toutefois, elle s'affaiblit lorsqu'on modélise le marché européen qui est dominé par des oligopoles, comme on l'a constaté.

D'autres modèles ont été développés en concurrence imparfaite (les acteurs sont donc *price-makers*) : ils permettent d'appréhender la structure économique actuelle en Europe. En outre, la théorie économique nous enseigne que la concurrence pure et parfaite est un cas particulier de l'oligopole. Pour passer d'une situation à l'autre, il suffit de rendre le nombre d'acteurs stratégiques très grand.

Concernant les contrats long-terme, la plupart des descriptions de marché les négligent ou les prennent en compte de manière exogène. Cependant, les problématiques que nous avons développées précédemment nous ont menés à tenter de rendre ces contrats endogènes (afin de pouvoir les comparer, entre autres, aux différents prix spot). Par conséquent, notre travail fera en sorte que les paramètres de ces contrats seront fournis en sortie de GaMMES.

Quant à la substitution énergétique, on constate qu'elle n'est pas encore prise en compte actuellement dans la modélisation des marchés, malgré l'importance qu'elle revêt pour les consommateurs en particulier. Plus généralement, cette substitution peut contrebalancer le pouvoir de marché des acteurs stratégiques car elle les forcerait par exemple à ne pas augmenter considérablement le prix du gaz naturel, si le prix du substitut est faible. [8] Par conséquent, la prise en compte de la substitution énergétique peut fortement changer l'économie du gaz naturel.

Au regard de ce que nous avons présenté, il nous est apparu que les innovations de notre travail sont les suivantes :

- La question de la sécurité d'approvisionnement en Europe est traitée dans un cadre assez large et général, pouvant s'appliquer à plusieurs pays vulnérables, où nous prenons en compte la structure économique actuelle des marchés. Ainsi, nous avons pu comparer notre étude à celles que l'on trouve dans la littérature. Nous nous sommes aidés des outils de modélisation pour quantifier l'impact de l'insécurité d'approvisionnement sur les marchés et

7. Cette classe de modèles comporte les études technico-économiques dont le but est de déterminer les différents coûts marginaux d'approvisionnement en gaz.

8. De manière similaire, la substitution énergétique peut exacerber le pouvoir de marché si le substitut est cher.

les moyens stratégiques de la limiter. Ainsi, nous avons trouvé des conditions favorables à la régulation du marché dans certains cas où les consommateurs deviennent particulièrement vulnérables.

- Nous avons développé une approche exploitant des techniques de systèmes dynamiques afin de prendre en compte la substitution énergétique dans la fonction de demande en gaz naturel. Notre approche permet notamment d'appréhender des effets de concurrence entre énergies dans la demande ainsi que des effets d'inertie de consommation liés aux investissements. En particulier, nous avons réussi à introduire, au sein même de la fonction de demande en gaz naturel, le prix du substitut (charbon ou pétrole).

- Nous avons utilisé cette fonction de demande dans un modèle dynamique d'équilibre partiel de description des marchés du gaz naturel en Europe. Notre modèle, nommé GaMMES pour Gas Markets Modeling with Energy Substitution, se veut réaliste dans la mesure où la structure économique qu'il prend en compte est assez complexe et englobe les principaux déterminants des marchés. La plupart des acteurs de la chaîne gazière sont représentés : producteurs, traders locaux intermédiaires, consommateurs et opérateurs de transport et de stockage. Les infrastructures de transport et de stockage sont modélisées ainsi que la chaîne GNL. La production est répartie selon les différents champs d'exploitation et les investissements (augmentation de capacités) en production, transport et stockage sont pris en considération de manière endogène par le modèle. La substitution énergétique est déduite de notre approche de type systèmes dynamique. Les contrats long-terme liant producteurs et traders sont modélisés de manière endogène également et le pouvoir de marché est donné aux producteurs et traders indépendants. L'interaction stratégique est modélisée de manière fine. En effet, les producteurs et traders se font concurrence à la Cournot et la dissymétrie d'exercice du pouvoir de marché entre ces protagonistes est représentée. A notre connaissance, GaMMES est le premier modèle gazier à prendre en compte une structure économique complexe en rendant endogènes les LTC et en considérant la substitution énergétique.

- La prise en compte de manière endogène de la contractualisation long-terme nous a conduits à écrire notre modèle sous forme de problème de Nash-Cournot généralisé. Cette catégorie de problèmes est particulièrement riche car elle englobe beaucoup de situations rencontrées en économie industrielle (notamment les problématiques de concurrence imparfaite). Elle a été introduite récemment en économie et à notre connaissance, aucun modèle gazier ne l'a encore utilisée. Cette problématique nous a poussés à étudier la théorie fondamentale des problèmes de Nash-Cournot généralisés et notamment à introduire les Inéquations et Quasi-Inéquations variationnelles (VI et QVI). Nous avons en particulier trouvé un moyen simple de résoudre ces problèmes, qui passe par la recherche d'une caractérisation de la solution de type VI, en faisant intervenir des relations entre différentes variables duales. A cet égard, cette thèse possède un apport sur le plan théorique.

- Puisque la substitution énergétique a été prise en compte dans notre modélisation, nous avons choisi de l'exploiter pour comprendre l'influence des fluctuations du prix du pétrole sur les marchés gaziers. Pour ce faire, nous avons développé une extension stochastique du modèle GaMMES, où la fluctuation des prix du pétrole est mise en avant. Par conséquent, nous avons modélisé le prix du gaz naturel comme une variable aléatoire, dont la spécification de la loi a été réalisée grâce à une étude économétrique. Ensuite, nous avons construit l'arbre des scénarii possibles d'évolution du prix du pétrole et développé ainsi le modèle stochastique. A notre connaissance, l'impact de la fluctuation des prix du pétrole sur l'évolution des marchés gaziers est une nouveauté dans le domaine. Ainsi, il nous a été possible de comparer les évolutions déterministes et stochastiques des marchés, notamment pour les contrats long-terme, et à quantifier, via des outils mathématiques, l'importance de prendre en compte l'aléa du prix du pétrole dans les décisions des différents acteurs de la chaîne. Notre travail a mené à l'élaboration de S-GaMMES, un modèle dynamique et stochastique de Nash-Cournot généralisé pour les marchés gaziers.

Notre étude s'est principalement concentrée sur le périmètre européen et sur l'horizon de temps 2000-2035. Toutefois, pour des questions de temps de calcul, nous avons préféré restreindre la description de la consommation au marché de l'Europe du nord-ouest, ce qui correspond à plus de 80% des échanges européens de gaz.

§ 0.6 Structure de la thèse

Cette thèse est divisée de la manière suivante :

La première partie présente les outils mathématiques de base que nous avons utilisés dans notre travail de recherche. Cette partie contient le chapitre 1 et développe les bases de la théorie de l'optimisation (Lagrangien, conditions de Karush Kuhn et Tucker, dualité etc.), la théorie des jeux non coopératifs (concurrence imparfaite et équilibres de Nash) et les problèmes de complémentarité (MCP).

La deuxième partie s'intéresse à la sécurité d'approvisionnement en Europe et comprend les chapitres 2 et 3. Le chapitre 2 s'efforce de fournir un cadre théorique à même de décrire le comportement réaliste d'un oligopole lorsqu'il est soumis à un risque de rupture d'approvisionnements de la part de certains producteurs. Le chapitre 3 applique le modèle à trois pays européens. Le premier cas tente d'expliquer les choix d'approvisionnement allemands dans les années 1980 et sert de point de comparaison entre notre modèle et ceux développés précédemment. Le deuxième cas s'inspire de la situation actuelle d'un pays fortement dépendant du gaz russe, à savoir la Bulgarie. Une fois la sensibilité des paramètres de marché par rapport au risque de rupture étudiée, nous avons cherché à proposer des moyens de réguler le marché à des fins d'optimisation de bien-être social et des conditions de leur application, en fonction de la probabilité de rupture. Le troisième cas tente de comprendre la régulation actuellement appliquée en Espagne afin de forcer les traders à diversifier leurs sources d'importations. Une régulation alternative est proposée dans le but de la comparer à la première et de savoir dans quelles circonstances (notamment en fonction du risque de rupture) le régulateur doit choisir l'une ou l'autre dans le but de maximiser l'utilité sociale.

Cette partie s'efforce de répondre à la question 1. Elle s'appuie sur l'article suivant : I. Abada, O. Massol, 2011. *Security of supply and retail competition in the European gas market. Some model-based insights.*, Energy Policy 39 (2011), 4077-4088.

La troisième partie concerne la substitution énergétique et contient les chapitres 4 et 5. Le chapitre 4 montre comment nous avons exploité une approche de type systèmes dynamiques pour modéliser le comportement stratégique des consommateurs dont l'objectif est de choisir leur mix optimal en fonction des prix de marché et de la demande fossile primaire. La calibration du modèle y est discutée et les résultats sont fournis pour huit pays consommateurs. Le chapitre 5 exploite notre approche pour définir, calculer et spécifier une fonction de demande de gaz naturel dynamique qui englobe la substitution énergétique. On y montre en particulier comment le prix du pétrole est intégré au sein de la fonction de demande et comment l'inter-temporalité de la consommation est prise en compte.
Cette partie tente de répondre à la question 4. Elle s'appuie sur l'article suivant : I. Abada, V. Briat & O. Massol, 2011. *Construction of a fuel demand function portraying interfuel substitution, a system dynamics approach.*, Economix Working Paper, available at http ://economix.fr/fr/dt/2011.php, (article en cours de révision dans Energy Policy).

La quatrième partie développe le modèle déterministe GaMMES afin de décrire l'évolution des marchés du gaz naturel. Elle comprend les chapitres 6, 7 et 8. Le chapitre 6 fournit la description économique des marchés et utilise les résultats de la troisième partie afin de décrire la demande. Les programmes d'optimisation de tous les acteurs sont détaillés et expliqués. Aussi, ce chapitre montre la manière dont nous avons rendu les contrats long-terme endogènes au modèle et choisi une formalisation de type Nash-Cournot généralisé. Le chapitre 7 applique le modèle au marché gazier de l'Europe du nord-ouest. Des prévisions de consommation, d'évolution de prix, de production et de dépendance énergétique sont fournies pour l'horizon 2035. Les prix et volumes des contrats long-terme y sont donnés, analysés et comparés aux prix spot dans les différents pays. Afin d'exploiter la substitution énergétique offerte par notre modélisation, nous avons étudié l'évolution du prix du gaz naturel par rapport au prix du pétrole lors d'une année de référence donnée. Le chapitre 8 est purement théorique. Il définit et analyse les problèmes de Nash-Cournot généralisés et les compare aux problèmes de Nash-Cournot standards. Une caractérisation mathématique des équilibres est donnée dans les deux cas ainsi qu'une discussion sur l'unicité de la solution d'équilibre. Finalement, un exemple simple d'un jeu à deux acteurs est présenté dans le but de mettre en pratique les résultats théoriques préalablement établis.
Cette partie tente de répondre aux questions 2, 3, 5, 6 et 7. Elle s'appuie sur l'article suivant : I. Abada, V. Briat, S. A. Gabriel & O. Massol, 2011. *A Generalized Nash-Cournot Model for the North-Western European Natural Gas Markets with a Fuel Substitution Demand Function : The GaMMES Model.*, Economix Working Paper, available at http ://economix.fr/fr/dt/2011.php, (article est en cours de révision dans Networks and Spatial Economics).

La cinquième et dernière partie introduit un aspect stochastique à GaMMES. Elle contient les chapitres 9 et 10. Le chapitre 9 montre comment le prix du pétrole est rendu aléatoire sur la base d'une étude économétrique que nous avons développée. Ensuite, l'arbre des scénarii est élaboré et

le modèle développé dans sa forme extensive. Ici aussi, les différents programmes d'optimisation sont détaillés et discutés. Le reste du chapitre se concentre sur des résultats théoriques démontrant des relations générales entre les prix des contrats long-terme et les prix spot, dans un contexte d'incertitude sur le prix du pétrole. Le chapitre 10 applique le modèle à l'Europe. Des prévisions d'évolution du marché sont données pour les différents scénarii du modèle. Les paramètres des LTC sont fournis et comparés aux résultats déterministes. Cela permet de comprendre comment les producteurs et traders couvrent leurs différents risques dans un environnement aléatoire. Enfin, une mesure de l'importance de prendre en compte le caractère aléatoire de la demande dans les programmes d'optimisation des différents joueurs est founie et analysée.
Cette partie tente de répondre à la question 8. Elle s'appuie sur les articles suivants : I. Abada, 2011. *A stochastic generalized Nash-Cournot model for the northwestern European natural gas markets with a fuel substitution demand function : The S-GaMMES model.* et I. Abada, 2011. *Study of the evolution of the the northwestern European natural gas markets using S-GaMMES.*, Chaire Economie et Climat Working Papers.

La conclusion revient sur les objectifs de la thèse et les moyens employés pour les atteindre. On y trouvera un développement des points faibles de nos modèles ainsi que les extensions possibles de notre travail.

Bibliographie

[1] BP Statistical Review of World Energy, 2010. www.bp.com.

[2] BP Statistical Review of World Energy, 2011. www.bp.com.

[3] R. Egging, S. A. Gabriel, F. Holtz, J. Zhuang, 2008a. *A complementarity model for the European natural gas market.* Energy Policy 36 (7), 2385-2414.

[4] R. Egging, F. Holz, S. A. Gabriel 2010. *The World Gas Model : A Multi-Period Mixed Complementarity Model for the Global Natural Gas Market.* Energy. 35 (10), pp. 4016-4029.

[5] C. von Hirschhausen, A. Neumann, 2008. *Long-Term Contracts and Asset Specificity Revisited : An Empirical Analysis of Producer-Importer Relations in the Natural Gas Industry.* Review of Industrial Organization, Vol. 32, No. 2, pp. 131-143.

[6] M. Hoel, S. Strm, 1987. *Supply security and Import diversification of Natural Gas.* in R. Golombek, J. Vislie and M. Hoel (Eds.) : Natural gas markets and contracts, North-Holland.

[7] F. Holz, 2009. *Modeling the European Natural Gas Market - Static and Dynamic Perspectives of an Oligopolistic Market.* Ph.D. Dissertation, Technische Universitat-Berlin.

[8] F. Holz, C. von Hirschhausen, C. Kemfert, 2008. *A Strategic Model of European Gas Supply (GASMOD).* Energy Economics 30, 766-788.

[9] IEA, 2008. *Natural Gas Information.*

[10] International Gas Union, Gastech Conference Amsterdam March 2011. *New Trends in Gas Price Formation.*

[11] B. Kruyt, D. P van Vuuren, H. J. M. de Vries, H. Groenenberg, 2010. *Indicators for energy security.* Energy Policy, Vol. 38, No. 4.

[12] N. Lefèvre, 2010. *Measuring the energy security implications of fossil fuel resource concentration.* Energy Policy, Vol. 38, No. 4 (April), pp. 1635-1644.

[13] W. Lise & B. F. Hobbs, 2008. *Future evolution of the liberalised European gas market. Simulation results with the dynamic GASTALE model.* Energy Policy 36 (6), 1890-1906.

[14] A. S. Manne, K. Roland, G. Stephan, 1986. *Security of supply in the Western European market for natural gas*. Energy Policy, Vol. 14, No. 1 (February), Pages 52-64.

[15] A. Markandya, M. Pemberton, 2010. *Energy security, energy modeling and uncertainty.* Energy Policy, Vol. 38, No. 4 (April), pp. 1609-1613.

[16] J. Percebois, 2006. *Dépendance et vulnérabilite : Deux façons connexes mais différentes d'aborder les risques énergétiques*. Cahier N 06.03.64, Centre de Recherche en Economie et Droit de l'énergie (CREDEN), Université de Montpellier, France.

[17] World Trade Gas Model, Rice, 2004.
www.rice.edu/energy/publications/docs/GSP_ WorldGasTradeModel_ Part1_ 05_ 26_ 04.pdf.

Deuxième partie

Mathematical tools

- Chapitre 1 -

Technical background.

§ 1.1 Introduction

This chapter presents the fundamental mathematical and economic tools that will be used in this thesis. The first part is dedicated to optimization theory, the second concerns non-cooperative game theory and the last part is about complementarity problems and market structures modeling.

§ 1.2 Optimization

1.2.1 Introduction

Optimization is a very old subject that has undergone a radical change and has gained in interest since the use of computers. Optimization concerns a lot of different fields such as economics, robotics, logistics, signal processing, optimal control, etc. One typical example is the optimization of a function over a space of finite dimension, let's say $\mathbb{R}^n$. This situation is the most frequent in operations research (where linearity is, in most cases, a required feature). Another possible situation is when the function to optimize depends on another function, such as the solution of a standard differential equation. This is typically the case in robotics and automatics where the aim is to look for the "optimal command." Basically, this situation corresponds to an optimization of a function definded on a space of infinte dimension, such as $C^\infty(\mathbb{R}^n)$, the set of functions that are infinitely differentiable over $\mathbb{R}^n$. The last field we will evoke is the optimal control : the function to optimize is defined on an infinite dimension space too, but the "variable" can be the solution of a partial differential equation. This is interesting in optimal control theory and has some applications in the optimal conception of structures in mechanics, for example. Other situations where the use of optimization is important are provided in Section 1.2.2.

More generaly, optimization problems can be divided into two types : the *continuous variables optimization* and the *integer variables optimization*. The first one has a continuous definition domain, most often $\mathbb{R}^n$. The second one has a integer definition domain, generaly $\mathbb{Z}^n$. Integer optimization problems may seem easier to solve because, intuitively, their domain is "smaller" than the continuous optimization ones. However, this is false because looking for the continuous solutions of optimization problems can exploit the objective function derivative properties. As we will see later, the main optimality conditions of the optimization theory concern the continuous variables optimization because of the use of the differential calculus theory.

1.2.2 Examples

This section presents some examples of the application of the branches of optimization.

1.2.2.1 The transport problem

This is the typical example of operation research and logistics. The idea is to optimize the delivery of a good. A firm owns a set of storehouses indexed by $i \in I$ and clients indexed by $j \in J$. Each storehouse has a stock of the good s_i and each client wants a quantity d_j. We suppose that the supply is greater than demand, so that the problem is feasible. The transportation unit cost from i to j's location is c_{ij}. The objective function to minimize is the total transportation cost while cntrolling the amount that goes from i to j, x_{ij}. The constraints concern the supply at each storehouse and the demand of each client. The problem can be written as follows :

$$\begin{array}{lll} \text{Inf} & \sum_{ij} c_{ij} x_{ij} & \\ \text{s.t.} & \forall i \in I \ \sum_j x_{ij} & \leq s_i \\ & \forall j \in J \ \sum_j x_{ij} & = d_j \\ & \forall (i,j) \in I \times J \ x_{ij} & \geq 0 \end{array}$$

This continuous optimization problem is a particular case of linear programming (LP) because both the objective function and the constraints are linear functions of the variables x_{ij}. The variables belong to a finite-dimension space.

1.2.2.2 The monopoly problem

This is a typical example derived from the industrial economics theory. Let us consider a firm that produces a good needed in a demand market. The firm has to decide the optimal quantity to produce x in order to optimize profit. The inverse demand function that characterizes the consumers' behavior in terms of consumption, regarding the good's price, is f. If amount x is brought to the market, the price will be set to $p = f(x)$. We will assume the existence of a technical production capacity constraint K. The production cost function is c. The firm's optimization progam can be written as follows :

$$\begin{array}{lll} \text{Sup} & \Pi(x) = & \\ & f(x)x - c(x) & \\ \text{s.t.} & x & \leq K \\ & x & \geq 0 \end{array} \tag{1.2}$$

Obviously, solving the simple monopoly's problem we have presented is straightforward. If we assume that the profit Π is concave, we know that we have a unique solution (see Section 1.2.4). We call m the optimum variable (if it exists) that maximizes Π over $\mathbb{R}$ and x^* the solution of (1.2). Figure 1.1 graphically solves the problem. It shows that (assuming that $\Pi'(0) > 0$) :

If $m \geq K, \quad x^* = K$
If $m \leq K, \quad x^* = m$

This continuous optimization problem is a particular case of non-linear programming (NLP) because the objective function is non-linear. If the inverse demand function decreases linearly, the monopoly problem is a particular case of quadratic programming, because function Π is quadratic. The variable x is such that $x \in \mathbb{R}$, a finite-dimension space.

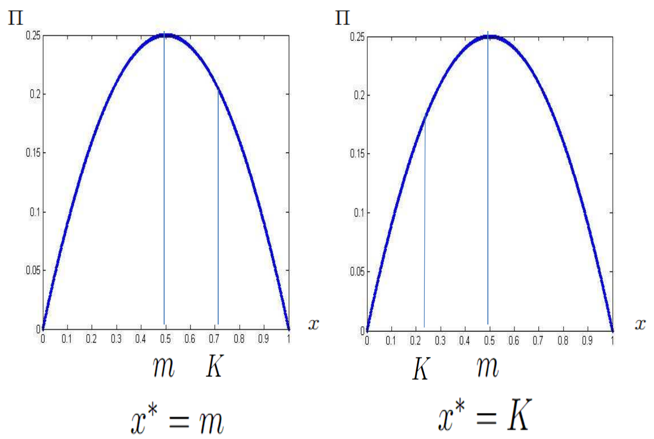

FIGURE 1.1 – *A graphical solution of the monopoly problem.*

1.2.2.3 The membrane vibration problem

We consider the vibration of an elastic membrane $\Omega \subset \mathbb{R}^n$ in mechanics. Its movement can be studied thanks to a differential equation given in equation 1.3a We denote by x the spatial variable $x \in \Omega$ and f the force applied on the membrane $f: \ \Omega \longrightarrow \mathbb{R}^n$. $\partial\Omega$ denotes the domain's border, which is assumed fixed.

$$\begin{aligned} \forall x \in \Omega \quad -\Delta u(x) &= f(x) \qquad (1.3a) \\ \forall x \in \partial\Omega \quad u(x) &= 0 \end{aligned}$$

It is easy to demonstrate (using the variational formulation) that equation (1.3a)'s solutions are exactly the solutions of the following optimization problem :

$$\begin{aligned} \text{Inf} \quad & E(v) = \int_\Omega |\nabla(u(x))|^2 dx - \int_\Omega f u dx \qquad (1.4a) \\ \text{s.t.} \quad & v \in H_0^1(\Omega) \end{aligned}$$

where $H_0^1(\Omega)$ is the Sobolev space defined by the functions u such that $u \in \mathcal{L}^2(\Omega)$, $\nabla u \in \mathcal{L}^2(\Omega)$ (the derivative is considered in the sense of the distributions theory) and $u = 0$ over $\partial\Omega$. Problem (1.4a) is a mechanical energy minimization, over an infinite dimension (functional) space. Here again, the optimization is continuous.

1.2.2.4 The knapsack problem

We present here the most famous integer optimization problem. We consider a knapsack that has a finite volume V and a set of objects I which need to be carried. Each object $i \in I$ has a weight w_i and a volume v_i. The idea is to minimize the total weight of the knapsack by selecting which object to carry. The variable x_i is such that $x_i \in \{0,1\}$ and $x_i = 1$ if object i is selected whereas $x_i = 0$ otherwise. The total weight carried in the knapsack is hence $\sum_i x_i w_i$ and the total volume $\sum_i x_i v_i$. The optimization problem we need to solve is given below :

$$\begin{aligned} \text{Inf} \quad & \textstyle\sum_i x_i w_i \\ \text{s.t.} \quad & \textstyle\sum_i v_i x_i \leq V \\ \text{s.t.} \quad & \forall i \in I,\ x_i \in \{0,1\} \end{aligned} \tag{1.5a}$$

Solving the knapsack problem is simple, in practice. Let's sort our objects such that $w_1 \leq w_2 \leq ... \leq w_n$ and denote by x_i^* the optimal solution. The first object to select is the lightest one : $i = 1$. If there is a remaining capacity in the knapsack (i.e. $V - v_1 > 0$), $x_1^* = 1$, otherwise $x_1^* = 0$. The second object to consider is $i = 2$, if the volume constraint is still not binding : $V - v_1 - v_2 > 0$, $x_2^* = 1$ otherwise $x_2^* = 0$ etc.

Obviously, since $\forall i,\ x_i \in \mathbb{Z}$, the knapsack problem is a particular case of integer optimization.

1.2.3 Definitions and notation

This section gives the principal notation, definition, and vocabulary that will be used in this manuscript. As said before, an optimization problem can be written as follows :

$$\begin{aligned} \text{Inf} \quad & f(x) \\ \text{s.t.} \quad & x \in K \subset E \end{aligned} \tag{1.6a}$$

The set E is usually a vectorial space provided with a norm $||.||$. The subset K will be called the **feasible region**. Usually, this region is issued from a set of equality or inequality constraints. If $K = \{\Phi\}$ (K is empty), we will say that the problem is infeasible. Function $f : K \longrightarrow \mathbb{R}$ will be called the **objective function**. We will assume that it belongs to $C^1(E)$ (continuously differentiable). The variable x will be called the **decision variable** (or decision variables in case of a finite dimension superior than 2). We will denote by f^* the infimum of f over K. If $\exists x \in K$ such that $f(x) = f^*$, we will say that infimum is reached. In that case, the infimum is actually a minimum and we will denote this by x^*. If we already know that the infimum will be reached, problem (1.6a) can be rewritten as follows :

$$\begin{aligned} \text{Min} \quad & f(x) \\ \text{s.t.} \quad & x \in K \subset E \end{aligned} \tag{1.7a}$$

In the following explanations, definitions, and theorems, we will consider only minimization problems. The maximization case can be solved using the following property :

$$\begin{array}{llcll} \text{Inf} & f(x) & = & -\text{Sup} & -f(x) \\ \text{s.t.} & x \in K & & \text{s.t.} & x \in K \end{array}$$

Definition 1. *x is a local minimum of f over K if $x \in K$ and*

$$\exists \eta > 0 \textit{ such that } \forall y \in K,\ ||y - x|| \leq \eta \Rightarrow f(y) \geq f(x).$$

Definition 2. *x is a (global) minimum of f over K if $x \in K$ and*

$$\forall y \in K,\ f(y) \geq f(x).$$

Obviously, in the optimization problems we are interested in (that can be generally described thanks to problem (1.6a)), we look for global optima. The main difficulty is hence to get rid of local optima because there is no simple theoretical result that allows one to numerically distinguish between local and global minima.

The main property that will allow us to ensure that a local minimum is actually a global one concerns the objective function f. This has to do with its graph's shape, which must be convex. First we need to define the convexity of the feasible region.

Definition 3. *The set K is convex if*

$$\forall x, y \ \in K,\ \forall \lambda \in [0, 1],\ \lambda x + (1 - \lambda) y \ \in K \tag{1.8}$$

A convex set is such that each line segment that links each pair of points, belonging to the set, remains in the set. Figure 1.2 shows the difference between a convex and a non-convex set.

FIGURE 1.2 – *Convex and non-convex sets.*

A convex function needs to be defined on a convex set.

Definition 4. *Let K be a convex set and f a function defined over K. f is convex over K if :*

$$\forall x, y \in K,\ \forall \lambda \in [0,1],\ f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda) f(y) \tag{1.9}$$

Definition 5. *Let K be a convex set and f a function defined on K. f is concave over K if :*

$$\forall x, y \in K,\ \forall \lambda \in [0,1],\ f(\lambda x + (1-\lambda)y) \geq \lambda f(x) + (1-\lambda) f(y) \tag{1.10}$$

A convex (concave) function is such that its graph is convex (concave). Figure 1.3 shows the difference between a convex and a non-convex function.

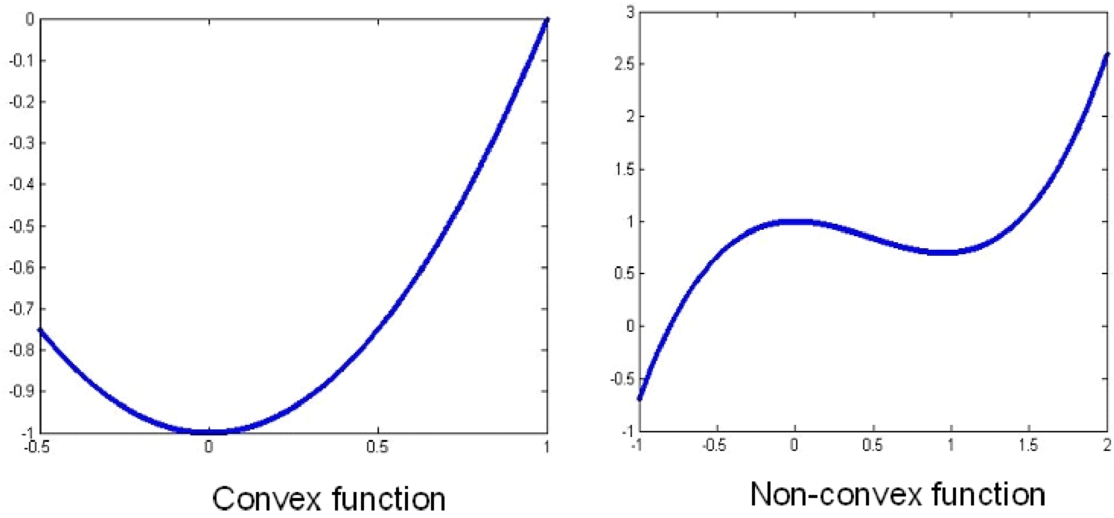

FIGURE 1.3 – *Convex and non-convex functions.*

As will be seen in Section 1.2.4, the convexity is a very useful property that will allow us to ensure that a local minimum is global. All the theoretical properties stated in Section 1.2.4 concern convex objective functions of minimization problems. **They can be generalized to concave objective functions of maximization problems**.

1.2.4 Existence and uniqueness properties

The first result we present is related to the finite dimension optimization. We remind that f is assumed to be differentiable over K.

Theorem 1. *If K is a closed and bounded set, problem (1.6a) has a solution*

Démonstration. If $K \subset \mathbb{R}^n$ is closed and bounded, it is compact (finite dimension). f is differentiable over K. In particular it is continuous. Hence, $f(K)$ is a compact set and therefore closed and bounded in $\mathbb{R}$. The infimum is thus reached. $\square$

Theorem 1 is an existence theorem. It guarantees the existence of a solution associated with the optimization problem.

If $K = \mathbb{R}^n$, which means that the optimization is carried out without constraints on a finite dimension space, it is easy to characterize the optimum thanks to the following gradient theorem :

Theorem 2. *If $K = \mathbb{R}^n$ and x is a local optimum, $\vec{\nabla} f(x) = \vec{0}$*

The gradient of f is defined as follows :

$$\vec{\nabla} f(x) = \begin{pmatrix} \frac{\partial f}{\partial x_1}(x) \\ \frac{\partial f}{\partial x_2}(x) \\ ... \\ \frac{\partial f}{\partial x_n}(x) \end{pmatrix}.$$

The optimization problem is therefore "easy" to solve. Indeed, solving equation $\vec{\nabla} f(x) = \vec{0}$ provides the candidates to the optimization problem's solutions. We then need to select the optimal solutions (global optima). This may be numerically difficult because a local optimum can be optimal over a large subset of K. To bypass this difficulty, it may be interesting to use the following theorem :

Theorem 3. *If f is convex over the convex set K, a local minimum is global.*

Démonstration. Let x be a local minimum of f over K and η be such that $\forall y \in K$ such that $||y - x|| \leq \eta,\ f(y) \geq f(x)$. Let us suppose that $\exists x' \in K$ such that $f(x') < f(x)$ and $x' \neq x$. We know that $\exists \lambda \in (0, 1)$ such that $||\left(\lambda x + (1-\lambda)x'\right) - x|| \leq \eta$. Therefore $f\left(\lambda x + (1-\lambda)x'\right) \geq f(x)$. However, by convexity of f, $f\left(\lambda x + (1-\lambda)x'\right) \leq \lambda f(x) + (1-\lambda) f(x') < \lambda f(x) + (1-\lambda) f(x) = f(x)$, which is absurd. x is hence a global optimum over K. □

Theorem 3 ensures the uniqueness of the optimum, if it exists.

Theorem 4. *f is convex if and only if*
$\forall x,\ y \in K,\ f(y) \geq f(x) + \vec{\nabla} f(x).(y - x)$

Theorem 5. *If f is twice differentiable. f is convex and if and only if*
$\forall x \in K$ the Hessian matrix of f, $H_f(x)$ is positive semi-definite.

The Hessian matrix of f, is defined by the following :

$$\forall x \in K,\ H_f(x) \in M_n(\mathbb{R}) \text{ and } \forall i, j,\ H_f(x)_{ij} = \frac{\partial^2 f}{\partial x_i \partial x_j}(x)$$

or

$$\forall x \in K,\ H_f(x) = \begin{pmatrix} \frac{\partial^2 f}{\partial x_1^2}(x) & \frac{\partial^2 f}{\partial x_1 \partial x_2}(x) & ... & \frac{\partial^2 f}{\partial x_1 \partial x_n}(x) \\ ... & ... & ... & ... \\ \frac{\partial^2 f}{\partial x_n \partial x_1}(x) & \frac{\partial^2 f}{\partial x_n \partial x_2}(x) & ... & \frac{\partial^2 f}{\partial x_n^2}(x) \end{pmatrix}$$

Definition 6. *A matrix $M \in M_n(\mathbb{R})$ is positive semi-definite if* $\forall x \in \mathbb{R}^n,\ {}^t x M x \geq 0$.

The Schwarz theorem allows us to assert that the Hessian matrix is symmetric in $M_n(\mathbb{R})$ and is therefore diagonalizable. It is easy to demonstrate that a symmetric matrix is positive semi-definite if and only if all its eigenvalues are nonnegative.

Theorem 6. ***(Euler Inequality)*** *If f is defined over a convex set K and if x is a local minimum of f then :*

$$\forall y \in K, \quad \nabla f(x).(y - x) \geq 0 \tag{1.11}$$

If f is convex and $x \in K$ is a point that verifies (1.11), then x is an optimum of f over K.

Euler's inequality presented in Theorem 6 is generally a necessary condition that characterizes a local optimum. It also becomes sufficient if f is convex.

1.2.5 Duality, Lagrangian multipliers and KKT conditions

This section presents the most useful results of optimization because they allow one to characterize, in a simple way, the optimum. It starts with the definition of the Lagrangian. Let us consider the following optimization problem which is a particular case of problem (1.7a) :

$$\begin{aligned} \text{Inf} \quad & f(x) \\ \text{s.t.} \quad & F(x) = 0 \\ & G(x) \leq 0 \end{aligned} \tag{1.12a}$$

where the mappings F and G are such that $F : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ and $G : \mathbb{R}^n \longrightarrow \mathbb{R}^p$. The inequality $G(x) \leq 0$ is a condensed notation for $\forall i \in \{1, 2..., p\},\ G_i(x) \leq 0$ and the equality $F(x) = 0$ is a condensed notation for $\forall i \in \{1, 2..., m\},\ F_i(x) = 0$. This means that we have m equality constraints and p inequality constraints. The Lagrangian $\mathcal{L}$ is a function defined over $\mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^{+p}$ by the following :

Definition 7.

$$\forall x \in \mathbb{R}^n, \lambda \in \mathbb{R}^m, \mu \in \mathbb{R}^{+p}, \quad \mathcal{L}(x, \lambda, \mu) = f(x) + \lambda F(x) + \mu G(x) \tag{1.13}$$

The link between problem (1.12a) and the Lagrangian is provided thanks to the following lemma :

Lemma 1.

$$Inf_{\{x \in \mathbb{R}^n\}}\ Sup_{\{\mathbb{R}^m \times \mathbb{R}^{+p}\}} \mathcal{L}(x, \lambda, \mu) = Inf_{\{F(x)=0\ \&\ G(x) \leq 0\}} f(x)$$

The following problem :

$$\underset{x \in \mathbb{R}^n}{\text{Inf}} \quad \underset{(\lambda, \mu) \in \mathbb{R}^m \times \mathbb{R}^{+p}}{\text{Sup}} \quad \mathcal{L}(x, \lambda, \mu) \tag{1.14a}$$

is called the primal problem. Lemma 1 states that solving the primal problem gives the exact solution to problem (1.12a).
It is easy to demonstrate the following inequality :

$$\underset{x \in \mathbb{R}^n}{\text{Inf}} \quad \underset{(\lambda,\mu) \in \mathbb{R}^m \times \mathbb{R}^{+p}}{\text{Sup}} \quad \mathcal{L}(x,\lambda,\mu) \;\geq\; \underset{(\lambda,\mu) \in \mathbb{R}^m \times \mathbb{R}^{+p}}{\text{Sup}} \quad \underset{x \in \mathbb{R}^n}{\text{Inf}} \quad \mathcal{L}(x,\lambda,\mu)$$

The following problem :

$$\underset{(\lambda,\mu) \in \mathbb{R}^m \times \mathbb{R}^{+p}}{\text{Sup}} \quad \underset{x \in \mathbb{R}^n}{\text{Inf}} \quad \mathcal{L}(x,\lambda,\mu) \tag{1.15a}$$

is called the dual problem. As it will be seen later, the dual and primal problems are correlated (with some assumptions) in case of the existence of the optimum.

In order to present the conditions that will characterize the optimum, we first need to define the constraint qualifications.

Definition 8. *Let x be a feasible point such that $G(x) \leq 0$. We denote by $B(x)$ the set $B(x) = \{i \in \{1, 2...p\}, G_i(x) = 0\}$ of the binding (or active) constraints in x.*

Note that $B(x) = \phi$ is empty if all the constraints are not binding in x.

Definition 9. *Let x be a feasible point such that $G(x) \leq 0$. We will say that the constraints are qualified in x if there exists a direction $u \in \mathbb{R}^n$ such that $\forall i \in B(x)$, $\nabla G_i(x).u < 0$ or $\nabla G_i(x).u = 0$ and G_i is affine.*

Constraint qualification is an important property that will be used to characterize the optimum. The definition given above is a particular (the most useful) case of constraint qualifications. More general constraint qualifications can be found in (3). In particular, if the inequality constraints are affine, constraint qualifications always hold.

We first present the necessary conditions that guarantee the optimum. These are the Karush-Kuhn-Tucker conditions

Theorem 7. ***(KKT necessary conditions)*** *If constraint qualifications hold in x^* and if x^* is optimal for problem 1.12a, then :*

$$\exists (\lambda^*, \mu^*) \in \mathbb{R}^m \times \mathbb{R}^{+p}, \qquad \nabla f(x^*) + \sum_{i=1}^{m} \lambda_i^* \nabla F(x^*) + \sum_{i=1}^{p} \mu_i^* \nabla G_i(x^*) = 0$$

$$F(x^*) = 0, \;\; G(x^*) \leq 0, \;\; \mu^*.G(x^*) = 0.$$

The KKT conditions we presented in Theorem 7 are necessary conditions of optimality. They require no assumptions on the objective function or the constraints (except their differentiability and constraint qualifications). The following theorem gives the sufficient conditions.

Theorem 8. ***(KKT necessary and sufficient conditions)*** *We assume that f and G are convex functions, F is affine and constraint qualifications hold in x^*. x^* is optimal for problem 1.12a if and only if*

$$\exists(\lambda^*, \mu^*) \in \mathbb{R}^m \times \mathbb{R}^{+p}, \qquad \nabla f(x^*) + \sum_{i=1}^{m} \lambda_i^* \nabla F(x^*) + \sum_{i=1}^{p} \mu_i^* \nabla G_i(x^*) = 0$$

$$F(x^*) = 0,\ G(x^*) \leq 0,\ \mu^*.G(x^*) = 0.$$

In that case, the primal and dual problems have the same solution, which is (x^*, λ^*, μ^*).

We can therefore write :

$$\underset{x \in \mathbb{R}^n}{\text{Inf}} \quad \underset{(\lambda,\mu) \in \mathbb{R}^m \times \mathbb{R}^{+p}}{\text{Sup}} \quad \mathcal{L}(x, \lambda, \mu) = \underset{(\lambda,\mu) \in \mathbb{R}^m \times \mathbb{R}^{+p}}{\text{Sup}} \quad \underset{x \in \mathbb{R}^n}{\text{Inf}} \quad \mathcal{L}(x, \lambda, \mu)$$

$$= \mathcal{L}(x^*, \lambda^*, \mu^*)$$

Theorem 8 is very useful because it provides necessary and sufficient conditions that characterize the optimum.

It is important to highlight the inherent assumptions required in the KKT conditions :
- the objective function must be convex.
- the inequality constraints need to be defined thanks to convex functions and the equality constraints thanks to affine functions.
- constraint qualifications.

In this thesis, these conditions will always be met. In particular, most of our optimization programs maximize profits and we will always demonstrate that the objective functions are concave. Our constraints are linear and therefore convex and constraint qualifications always hold. The KKT conditions are thus necessary and sufficient conditions for optimality.

Example 1. *Let's solve the following maximization problem :*

$$\begin{array}{lll} \textit{Sup} & f(x_1, x_2) = x_1 & \\ \textit{s.t.} & g_1(x_1, x_2) = x_1 + x_2 - 1 & \leq 0 \\ & g_2(x_1, x_2) = -x_2 & \leq 0 \end{array}$$

In this particular example, the decision variable is a vector of $\mathbb{R}^2$. The constraint function G is such that $G(x_1, x_2) = (g_1(x_1, x_2), g_2(x_1, x_2))$, $n = 2$ and $p = 2$. The objective function f is linear and in particular convex. The function G is linear, too. Hence, constraint qualifications hold. The KKT conditions are, in this example, necessary and sufficient for optimality. The dual variable $\mu = (\mu_1, \mu_2)$ is such as $\mu \leq 0$ and we must solve the following system :

$$\begin{aligned}
&\frac{\partial f}{\partial x_1}(x) + \mu_1 \frac{\partial g_1}{\partial x_1}(x) + \mu_2 \frac{\partial g_2}{\partial x_1}(x) = 0 \\
&\frac{\partial f}{\partial x_2}(x) + \mu_1 \frac{\partial g_1}{\partial x_2}(x) + \mu_2 \frac{\partial g_2}{\partial x_2}(x) = 0 \\
&\mu_1 \leq 0 \\
&\mu_2 \leq 0 \\
&g_1(x) \leq 0 \\
&g_2(x) \leq 0 \\
&\mu_1 g_1(x) + \mu_2 g_2(x) = 0
\end{aligned}$$

or

$$\begin{aligned}
&1 + \mu_1 = 0 \\
&\mu_1 - \mu_2 = 0 \\
&\mu_1 \leq 0 \\
&\mu_2 \leq 0 \\
&x_1 + x_2 - 1 \leq 0 \\
&-x_2 \leq 0 \\
&\mu_1(x_1 + x_2 - 1) = 0 \\
&\mu_2 x_2 = 0
\end{aligned}$$

The optimal solution is $(x_1, x_2) = (1, 0)$ *and* $(\mu_1, \mu_2) = (-1, -1)$.

§ 1.3 Non-cooperative game theory

1.3.1 Introduction

Game theory is a mathematical way to study the interaction of a certain number of players where each actor's fate depends on what he does and what the others do. This situation is often referred to as "strategic interaction." Intuitively, each player, in order to optimize his utility, will have to anticipate the decisions of the others. The economic rationality is a common hypothesis to the game theory frame. The oligopoly is a particular application of game theory. Let us consider a set of producers of a homogenous good needed in a demand market. Each player has to choose the quantity to produce in order to maximize his profit. His decision variable will influence the global amount produced and therefore the market price. Consequently, he has the ability not only to influence his profit, but also all the other producers' payoff.

Economists commonly consider that game theory was founded in 1944 in the book "Theory of games and economic behaviour" (10) by von Neumann and Morgenstern. There were some precursory works, in particular those of Antoine-Augustin Cournot in 1838, who studied the duopoly outcome. von Neumann and Morgenstern studied situations where the benefit of one player is equal to the loss of the others. This situation is called a "zero sum game". In 1951, Nash generalized Cournot's ideas of the duopoly and von Neumann and Morgenstern' work to define and study the equilibrium of a variable sum-game. The applications of his theory started to emerge in the 1970s especially in industrial economics, where it allowed the modeling of imperfect competition where market power is exerted.

It is important to note that the game theory principally relies on three assumptions :
- Each player strives to optimize, rationally, his payoff.
- Each player knows that all the other players do the same.
- Each player knows exactly the game rules and frame (number of players, their possible actions, etc.).

1.3.2 Definitions and notation

It is common to distinguish between two kinds of games :
- Cooperative games.
- Non-cooperative games.

In a cooperative game, a set of players can form coalitions and act as a unique player. Their decision variables are correlated. On the contrary, no coalition can be formed when the game is non-cooperative. In that case, each player has a certain number of actions, called strategies that influence both his and the other player's payoff. A game can be described in a **strategic** or **extensive** form. When the description is strategic, all the strategies are known by all the players and each player has the same information about the influence of each other player's actions over the payoff. Therefore, each player has to select a strategy, in order to optimize his payoff, given this information. When the game is extensive, it can be defined thanks to a tree, where each node represents the status of the game at a given moment, *i.e.*, the player who has to play, his possible strategies, and the information he has. The final payoff can be calculated at the level of the tree's leaves.

Each player's possible action that influences the payoff is called a **strategy**. A set of **pure** strategies is a set where each strategy is chosen with certainty. On the opposite, a set of **mixed** strategies is a set where each strategy is played, randomly, with a given law probability.

Non-cooperative game theory is very useful for modeling the behavior of oligopolistic economic agents, who can influence each other's profit while exerting market power. It can also be used to represent international negotiations between governments, electoral competition, etc. In this thesis, non-cooperative game theory will allow us to model the strategic interactions of gas chain players who can exert market power (producers, independent traders etc.).

1.3.3 Strategic games, definition and notation

In this thesis, we will consider only the situation where the number of players is finite N. In this chapter, the set of players will be denoted by $P = \{1, 2...N\}$. For each player $p \in P$, we define the set of possible strategies S_p. The joint set of all possible strategies is $S = \Pi_p S_p$. The product Π is a notation for the Cartesian product $S_1 \times S_2 ... \times S_N$. For each player $p \in P$, we define the payoff f_p, as a function of S. Player p's payoff is such that $f_p : S \longrightarrow \mathbb{R}$. This payoff is a function of the variable $s \in S$, which means that f_p depends on the the joint strategies (of all the players), or that each player can influence all the other players' payoff.

For a two-player game where S is finite, we can represent the game thanks to two matrices M_1 and M_2. If player 1's (respectively player 2) strategies set is $\{s_1^1, s_2^1 ... s_n^1\}$ ($\{s_1^2, s_2^2 ... s_m^2\}$), then M_1 and M_2 are in $\mathbb{R}^{n \times m}$, $M_1(i,j) = f_1(s_i^1, s_j^2)$ and $M_2(i,j) = f_2(s_i^1, s_j^2)$, as shown in figure 1.4.

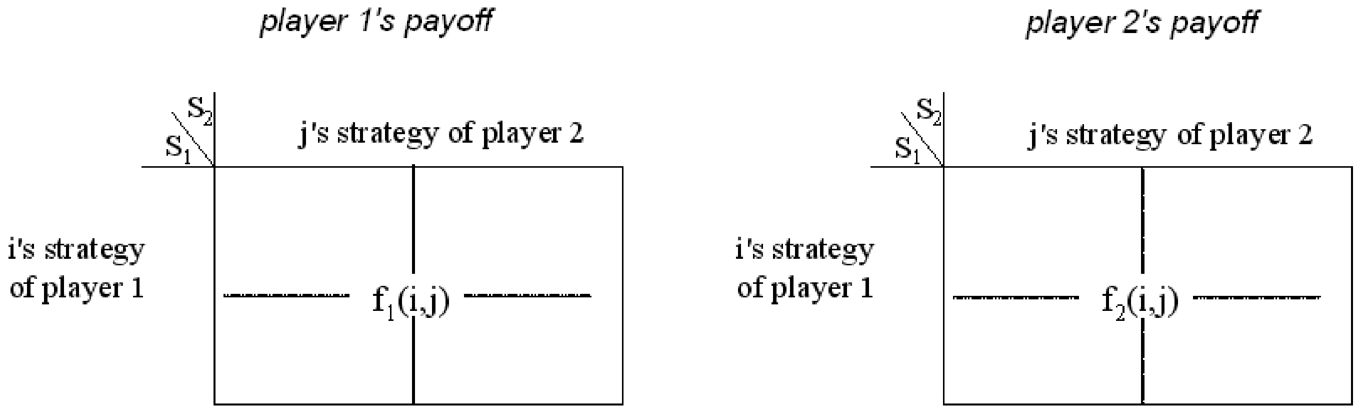

FIGURE 1.4 – *A two-player strategic game.*

Example 2. *Figure 1.5 gives the payoff matrix of the Rock-Paper-Scissors game. There are two players and the set of strategies is $S_1 = S_2 = \{Rock, Paper, Scissors\}$. Each strategy of $S = S_1 \times S_2$ leads to a couple of gains, written in parenthesis. The left member is the first player's gain whereas the right member is the second player's payoff.*

		player 2		
		Rock	Paper	Scissors
player 1	Rock	(0,0)	(-1,1)	(1,-1)
	Paper	(1,-1)	(0,0)	(-1,1)
	Scissors	(-1,1)	(1,-1)	(0,0)

FIGURE 1.5 – *The Rock-Paper-Scissors game.*

One can notice that in the Rock-Paper-Scissors game, the sum of the players' payoffs is always 0, no matter the strategies chosen by the players. This can be rewritten as follows : $\forall s \in S,\ \sum_p f_p(s) = 0$. This remark leads us to the following definition :

Definition 10. *A constant-sum game is a game where :*

$$\exists C \in \mathbb{R},\ \textit{such as}\ \forall s \in S,\ \sum_p f_p(s) = C \tag{1.18}$$

A zero-sum game is a game where :

$$\forall s \in S,\ \sum_p f_p(s) = 0 \tag{1.19}$$

A constant-sum game is the typical example of a sharing game situation, where the gain C must be shared between the players, regarding the strategies they decide to go for. The zero-sum game is the winner/loser situation (in the case of two players), where the gain obtained by one player is exactly the loss of the other.

Let us now consider the two-player game given in figure 1.6. Each player has two possible strategies, a and b.

		player 2	
		a	b
player 1	a	(1,1)	(0,3)
	b	(3,0)	(2,2)

FIGURE 1.6 – *A two-player game.*

If player 1 selects strategy a, then player 2 must choose strategy b, because it is the one that maximizes his payoff. Similarly, if player 1 tries strategy b, then player 2 must also select strategy b, for the same reason. No matter what player 1 chooses, player 2 should always go for strategy b. We will say that b is a dominant strategy for player 2. Strategy b is also a dominant strategy for player 1. To correctly define the notion of dominant strategy, it is better to use the following notation : if $s \in S$ is a joint strategy and p a player, we will denote by S_{-p} the joint set of the other players : $S_{-p} = \Pi_{j \neq p} S_j$ and we will write $s = (s_p, s_{-p})$ where $s_p \in S_p$ is the strategy wanted by p and s_{-p} the joint strategy decided by the others.

Definition 11. *Strategy s_p^* is a dominant strategy for player p if*

$$\forall s_{-p} \in S_{-p}, \forall s_p \in S_p,\ f_p(s_p, s_{-p}) \leq f_p(s_p^*, s_{-p}) \tag{1.20}$$

A dominant strategy for one player is a strategy he needs to play in order to optimize his payoff, no matter what the other players do.

1.3.4 Equilibria definition

It is reasonable to assert that for the situation presented in figure 1.6, the outcome of the game will be (b,b), where each player plays his dominant strategy. The corresponding payoff is (2,2). This introduces the notion of equilibrium. This remark leads to the following definition :

Definition 12. ***(The dominant strategies equilibrium)*** *A joint strategy $s^* \in S$ is a dominant strategies equilibrium if :*

$$\forall p \in P,\ s^*_p \text{ is a dominant strategy for } p \tag{1.21}$$

or similarly :

$$\forall p \in P,\ \forall s_{-p} \in S_{-p}, \forall\ s_p \in S_p,\ f_p(s_p, s_{-p}) \leq f_p(s^*_p, s_{-p}) \tag{1.22}$$

In the game given in figure 1.6, (b, b) is an equilibrium in dominant strategies. In the Rock-Paper-Scissors game, there is no dominant strategy for any player. Therefore, the existence of a dominant strategies equilibrium is not always ensured. This is principally due to the fact that games having dominant strategies are actually "pseudo-games" [1] where the outcome is straightforward. It may be interesting to weaken the definition of the quilibrium of a non-cooperative game.

Definition 13. ***(The Nash-Cournot equilibrium)*** *A joint strategy $s^* \in S$ is a Nash-Cournot equilibrium if, starting from that strategy, no player has an incentive to change his strategy in order to increase his payoff.*

$$\forall p \in P,\ \forall s_p \in S_p,\ f_p(s_p, s^*_{-p}) \leq f_p(s^*_p, s^*_{-p}) \tag{1.23}$$

It is straightforward that an equilibrium in dominant strategies is a Nash-Cournot equilibrium and there is no other possible Nash-Cournot equilibria.

Let's consider the two-player game given in figure 1.7. Each player has two strategies $\{a, b\}$.

It is easy to note that this game cannot have possible Nash-Cournot equilibria and therefore has no dominant strategies equilibrium. Thus, one can conclude that the existence of a Nash-Cournot equilibrium is not always ensured. This is also the case of the uniqueness. [2] This is one motivation of the introduction of mixed strategies. A mixed strategies equilibrium describes the situation where each player has the possibility to choose his strategy with a given probability. Each player's problem is to find the optimal probabilities associated with his strategies set, in order to maximize the payoff, while trying to take into account, by anticipation, his opponents' actions.

1. A pseudo-game is a game where the players always have a "winning strategy", no matter what the opponents do.

2. It is easy to note that a game whose payoffs are equal regardless of the joint strategy chosen by the players will lead to multiple Nash-Cournot equilibria.

		player 2	
		a	b
player 1	a	(-3,-3)	(0,-2)
	b	(-2,0)	(-1,-1)

Figure 1.7 – *A two-player game.*

Such situations are not used in this thesis where we focus mainly on the Nash-Cournot equilibria characterization, in a deterministic strategies choice. More details and advantages regarding the mixed strategies equilibrium can be found in (10). [3]

§ 1.4 Oligopolistic markets and mixed complementarity problems

1.4.1 Introduction

An oligopoly is a market situation where a small number of firms compete in order to sell a specific product. If their number is small enough, they can become price makers and exert market power. Market power is the economic action of a firm that can influence its profit by reducing the quantity of the product it brings to the market in order to force the price up. Such a situation characterizes the European natural gas markets. Indeed, the current European natural gas trade is dominated by oligopolies in the upstream. Since the production is very concentrated geographically, market power can be exerted by the different producers, in the downstream. The conventional natural gas producing countries that sell in Europe are : [4] Russia, Algeria, the United Kingdom, Norway, the Netherlands and Quatar (LNG). Their exports represent more than 90% of the European natural gas consumption. Since their number is not large, they may have an incentive to exert market power.

Mathematically speaking, an oligopoly is often modeled by considering the strategic interactions between the producers, who compete a la Cournot. A Cournot competition represents

3. Such advantages include for instance more simple results on the existence or uniqueness of the equilibria.
4. The list is not exhaustive. It includes the biggest producers (in volume).

the situation where the producers have to decide their optimal volumes, or quantities, they will bring to the market. The price is hence an output of this interaction. On the contrary, a situation where the producers fix the price (as a decision variable) is called a Bertrand competition. In both cases, the consumers are modeled thanks to their demand or inverse demand function, which can be seen as their reaction function. Considering the inverse demand function is necessary in the Cournot competition case. It relates the market price to the total volume decided by the producers.

As we will see later, an oligopoly can be studied thanks to non-cooperative game theory where the market outcome will be characterized by a Nash-Cournot equilibrium. Mathematically, such a situation is formulated using a mixed complementarity problem.

The different economic market structure descriptions can be summarized in the following figure :

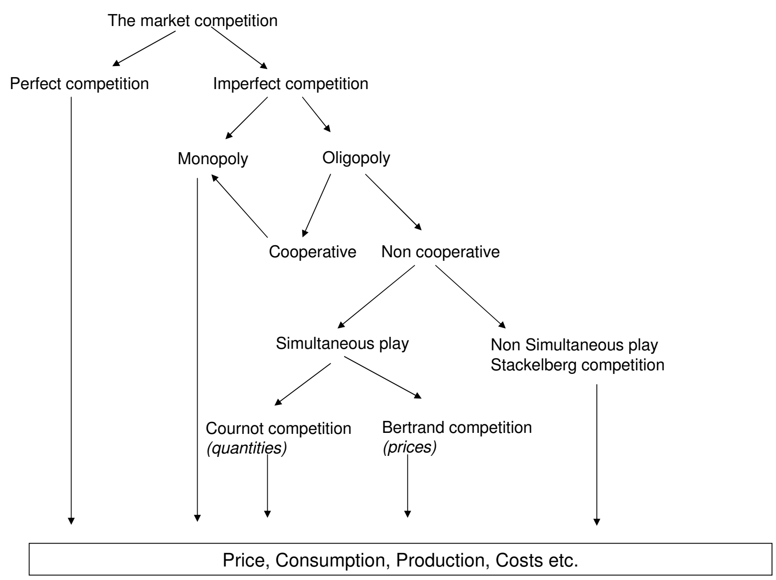

Figure 1.8 – *Possible market structure modeling.*

The competition can either be perfect (price-taking producers) or imperfect (price-making producers). In an imperfect competition context, some actors can influence a market parameter (in most situations the price) by changing their actions (production, for instance) in order to increase their profit. Such a situation is not possible in a perfect competition market (when the price is regulated, for instance). When there is market power, it can be exerted by one or more players.

1.4.2 A simple oligopoly

Let us present, as an example, a simple interaction between gas producers, under non-negativity constraints. The set of producers is I and each producer $i \in I$ will have to decide its optimal output x_i to sell to the consumers. These consumers have a welfare maximization program that links the gas price p to the quantity consumed q by the inverse demand function, which will be denoted by f. f is a decreasing function. To simplify the calculations, we will assume that it is linear : $f(q) = a - bq$ where a and b are positive. $b \neq 0$ reflects the fact that the demand is elastic to the price. A situation where $b = 0$ induces that the demand is inelastic, or fixed exogenously by the consumers. The unit production marginal cost of a producer i is constant : c_i.

$$p = f(q) = a - bq \tag{1.24}$$

Producer i's profit maximization program is given by the following :

$$\begin{array}{ll} \text{Max} & \Pi_i = (a - b\sum_{j \in I} x_j)x_i - c_i x_i \\ \text{s.t.} & \qquad\qquad x_i \geq 0 \end{array}$$

$\sum_{j \in I} x_j$ denotes the total quantity q of gas brought to the consumers. Because of the term $a - b\sum_{j \in I} x_j$, producer i's payoff Π_i depends not only on his decision variable x_i but also on the other producers' decision variables $x_{j \neq i}$. This situation can be seen as a non-cooperative game. The players are the producers and each player's strategy set S_i is $\mathbb{R}^+$. The Nash-Cournot equilibrium is the market situation where the quantities x_i^* are such as :

$$\forall i \in I,\ x_i^* \in \mathbb{R}^+ \text{ and } \forall i \in I,\ \Pi_i(x_i, x_{j \neq i}^*) \text{ is optimal when } i \text{ produces } x_i^* \tag{1.25}$$

Since the inverse demand function is linear, it is easy to notice that each producer's objective function is concave with respect to his decisions variables. In other words, taking the other producers's decision variables as exogenous in his optimization program, each producer deals with a concave profit function. Since the constraints are all linear, constraint qualifications hold and the KKT conditions (or the first-order conditions) are necessary and sufficient to characterize the optimum. Therefore, if λ_i are the dual variables associated with the non-negativity constraints, we have to look for the primal and dual variables x_i^* and λ_i such that :

$$\forall i \in I, \quad \frac{\partial \Pi}{\partial x_i}(x^*) - \lambda_i = 0 \tag{1.26}$$
$$\forall i \in I, \quad x_i^* \geq 0 \tag{1.27}$$
$$\forall i \in I, \quad \lambda_i \leq 0 \tag{1.28}$$
$$\forall i \in I, \quad \lambda_i x_i^* = 0 \tag{1.29}$$

These equations are often rewritten by getting rid of the non-negativity constraints dual variables. This leads to the following formulation : looking for the primal variables x_i^* such that :

$$\begin{aligned}
\forall i \in I, \quad & \tfrac{\partial \Pi}{\partial x_i}(x^*) && \leq 0 && (1.30)\\
\forall i \in I, \quad & x_i^* && \geq 0 && (1.31)\\
\forall i \in I, \quad & \tfrac{\partial \Pi}{\partial x_i}(x^*)x_i^* && = 0 && (1.32)
\end{aligned}$$

or, in a more condensed notation :

$$\forall i \in I, \quad 0 \leq x_i \perp \tfrac{\partial \Pi}{\partial x_i}(x^*) \quad \leq 0 \tag{1.33}$$

The $\perp$ sign is the orthogonality sign. It means that the product of the two corresponding terms is nought.

Equation 1.33 is the basic instance of a mixed complementarity problem, that will be presented later on. This orthogonality condition is often called the slackness condition. If we use the linear formulation of the inverse demand function, equation 1.33 becomes :

$$\forall i \in I, \quad 0 \leq x_i^* \perp \left(a - b\textstyle\sum_j x_j^* - c_i - bx_i^*\right) \quad \leq 0 \tag{1.34}$$

If $\forall i \in I,\ a > (n+1)c_i - \sum_j c_j$, then the Nash-Cournot equilibrium solution to equation 1.34 is such that :

$$x_i^* = \frac{a + \sum_j c_j - (n+1)c_i}{(n+1)b} \tag{1.35}$$

The consumption is

$$q^* = \frac{na - \sum_j c_j}{(n+1)b} \tag{1.36}$$

and the price

$$p^* = \frac{a + \sum_j c_j}{(n+1)} \tag{1.37}$$

The first observation one can draw from the oligopoly study is that this situation is more general than the pure and perfect competition frame. Indeed, if the players are symmetric (*i.e.*, all the players support the same cost $c : \forall i,\ c_i = c$), the price converges toward c when the number of strategic players is big enough. The economic theory shows that the pure and perfect competition price is always equal to the marginal production cost c, otherwise, the producers would trigger a price war that would decrease the price to the marginal cost (Bertrand's paradox). The second observation to draw from our study is the following : the market price (quantity) induced by a strategic interaction is higher (lower) than the pure and perfect competition price (quantity). This is a result of the market power exercice : the producers have a strong incentive to reduce the volumes to force the price up, because they know they can influence the price.

To introduce mixed complementarity problems, let us write the general formulation of an oligopoly : the set of players is I. Each player has a set of decision variables that can be summarized in one vector variable $x_i \in \mathbb{R}^n$. The global decision vector x is $x = (x_1, x_2 ... x_N)$. Each player faces feasibility constraints of the form : $g_i(x_i) \leq 0$ and $h_i(x_i) = 0$. His payoff is $\Pi_i(x)$. This notation holds an implicit assumption which is the following : As opposed to the payoffs, the players do not influence each other's feasibility set. Such a situation is referred to as the **Standard Nash-Cournot game**, as opposed to the **Generalized Nash-Cournot game**, where the players have the possibility to change, via their decision variables, the other players' constraints sets. Generalized Nash-Cournot (or GNC) games will be presented in chapter 8.

Player i's profit maximization program is given by the following :

$$\begin{array}{rl} \text{Max} & \Pi_i(x) \\ \text{st} & g_i(x_i) \geq 0 \quad (\lambda_i) \\ & h_i(x_i) = 0 \quad (\mu_i) \\ & x_i \geq 0 \end{array}$$

The dual variables (vector variables) are written in parenthesis. If the profits Π are concave functions of the vector variables x_i (regardless of the other variables x_i^-) and constraint qualification holds, then the KKT conditions are necessary and sufficient to characterize the optima. The Nash-Cournot equilibrium is given by the following equations :

$$\forall i \in I, \quad 0 \leq x_i \perp (\nabla_{x_i}(\Pi_i)(x) + \lambda_i \nabla_{x_i}(g_i)(x_i) + \mu_i \nabla_{x_i}(h_i)(x_i)) \leq 0 \tag{1.38}$$

$$\forall i \in I, \quad 0 \geq \lambda_i \perp g_i(x_i) \leq 0 \tag{1.39}$$

$$\forall i \in I, \quad \text{free } \mu_i \quad h_i(x_i) = 0 \tag{1.40}$$

This is a typical formulation of a mixed complementarity problem.

1.4.3 Complementarity Problems

A Non-Linear Complementarity Problem (or NCP) is a mathematical class of problems that can be formulated by the following :

Definition 14. ***(MCP)*** *Given a point-to-point mapping $F : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ and $n_1,\ n_2 \in \mathbb{N}$ such as $n_1 + n_2 = n$, we want to solve the following problem :*
Find $x \in \mathbb{R}^{n_1}$ and $y \in \mathbb{R}^{n_2}$ such that :
$\forall i \in 1, ... n_1, \quad F_i(x, y).x_i = 0$
$\forall i \in 1, ... n_1, \quad x_i \geq 0$
$\forall i \in 1, ... n_1, \quad F_i(x, y) \geq 0$
$\forall j \in 1, ... n_2, \quad F_{n_1+j}(x, y) = 0$
$\forall j \in 1, ... n_2, \quad y_j$ *free*

The previous equations can be re-written using the $\perp$ sign :
$\forall i \in 1, ...n_1, \quad 0 \leq x_i \perp F_i(x, y) \geq 0$
$\forall j \in 1, ...n_2, \quad \text{free } y_j, \; F_{n_1+j}(x, y) = 0$

If the function F is linear, the problem is called a Linear Complementarity Problem (LCP). A Mixed Complementarity Problem (MCP) is a more subtle problem where the variables x_i can be constrained by other than 0 upper and lower bounds. More particularly, if we introduce new bounds u_i and l_i, we have to replace the equation $\forall i \in 1, ...n_1, \quad 0 \leq x_i \perp F_i(x, y) \geq 0$ by the following :

$$\begin{array}{rl} \text{if } x_i = l_i & F_i(x,y) \geq 0 \\ \text{if } l_i < x_i < u_i & F_i(x,y) = 0 \\ \text{if } x_i = u_i & F_i(x,y) \leq 0 \end{array}$$

Considering equations (1.17a), it is straightforward that a standard optimization problem, whose solution can be obtained thanks to the KKT conditions, is a particular case of MCPs. Similarly, a non-cooperative game's Nash-Cournot equilibrium, whose strategy sets are continuous, is also a particular case of MCPs. A market where strategic interactions are exerted can therefore be modeled as an MCP.

Figure 1.9 gives the different market structure situations that can be modeled in a complementarity form.

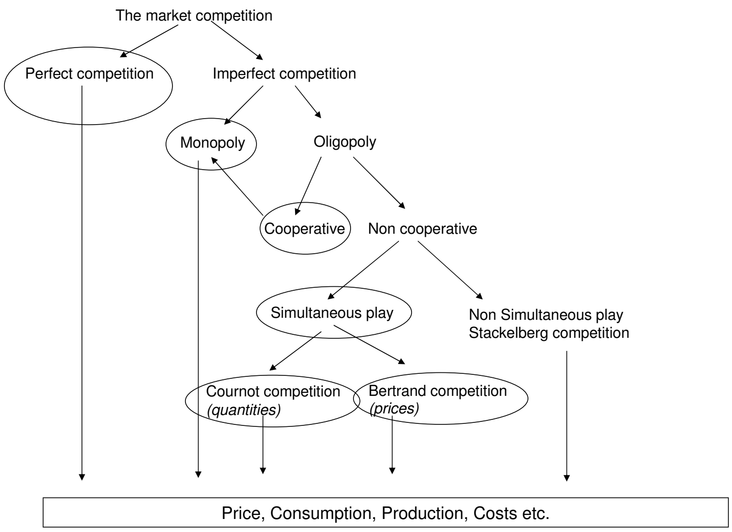

FIGURE 1.9 – *Market structure situations that can be modeled thanks to MCPs.*

All the natural gas markets models we develop in this manuscript will be presented in a complementarity form, using the KKT conditions.

§ 1.5 Conclusion

This chapter presents the main mathematical and economic tools that will be used in the manuscript : optimization theory, non-ccoperative game theory and mixed complementarity problems (MCP).

First we defined general optimization problems and presented the main theorems that ensure the existence/uniqueness of the optimal solution. The KKT conditions allow one to characterize easily the solution. The second part of the chapter focuses on non-cooperative game theory that allows us to take into account strategic interactions in economic structures modeling. We have defined in particular Nash-Cournot equilibra that may characterize imperfect competition markets outcomes. Finally, we applied optimization and non-cooperative game theories to standard imperfect competition markets modeling, in order to find necessary and sufficient conditions allowing us to calculate the Nash-Cournot equilibrium. This led us to define and study mixed complementarity problems.

All these notions will be used in our natural gas markets modeling.

Bibliographie

[1] R. K. Ahuja, T. L. Magnanti, and J. B. Orlin, 1993. *Network Flows : Theory, Algorithms, and Applications.* Network Flows : Theory, Algorithms, and Applications.

[2] B. Allaz, J. L. Vila, 1993. *Cournot Competition, Forward Markets and Efficiency.* Journal of Economic Theory 59(1), 1-16.

[3] G. Allaire, 2005. *Analyse numérique et optimisation.* Editions de l'Ecole Polytechnique, France.

[4] J. F. Bonnans, J. C Gilbert, C. Lemaréchal, C. A. Sagastizl, 2006. *Numerical Optimization - Theoretical and Practical Aspects.* Springer-Verlag.

[5] M. S. Bazaraa, H. D. Sherali, C. M. Shetty, 1993. *Nonlinear programming theory and algorithms.* John Wiley and Sons, New York.

[6] R. W. Cottle, J. -S. Pang, R. E. Stone, 1992. *The Linear Complementarity Problem.* Academic Press.

[7] A. A. Cournot, 1838. *Recherches sur les principes mathématiques de la théorie des richesses.* France.

[8] F. Facchinei & J. -S. Pang, 2003. *Finite-Dimensional Variational Inequalities and Complementarity Problems.* Springer, New York.

[9] D. Fudenberg, J. Tirole, 1991. *Game theory.* MIT Press.

[10] von Neumann & Morgenstern, 1944. *Theory of games and economic behaviour.* Princeton University Press, USA.

[11] J. F. Thisse, *Théorie des jeux, une introduction.* Available at http ://www.core.ucl.ac.be/staff/thisse(micro_licence).pdf.

Troisième partie

Security of supply in the European gas markets

- Chapitre 2 -

Security of supply and retail competition in the European gas market. A theoretical approach.

§ 2.1 Introduction

The security of energy supply is all but a new concern for energy importing countries. However, this concern has certainly been rising in importance since the 1970s. It is not anticipated that this trend is going to stop as an increasing dependence on imported energy is expected in the coming decades (12). Among the different energy sources, natural gas constitutes a particular case that attracts a lot of attention.

In this part, though we explicitly focus on the European situation, the framework developed herein remains general and can be adapted to analyze the situation of large importing countries (such as Japan, South Korea, Taiwan, China, India, to name but a few) without loss of generality. Nowadays, there are several factors at work which explain the rekindling debate on the security of gas supplies in those countries. Firstly, on the supply side : a growing reliance on imports over longer distances is observed and a significant increase in the concentration of foreign supplies is expected for some regions like Europe (4). Secondly, speculation about the future behavior of the Gas Exporting Countries Forum (GECF) refer to a possible cartelization (17). Thirdly, the recent supply interruptions observed in a number of OECD regions (11) suggest that, whatever the causes (international tensions, terrorism or technical hazards impacting unreliable infrastructures), low but positive probabilities of interruption have to be considered as likely risks. And, last but not least, natural gas plays an ever-increasing role in the energy mix : in most OECD countries, natural gas is the fastest growing fuel in the power generation mixes. Given the rigidities of power generation in the short-run, this growing interdependence between gas and electricity also raises concerns about both the security and the reliability of electricity supplies (11).

Before going further, we need to discuss how the downstream part of the gas industry usually manages the possible shortfall in upstream gas. Possible remedies include : large-scale commercial storage, strategic "stockpiles" (if any), re-routing of existing gas flows, increased production from other suppliers that may compensate the shortfall of others. In any case, these instruments might be unavailable. For example, local geological conditions can impede the construction of large underground gas storages (e.g., Belgium), capacity constraints on existing transmission networks can prevent the suitable re-routing of existing gas flows (e.g., South Eastern European countries), local production can be inexistent (e.g., Bulgaria). Until now, strategic stockpiling, a well-known measure implemented to increase the security of oil supplies ((20) ; (18)), has not been viewed as a workable solution in the case of natural gas supplies ((11), pp. 67-83). Now, the possibility to create some kind of precautionary storage is currently being discussed in Europe. However, given the costs of these measures, it is not certain that the stored volumes will be sufficient to fully replace the disrupted supplies. As a result of these disruptions, retailers may have no alternative but to pass along the shortfall to end-users through selective interruptions. In this chapter, we analyze how these disruptions influence the retailers' contracting behavior since they can try to minimize the impact of those interruptions using diversified import sources.

Because of this perceived vulnerability, the security of gas supplies has inspired a huge amount of literature that can be roughly divided into two categories. The first one is by far the largest and gathers all the contributions dominated by purely geopolitical concerns. [1] The second category uses a microeconomic framework to analyze energy security. Apart from some rare contributions (e.g., (15) ; (9) ; (16)), the literature dedicated to the particular case of the gas industry is not tremendously developed. Moreover, most of these contributions refer to a now outdated institutional context. Until the 1990s, the European natural gas industry was subject to government regulations and controls. In most countries, regulated state-owned or state-controlled corporations were responsible for most of the purchase, transport, and sale of natural gas to the distributors. [2] As far as economic analysis is concerned, the decisions of those firms regarding supply security were captured in (15) or (9). From an economic policy perspective, this previous organization was suspected to provide a "cosy arrangement" : import contracts did not matter because the rate-of-return regulation provided a guarantee that costs would be met and, hence, the guarantor would not be potentially stranded (7).

Following the UK's liberalization and privatization reforms of the late 1980s (e.g., (23) ; (19)), a complete transformation of the regulatory regime started in Continental Europe in the early 2000s. Non-discriminatory access provisions to the gas infrastructures (transportation, storage, and LNG terminals) were introduced so as to guarantee equal opportunities to all players (10). As a result, competition emerged among importers, now privately-owned firms. These firms, named retailers, purchase various inputs (gas from local and foreign upstream producers, transport services and services necessary to meet fluctuations in demand) and sell gas to end-users. Customers are no longer committed to any particular retailer, creating the conditions for a competitive rivalry among these firms.

This reform suggests a thought-provoking research question : does competition among gas retailers influence their choices of inputs ? Framed differently, it simply asks for an investigation of retailer's contracting behavior in a gas market dominated by long-term import contracts. How do the retailers' contracting choices influence the market outcomes (gas price, social welfare in the importing market, retailers profits, etc.) regarding the degree of supply insecurity.

In this part, we provide an extension of the models developed by (15) and (9). In these contributions, the authors study the decisions taken by a representative central gas buyer whose objective was to maximize the expected utility of gas consumption net of the purchaser cost of buying gas. The objective functions used here explicitly take into account possible interruptions whose occurrences are captured thanks to perceived probabilities. Both long- and short-run issues were jointly considered. The costs attached to each of these disruption states were valued thanks to short-run consumer surplus concepts while both energy purchases and consumptions under normal conditions were related to the long-run demand curve. Both papers provided a very effective formulation but captured the essence of a now outdated institutional arrangement. Compared to these early papers, we explicitly model retailers as profit-maximizing firms engaged in a Cournot competition. This chapter presents and justifies the theoretical framework developed for analyzing their import diversification strategies. To illustrate the possibilities offered by this model, three

1. For example, several recent articles propose measures of energy security ((21) ; (14) ; (13)).
2. In some countries (France, for example), a legal import monopoly was even granted to one particular firm.

empirical illustrations based on real case studies are successively presented and commented on chapters 3. In the first example, a historical analysis of the German situation in the early 1980s is provided. In the second, the case of South Eastern Europe is studied to analyze the possible disruptions of Russian imports and the consequences on the importer's behavior. In the third one, a comparison between two kinds of market regulations in Spain is provided in order to quantify their impact on the social welfare.

The same notation will be used in chapters 2 and 3.

§ 2.2 Formulation of the problem

2.2.1 Preliminary remarks and notations

As this chapter explicitly addresses the particularities of the Continental European gas industry, some definition is needed to justify the assumptions chosen in our theoretical model. In this work, we assume a Cournot competition among the natural gas retailers of a given country and we study a hypothetical long-run equilibrium. To be more specific, the model corresponds to a static long-run equilibrium in which costs reflect a typical year.

Moreover, our analysis is focused on long-run aspects. The gas infrastructure required to supply gas to end-users is not explicitely modeled. This may be interpreted as assuming a fully accessible gas infrastructure without bottlenecks. This assumption may perfectly reflect European gas infrastructure conditions in the long-run, when short-run regulatory and investment issues are resolved. Thus, the retailer's costs can be summarized as the total cost of the natural gas purchased from the different upstream producers.

We will use these notations :

i index for retail firms in the country under study,
I the set of retailers in the country under study,
j index for upstream gas producers,
J the set of upstream gas producers.

Here we assume that all possible supply disruption states can be enumerated and we simply note Ω the (finite) set of all these random events named ω. For simplicity, the particular state ω of no-disruption is named 0. Whatever the disruption state ω, its occurrence can be appraised thanks to a probability $\theta(\omega)$. Obviously, we have $\sum_{\omega\in\Omega}\theta(\omega) = 1$. We also assume that a consensus exists in the country on both the definition of the discrete set Ω and on the value of the probability of all the different events. Thus, those probabilities constitute common knowledge for the retailers. This assumption seems reasonable as a consensus is generally observed in most importing countries regarding the disruptive nature of the various importing schemes. Therefore, we do not model either the individual firms' subjective perception of the disruption risk, or the difference between real risks and risk perceptions. From a practical perspective, applied procedures like the one presented in (2) can be used to evaluate those probabilities.

We now have to explain how a retailer $i \in I$ acquires its gas. We assume that there are no wholesale markets and the volumes purchased are supposedly entirely obtained thanks to pre-existing bilateral contracts. At first sight, this assumption might look surprising since the pro-competitive move of the early 2000s was expected to be accompanied by the rapid development of wholesale spot markets in Continental Europe (10). But, this emergence has been far slower than expected and the long-term bilateral arrangements are still dominant. The need for a transition period to phase out pre-existing oil products indexed long-term contracts is not a sufficient ground to explain the continuing pre-eminence of these long-term contracts, and industrial observations suggest that retailers are still ready to engage in long-term bilateral trade. Despite early barriers to entry concerns that motivated an in-depth sectoral analysis by the European Commission (5), those long-term arrangements are now fully admitted by the European authorities and all juridical actions against long-term contracts have been withdrawn.[3] According to gas experts, the dominance of long-term supplies is fading in western Europe but this affirmation does not hold for Eastern Europe where the upstream market structure is much more concentrated. Hereafter, we focus on the case of Eastern European gas markets.

In this study, we do not model the competitive interactions among suppliers who compete in both price and quality of their supplies (in this context, quality would be the security of their supplies). As a result, we assume that the upstream prices of natural gas are set exogeneously.[4] Our assumptions are based on the results of the sectoral inquiry led by the European Commission (5). Firstly, gas prices may differ across sources $j \in J$ as evidence suggests that price indexation formulas used in long-term contracts can differ from one producer to another ((5), p. 103, fig. 32). Secondly, the European Commission noted that price indexation formulas are quite homogeneous among buyers located in a given region : either the UK, western or Eastern Europe ((5), p. 104, fig. 33). Thus, we assume no-discriminatory pricing : the price of a given source $j \in J$ is unique and proposed to all the potential buyers $i \in I$. Lastly, this inquiry clarifies the price provisions used in these bilateral long-term arrangements. In these contracts, the price of gas is settled thanks to predetermined indexation formulas that establish a direct linkage with the wholesale spot price of oil products. Given the limited short-run interactions among gas and oil products, we can assume that a disruption of gas supplies has no impact on the prices of oil products and hence on gas prices. Moreover, oil products price uncertainty is not modeled here. Thus, upstream prices are assumed to be constant across all the possible disruption states. In sum, upstream prices can be viewed as an exogeneously determined vector of prices $(p_j)_{j \in J}$, where each component corresponds to the price p_j proposed by the producer j.

The amount of gas purchased by the retailers i from the producer j is named x^0_{ij}. This quantity

3. In fact, the conclusions of this sectoral analysis were published just after the first Russo-Ukrainian dispute. Thus, they emphasize the capability of long-term contracts to provide a workable solution to the well-known "hold up" problem caused by *ex post* opportunism on the supply side.

4. A complete discussion on the fixation of this contractual price can be found in the interesting collection of papers presented in (6).

corresponds to the volume of gas supplied by j to i under a no-disruption state. For a retailer, this quantity can obviously be considered as a decision variable.

Under a given disruption state $\omega \in \Omega$, the subgroup of producers whose supplies are disrupted is named S_ω. The quantity of gas delivered to a retailer i by a gas producer j under a particular disruption state $\omega \in \Omega$ is equal to $x_{ij}^{\omega} = (1 - \delta_{S_\omega}(j))x_{ij}^0$ where $\delta_{S_\omega}(j)$ takes the value 1 if the gas producer j belongs to the collection of disrupted producers S_ω and 0 otherwise. We observe here that the disruption state index $\delta_{S_\omega}(j)$ attached to the producer j does not depend on i, which means that a disruption from this producer corresponds to a total disruption of all the volumes purchased by the different retailers. Stated differently, this means that there is no discrimination among retailers : if a producer decides to cut its supplies and stop deliveries to an infrastructure then those supplies are simultaneously cut for all the retailers. This assumption implies that either for technical or geopolitical reasons, all retailers are affected to the same degree by the disruption. It is important to note that our framework assumes that there is no supply-side response to a disruption : the occurrence of a disruption does not modify the bahavior of the non-disrupted producers. In particular, we do not model the flexibility provisions that can partially relieve the buyers' "Take or Pay"' obligations.

To simplify, the total amount of gas purchased and consumed under a given disruption state $\omega \in \Omega$ is named $x^\omega = \sum_{(i,j)\in I\times J} x_{ij}^{\omega}$. In particular, x^0 is the total volume of gas purchased under a no-disruption state. Similarly, we note $x_i^\omega = \sum_{j\in J} x_{ij}^{\omega}$ the total amount of gas purchased by a given retailer under the state ω.

Added to that, two inverse demand functions are needed. In the following, we first stick to a general formulation and denote : $f(k)$ the long-run willingness to pay for the gas where f is twice differentiable and $f'(k) < 0$, and $g(k,q)$, the short-run willingness to pay for quantity q, parametrically depending on the long-run consumption k. We assume that $g(k,q)$ is twice differentiable with $\partial g/\partial k > 0$ and $g(k,k) = f(k)$, $(\forall k \in \mathbb{R}^{+*})$. Indeed the short-run willingness to pay for the long-run quantity is equal to the long-run willingness. In the rest of the chapter k (respectively q) will denote the long (respectively short)-run quantity of gas. We also use a dumb variable t to denote the long- or short-run volume when needed in an integral. The description of f and g is general, a particular specification of the inverse demand functions will be detailed later on.

2.2.2 A formal representation of disruption costs

In this study, we assume that gas retailers only sign firm supply contracts with their customers. Moreover, we assume that the retail price of gas cannot be adjusted in the case of a sudden short-run disruption of gas supply (cf. the previous presentation of the rigidities of the natural gas industry). Besides, consumers are supposed to ignore the possible occurrence of sudden disruptions. Therefore, they assume that the total contracted amount of gas x^0 will be delivered. Should there be an interruption in deliveries, we assume that a retailer is required to make compensation payments to its disrupted customers (for example, with claims). As we are dealing with brief events, the compensation has to take into consideration the limited responsiveness of the short-run demand. Thus, the corresponding consumer unease can be approximated thanks to the short-run inverse demand function. For a disruption state ω, the total disrupted quantity is $x^0 - x^\omega$ and the corresponding consumers surplus variation is equal to : $\int_{x^\omega}^{x^0} g(x^0, t)dt$.

Of course, retailers are free to decide their upstream supply mixes. The composition of the input mix may thus vary from one retailer to another. In the event of a disruption, requiring the virtuous retailers to pay for the consequences of risky choices made by others would obviously create an incentive for the retailers to select the lowest cost, higher risk choice of input. Such a mechanism is both unjustifiable and unfair. For each disruption case, each retailer's payment to consumers is thus assumed to be set in proportion to its own responsibility in the total disruption. Formally, it means that under a disruption state $\omega \in \Omega \setminus \{0\}$, a given retailer i incurs a positive disruption cost $DC_i(x^0, \omega)$ equal to the payment required to its disrupted consumers :

$$DC_i(x^0, \omega) = \frac{\sum_{j \in J} (x^0_{ij} - x^\omega_{ij})}{x^0 - x^\omega} \int_{x^\omega}^{x^0} g(x^0, t)dt \tag{2.1}$$

Besides, we assume that a retailer is not required to pay the producers involved in S_ω for the disrupted volumes of gas observed under a state $\omega \in \Omega \setminus \{0\}$. Under that particular state, retailer i's profits are thus equal to the profits earned under the no-disruption state named 0, minus the disruption costs $DC_i(x^0, \omega)$ plus $\sum_{j \in S_\omega} p_j x^0_{ij}$.

2.2.3 The model

This section presents the agents' objectives. We reiterate that we need two inverse demand functions. The first one, $f(k)$ is the long-run willingness to pay for the gas. The second, $g(k, q)$ is the short-run willingness to pay for quantity q, parametrically depending on the long-run consumption k. In the following, we use a dumb variable t to denote the long- or short-run volume when needed in an integral.

Consumer : here, the decisions of the end-users are based solely on the retail price of gas named $P^\star$. We assume that gas end-users strive to maximize the value received from consumption minus the payments to retailers, assuming they cannot affect $P^\star$. Besides, they do not take into account the propensities of possible sudden disruptions. This assumption seems consistent with the industrial reality since most end-users completely ignore the details of the supply mix decided by the retailers and know almost nothing about the origin of the natural gas they are burning. As a result, their decisions cannot consider these disruption states. This behavior is thus represented by :

$$\begin{aligned} CONS(P^\star) : \quad & \underset{\{k\}}{Max} \quad \int_0^k f(t)dt - P^\star k \\ & \qquad k \geq 0 \end{aligned}$$

If the problem has an interior solution it is characterized with levels of consumption k by : $f(k) = P^{\star}$.

Gas retailer : here, we model the contracting behavior of a risk-neutral firm. To keep the model simple, we will not consider the case of a risk-averse firm. Thus, its optimization problem is to choose a purchase policy $(x^0_{ij})_{j\in J}$ under a no-disruption state so as to maximize its expected profit across all possible disruption states. Since we do not model possible recourse actions in case of disruption, the only decision variables are the contractual long-term volumes decided by the retailers.

$RETAILER_i$:

$$\underset{\left\{x^0_{ij},\, j\in J\right\}}{Max} \quad \bar{\Pi}_i(x^0_{ij}, (x^0_{lj})_{l\neq i}) = \sum_{j\in J}\left(f(x^0) - p_j\right) x^0_{ij} - \sum_{\omega\in\Omega\setminus\{0\}} \theta(\omega)\left(DC_i(x^0,\omega) - \sum_{j\in S_\omega} p_j x^0_{ij}\right)$$

$$x^0_{ij} \geq 0 \quad (\forall j \in J)$$

To simplify, the retailer's i expected profits can hence be rewritten as follows : $\bar{\Pi}_i(x^0_i) = A+B+C$ where :

$$A = \sum_{j\in J}\left(f(x^0) - p_j\right) x^0_{ij} \tag{2.2}$$

$$B = -\sum_{\omega\in\Omega\setminus\{0\}} \theta(\omega) DC_i(x^0,\omega) \tag{2.3}$$

$$C = \sum_{\omega\in\Omega\setminus\{0\}} \theta(\omega) \sum_{j\in J} p_j x^0_{ij} \delta_{S_\omega}(j) \tag{2.4}$$

The partial derivative of $\bar{\Pi}_i$ with respect to the decision variable x^0_{ik} is given in Appendix 1.

If the problem has an interior solution, the associated KKT conditions are :

$$\text{For } x^0_{ik} : \quad 0 \leq x^0_{ik} \quad \perp \quad \frac{\partial \bar{\Pi}_i}{\partial x^0_{ik}}(x^0_i) \leq 0 \tag{2.5}$$

where the derivative $\dfrac{\partial \bar{\Pi}_i}{\partial x^0_{ik}}(x^0_i)$ is given in Appendix 1. Once the KKT conditions are written, it is possible to solve the model and find the traders' strategic import choices.

§ 2.3 The model's application

The framework at hand seems suitable for capturing the key elements of some of the situations observed in the European natural gas industry. To illustrate this capability, it is worthwhile to choose a particular functional form for the long- and short-term inverse demands. In this section, we present some illustrations based on an iso-elasticity assumption for both the short-run and the long-run inverse demand functions. The long-run (respectively short-run) price elasticity is named ϵ_0 (respectively ϵ_1).

2.3.1 The iso-elasticity assumption

Here we follow (15) and (9) by assuming that, in the long-run, the inverse demand function is $f(k) = ak^{-\frac{1}{\epsilon_0}}$ where a is a constant parameter and k represents the long-run consumption amount. As a result, the short-run demand function associated with this particular long-run consumption k is given by $g(k,q) = ak^{\frac{1}{\epsilon}}q^{-\frac{1}{\epsilon_1}}$ where q is the amount of natural gas effectively consumed in the short-run and ϵ is a parameter defined so that :

$$\forall k \in \mathbb{R} \quad g(k,k) = f(k). \tag{2.6}$$

Thus, we have

$$\frac{1}{\epsilon} = \frac{1}{\epsilon_1} - \frac{1}{\epsilon_0} \tag{2.7}$$

We also assume that the long-run inverse demand is more elastic than the short-run one, *i.e.*, $\epsilon_0 > \epsilon_1$.
After some algebraic developments, we derive the KKT conditions for each retailer i :

$$\forall k \in J, \quad 0 \leq x^0_{ik} \perp (\alpha + \beta + \gamma + \eta) \leq 0 \tag{2.8}$$

where

$$\alpha = x^0 - \frac{1}{\epsilon_0} x^0_i \tag{2.9}$$

$$\beta = -p_k \left(1 - \Theta(k)\right) \frac{x0^{\left(1+\frac{1}{\epsilon_0}\right)}}{a} \tag{2.10}$$

$$\begin{aligned} \gamma = \quad & -x0^{\left(\frac{1}{\epsilon_1}\right)} \frac{\epsilon_1}{\epsilon_1 - 1} \frac{1}{\epsilon} \sum_{\omega \in \Omega} \theta(\omega) \frac{x^0_i - x^\omega_i}{x^0 - x^\omega} \left(x0^{\left(-\frac{1}{\epsilon_1}+1\right)} - x\omega^{\left(-\frac{1}{\epsilon_1}+1\right)} \right) \\ & -x0^{\left(\frac{1}{\epsilon_1}+1\right)} \sum_{\omega \in \Omega \setminus k \notin S_\omega} \theta(\omega) \frac{x^0_i - x^\omega_i}{x^0 - x^\omega} \left(x0^{\left(-\frac{1}{\epsilon_1}\right)} - x\omega^{\left(-\frac{1}{\epsilon_1}\right)} \right) \end{aligned} \tag{2.11}$$

$$\begin{aligned} \eta = \quad & -x0^{\left(\frac{1}{\epsilon_1}+1\right)} \frac{\epsilon_1}{\epsilon_1 - 1} \sum_{\omega \in \Omega \setminus k \in S_\omega} \theta(\omega) \frac{(x^0 - x^\omega) - (x^0_i - x^\omega_i)}{(x^0 - x^\omega)^2} \left(x0^{\left(-\frac{1}{\epsilon_1}+1\right)} - x\omega^{\left(-\frac{1}{\epsilon_1}+1\right)} \right) \\ & -x_0 \sum_{\omega \in \Omega \setminus k \in S_\omega} \theta(\omega) \frac{x^0_i - x^\omega_i}{x^0 - x^\omega}. \end{aligned} \tag{2.12}$$

Here, $\Theta(k)$ is simply $\sum_{\{\omega \in \Omega, k \in S_\omega\}} \theta(\omega)$, the overall probability that producer k cuts its supplies.

This setting allows us to study some interesting situations observed in the European natural gas industry. The coming chapter present some of these case studies.

§ 2.4 Conclusion

The main goal of this chapter is to study the impacts on the natural gas market of supply disruption risks. For that purpose, we develop a static model (over a typical period of one year) based on a Cournot game between different retailers who buy gas from possibly risky producers and bring it onto the market. The previous models found in the literature do not take into account the current economic situation of the energy markets in Europe because they assume a pure and perfect competition structure. Since their liberalization, an oligopolistic description that takes into consideration market power exerted through the gas chain is more suitable to study the European natural gas markets. In our model, the upstream market is represented as follows : the retailers sign long-term contracts with producers (e.g., Gazprom) that fix the gas selling price. We take into account the recent market liberalization by assuming that all the retailers have the same access to transport means. We also suppose that producers sell their gas at the same price to all the retailers. In the downstream market, the retailers' interaction is modeled by a Cournot game, with an assumption of market transparency, when all the actors maximize their expected profit, taking into consideration specific disruption costs they have to pay to consumers in the event of supply interruption from risky producers. Disruption costs can be quantified by introducing a short-run demand function. We were able to study in details some particular western European markets by making an iso-elasticity assumption on the long- and short-run inverse demand functions.

The following chapter presents some of the possible applications of our model.

- Chapitre 3 -

Security of supply and retail competition in the European gas market. Cases studies.

§ 3.1 Introduction

The previous chapter presented a Nash-Cournot model in order to understand and study the natural gas trade under disruptions risks. This model has been applied to three concrete situations in Europe. The first one focuses on the German natural gas market of the 1980s, where Ruhrgas had the possibility to import gas from the USSR and Norway. Knowing the geopolitical climate of that period (cold war), we have assumed that Norway is a secure producer whereas the USSR a risky one, who can interrupt its gas supplies with time. Our theoretical framework is therefore well suited to describe such a situation. A social welfare and firms's profit analysis are carried out in order to understand Ruhrgas's strategic behavior in that time. The second case study is of particular interest for the current southwestern Europe gas situation. The Bulgarian gas market, which is mainly dependent on the Russian supplies, can be described thanks to our model, where Russia is perceived as a risky supplier. We derive in particular interesting conclusions regarding the necessity to regulate the market, via volumes control, as a function of the disruption probability for social welfare optimization concerns. The last case study concerns the current Spanish market regulation : for supply diversification purposes, the Spanish government imposes on each gas trader not to buy from any producer more than 60% of the total amount purchased. However, with the Medgaz project, the Algerian gas exports toward Spain are expected to increase significantly in the forthcoming decades, which will probably bring the market to its regulation limits. Our model has been applied to describe such a regulation in Spain, taking into account the dominance of the Algerian supply.

In this chapter, we will use chapter 2's notation.

§ 3.2 Case 1 : The German situation in the 1980s

The study (9) is the first work to analyze the diversification issue in Continental Europe before the liberalization reforms described earlier. But even if we limit ourselves to the situation observed during the mid-1980s, there could be some doubt on the ability of this model to fully represent the situation observed in the Federal Republic of Germany (FRG), the largest gas importing country in Europe at that time. In (9), a representative gas buyer decides jointly its purchase of gas and its long-run capacity level so as to maximize the expected utility of gas consumption net of the purchaser cost of buying gas. Such an argument seemed reasonable for countries where price regulation consciously limited the profitability of monopoly importers. As was the case for Distrigaz in Belgium or Gaz de France ((22), p.99). But in the FRG, Ruhrgas AG, a privately-owned firm, was not explicitly regulated and earned comfortable profits. [1] As mentioned above, these early models posited a quasi-virtuous behavior for the importer, an assumption that hardly captures Ruhrgas's

1. Ruhrgas returned a net profit of between 16% and 19% of its own capital between 1984 and 1988. Those profit levels were particularly comfortable compared to those exhibited by both Distrigas and Gaz de France ((22), p.99).

past behavior.[2] A profit-maximizing behavior looks more appropriate to model Ruhrgas at that time.

In the following, we study the decisions made by Ruhrgas in the early-1970s regarding future imports planned for the 1980s. At that time, Ruhrgas knew that the small volumes of natural gas produced in the FRG and the much larger volumes of gas imported from the Netherlands would be insufficient to serve the future demand. Those volumes had already been purchased under pre-existing long-term bilateral agreements and were considered as both known and fixed in the coming decade. Thus, imports from two resource-rich countries, Norway and the USSR, had to be considered to serve this future demand. Here, we assume perfect foresight and apply the previous model to analyze Ruhrgas's decision. Ruhrgas's objective was to select its import policy so as to maximize its expected profit for a typical year in the 1980s.

We assume that there is only one large retailer, $I = \{1\}$. For simplicity, the index $i = 1$ is dropped in the following formulas. The volumes coming from either the Netherlands or the local FRG production are assumed to be kept constant whatever the circumstances and are simply named l. The supplies from these two sources located within the EEC were perceived as secure. Both are thus characterized by a zero probability of a disruption. Hence, the Ruhrgas decision can be simplified as choosing the imported volumes $(x_j)_{j \in J}$ from a set of two sources $J = \{1, 2\}$ where Norway is indexed 1 and the USSR is indexed 2. We assume that both for Norway and the USSR, there is a non-negligible risk of disruptive behavior. We denote by θ_1 (respectively θ_2) the disruption probability of Norway (respectively the USSR) and p_1, p_2 the prices charged by these producers. For Ruhrgas, the optimization problem is :

$$\begin{array}{ll} \text{Max} & \bar{\Pi}(x_1, x_2) \\ & x_1 \geq 0 \quad x_2 \geq 0 \end{array}$$

where

$$\begin{aligned} \bar{\Pi}(x_1, x_2) = \quad & f(x_0 + l)(x_0 + l) - p_1 x_1 - p_2 x_2 - \theta_1(1 - \theta_2) \int_{x_2+l}^{x_0+l} g(x_0, t)dt \\ & -\theta_2(1 - \theta_1) \int_{x_1+l}^{x_0+l} g(x_0, t)dt - \theta_1\theta_2 \int_{l}^{x_0+l} g(x_0, t)dt \\ & +\theta_1(1 - \theta_2)p_1 x_1 + \theta_2(1 - \theta_1)p_2 x_2 + \theta_1\theta_2(p_1 x_1 + p_2 x_2) \end{aligned} \tag{3.1}$$

x_1 (resp. x_2) is the quantity bought by the retailer from Norway (resp. the USSR) and $x_0 = x_1 + x_2$. The local production costs are assumed to be well-known. Hence, the variable l is not a decision variable. In the iso-elasticity context, we can easily calculate $\Pi(x_1, x_2)$.

2. Ruhrgas's prices were so high at that time that BASF, the largest gas user in Germany, decided to actively search for alternative supplies to bypass the monopoly. This situation led BASF to create an alternative gas retailer, Wingas (established as a joint-venture with the Russian Gazprom), and led them to play a major role in the construction of an import infrastructure between Russia and Germany (24).

$$
\begin{aligned}
\bar{\Pi}(x_1,x_2) = \quad & \mu(x_0+l)^{-\frac{1}{\epsilon_0}+1} \\
& +\nu(x_0+l)^{\frac{1}{\epsilon}}\left(\theta_1(1-\theta_2)(x_2+l)^{-\frac{1}{\epsilon_1}+1}+\theta_2(1-\theta_1)(x_1+l)^{-\frac{1}{\epsilon_1}+1}+\theta_1\theta_2 l^{-\frac{1}{\epsilon_1}+1}\right) \\
& -(1-\theta_1)p_1x_1-(1-\theta_2)p_2x_2
\end{aligned} \tag{3.2}
$$

where

$$\mu = a\left(1-(\theta_1+\theta_2-\theta_1\theta_2)\frac{\epsilon_1}{\epsilon_1-1}\right) \tag{3.3}$$

$$\nu = a\frac{\epsilon_1}{\epsilon_1-1}. \tag{3.4}$$

We can show that the profit is a strictly concave function of the variables x_1 and x_2 [3]. Hence the uniqueness of the solution is guaranteed, because the feasibility set is convex.
The profit's gradient depends on the variables as follows :

$$
\begin{aligned}
\frac{\partial\bar{\Pi}}{\partial x_1}(x_1,x_2) = \quad & \left(1-\frac{1}{\epsilon_0}\right)\mu(x_0+l)^{-\frac{1}{\epsilon_0}} \\
& +\frac{\nu}{\epsilon}(x_0+l)^{\frac{1}{\epsilon}-1}\left(\theta_1(1-\theta_2)(x_2+l)^{-\frac{1}{\epsilon_1}+1}+\theta_2(1-\theta_1)(x_1+l)^{-\frac{1}{\epsilon_1}+1}+\theta_1\theta_2 l^{-\frac{1}{\epsilon_1}+1}\right) \\
& +\nu(x_0+l)^{\frac{1}{\epsilon}}\left(1-\frac{1}{\epsilon_1}\right)\theta_2(1-\theta_1)(x_1+l)^{-\frac{1}{\epsilon_1}}-(1-\theta_1)p_1
\end{aligned} \tag{3.5}
$$

$$
\begin{aligned}
\frac{\partial\bar{\Pi}}{\partial x_2}(x_1,x_2) = \quad & \left(1-\frac{1}{\epsilon_0}\right)\mu(x_0+l)^{-\frac{1}{\epsilon_0}} \\
& +\frac{\nu}{\epsilon}(x_0+l)^{\frac{1}{\epsilon}-1}\left(\theta_1(1-\theta_2)(x_2+l)^{-\frac{1}{\epsilon_1}+1}+\theta_2(1-\theta_1)(x_1+l)^{-\frac{1}{\epsilon_1}+1}+\theta_1\theta_2 l^{-\frac{1}{\epsilon_1}+1}\right) \\
& +\nu(x_0+l)^{\frac{1}{\epsilon}}\left(1-\frac{1}{\epsilon_1}\right)\theta_1(1-\theta_2)(x_2+l)^{-\frac{1}{\epsilon_1}}-(1-\theta_2)p_2.
\end{aligned} \tag{3.6}
$$

We cannot find simple analytical expressions of the optimal imports x_1 and x_2 to guarantee a maximum benefit for the German company. Hence, we have to use numerical means to solve our two-dimensional problem. Let's assume for instance that $\theta_1 = 0$, which is to say that the Norwegian supply is secure and $\theta_2 > 0$. It would be interesting to study the economic conditions that make the German retailer choose its supplies exclusively from the secure supplier. These conditions obviously take into account the relative gas prices and the disruption probability. We can derive from this situation simple conditions that ensure the equilibrium gas amount to be $x_1^{eq} > 0$ and $x_2^{eq} = 0$. In that situation, using the KKT theorem, we can derive that :

3. The demonstration is numerical and uses the following values : $\epsilon_0 = 1.2$, $\epsilon_1 = 0.3$. The use of these values will be justified later.

$$\begin{array}{l} (x_0+l)^{eq} = (x_1+l)^{eq} \\ \frac{\partial \bar{\Pi}}{\partial x_1}(x_1^{eq}, x_2^{eq}) = 0 \\ \frac{\partial \bar{\Pi}}{\partial x_2}(x_1^{eq}, x_2^{eq}) \leq 0. \end{array} \tag{3.7}$$

Hence, we can calculate x_1^{eq} and find conditions on the parameters θ_2, p_1 and p_2 so that $x_2^{eq} = 0$:

$$\begin{array}{l} x_1^{eq} + l = \left(\frac{p_1}{a\left(1-\frac{1}{\epsilon_0}\right)}\right)^{-\epsilon_0} \\ l \leq \left(\frac{p_1}{a\left(1-\frac{1}{\epsilon_0}\right)}\right)^{-\epsilon_0} \\ (1-\theta_2)\left(p_1 - p_2\left(1-\frac{1}{\epsilon_0}\right)\right) \leq \frac{p_1}{\epsilon_0} \end{array} \tag{3.8}$$

Therefore, if the Norwegian supply is assumed to be secure and the local demand such as $l \leq \left(\frac{p_1}{a\left(1-\frac{1}{\epsilon_0}\right)}\right)^{-\epsilon_0}$, no Soviet gas is to be brought to FRG if (and only if) :

$$\begin{array}{ll} p_2 > \frac{p_1}{1-\frac{1}{\epsilon_0}} & \text{the Soviet Gas is too expensive or} \\ p_2 \leq \frac{p_1}{1-\frac{1}{\epsilon_0}} \text{ and } \theta_2 > \theta_2^{lim} = 1 - \frac{p_1}{\epsilon_0\left(p_1 - p_2\left(1-\frac{1}{\epsilon_0}\right)\right)} & \text{the Soviet supply is too risky.} \end{array} \tag{3.9}$$

We can now run some numerical simulations for a given set of values for the problem's parameters. Here, the following values were used : $\epsilon_0 = 1.2$, $\epsilon_1 = 0.3$, $a = 10$ and $l = 0.04$ in arbitrary units. The values of the long- and short-run elasticities are those used in (15).

To keep the discussion general, this numerical study has been conducted using arbitrary units for the prices and volumes.

Figure 3.1 gives the evolution of θ_2^{lim} over the Norwegian gas price p_1 for $p_2 = 5$ in arbitrary units. This function increases with the price p_1, for it may become interesting to buy risky gas if the secure option becomes very expensive.

Figure 3.2 gives the evolution of the amounts x_1^{eq} and x_2^{eq} over θ_2 for $p_1 = 6$, $p_2 = 2$, in arbitrary units. θ_1 takes the value 0. It is reasonable to assume that the secure gas is more expensive than the insecure one. Otherwise Germany would not have any incentive to purchase the riskiest gas. [4]

For $\theta_1 = 0$, we notice that if the probability of a Soviet disruption remains moderate ($\theta_2 < 0.12$), then the Soviet gas becomes attractive and has a higher share in the Ruhrgas supply mix.

4. Obviously, the validity of this assertion is subject to the availability of an appropriate transmission infrastructure. This point lies beyond the scope of this study that assumes no infrastructure constraints.

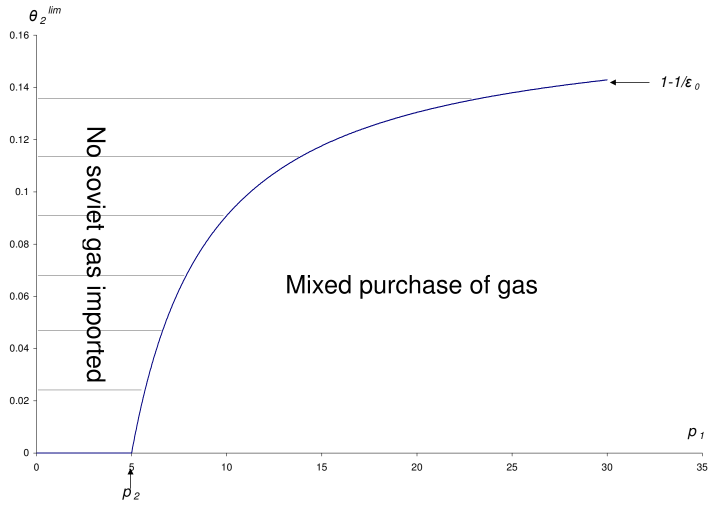

FIGURE 3.1 – *Evolution of θ_2^{lim} over p_1 (arbitrary unit). $p_2 = 5$ (arbitrary unit), $\theta_1 = 0$.*

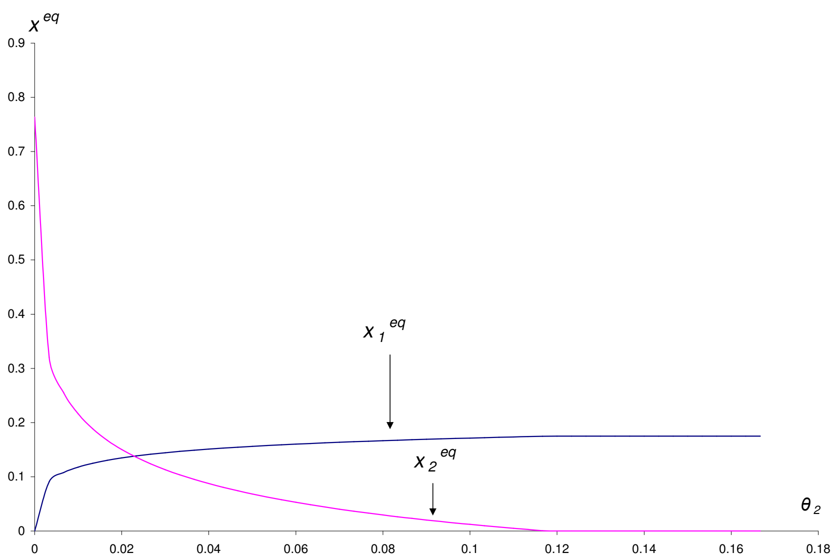

FIGURE 3.2 – *Evolution of $x_{1,2}^{eq}$ over θ_2 (arbitrary unit). $p_1 = 6$, $p_2 = 2$, $\theta_1 = 0$ or 0.1.*

Whereas, if $\theta_2 > 0.12$, the cost of the possible disruptions induces a relative shift towards the Norwegian gas and the Soviet gas becomes too risky ($x_2^{eq} = 0$). In that situation, the amount bought from Norway no longer depends on the disruption probability θ_2.

Figure 3.3 represents the dependence of the gas price in the FRG market on the disruption probability of the Soviet gas θ_2 for $\theta_1 = 0$, $p_1 = 6$, and $p_2 = 2$.

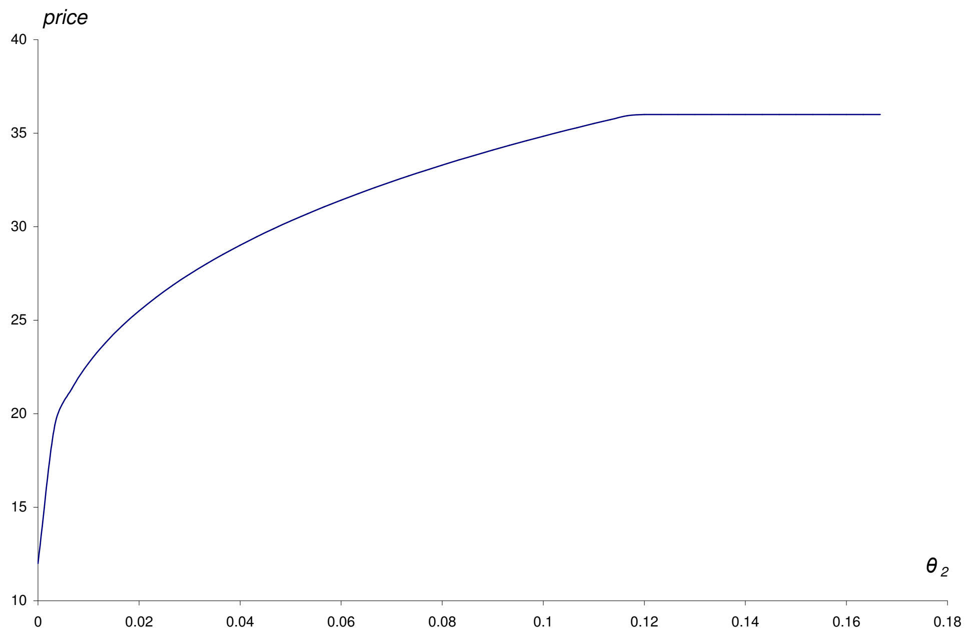

FIGURE 3.3 – *Evolution of the price over θ_2 (arbitrary units). $p_1 = 6$, $p_2 = 2$, $\theta_1 = 0$.*

Obviously, the price charged by the retailer increases with θ_2 to balance the possible impact of any gas disruption and reduce its inherent costs. Besides, for $\theta_2 > 0.12$, the retailer does not buy anymore gas from the USSR and there is, in that case, no risk of disruption. Hence, the price in the market no longer depends on θ_2. However, one can wonder whether it would be better for Ruhrgas to deal with risky producers if their selling price is low. Therefore, it may be interesting to study the impact of disruption on the retailer's profit and on the social welfare observed in the FRG. The social welfare obtained in West Germany W_{FRG} can be measured as the sum of the surplus obtained by the German consumers S_c and the profit obtained by the sole retailer :

$$W_{FRG}(x_1, x_2) = S_c(x_1, x_2) + \bar{\Pi}(x_1, x_2) \tag{3.10}$$

where the consumer surplus is :

$$S_c(x_1, x_2) \quad = \int_0^{x_0+l} f(t)dt - f(x_0 + l)(x_0 + l) \tag{3.11}$$

Therefore :

$$W_{FRG}(x_1, x_2) = a\frac{1}{\epsilon_0 - 1}(x_0 + l)^{1-\frac{1}{\epsilon_0}} + \bar{\Pi}(x_1, x_2). \tag{3.12}$$

The retailer's profit is given by expression (3.2).

Figure 3.4 shows how the retailer's profit and the social welfare evolve with θ_2 when $p_1 = 6$, $p_2 = 2$ (arbitrary units) and $\theta_1 = 0$.

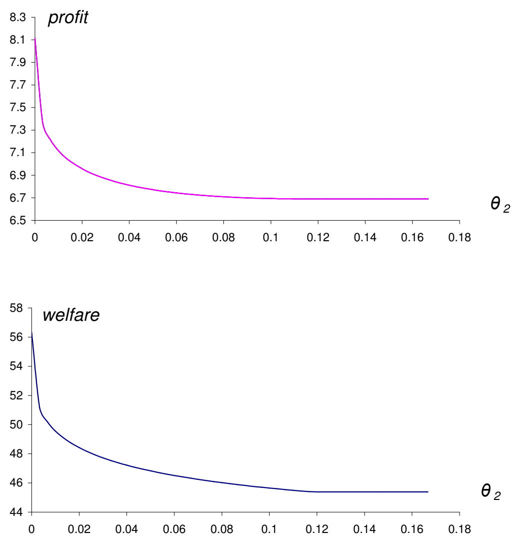

FIGURE 3.4 – *Evolution of the profit and social welfare over θ_2 (arbitrary unit). $p_1 = 6$, $p_2 = 2$ (arbitrary units), $\theta_1 = 0$.*

The profit decreases with the disruption probability, which suggests that it is better for the retailer to deal with secure gas suppliers. This preference is also suitable for the consumer : the social welfare decreases with the disruption probability.

It is now time to make a comparison between our model and the situation studied in (15). In this paper, Ruhrgas is described as a social welfare-maximizing firm. We can easily study this situation in our iso-elasticity framework : the retailer optimization program is given as follows :

$$\begin{array}{ll} \text{Max} & W_{FRG}(x_1, x_2) = \frac{a}{\epsilon_0 - 1}(x_0 + l)^{1-\frac{1}{\epsilon_0}} + \bar{\Pi}(x_1, x_2) \\ & x_1 \geq 0,\ x_2 \geq 0 \\ & \bar{\Pi}(x_1, x_2) \geq 0. \end{array}$$

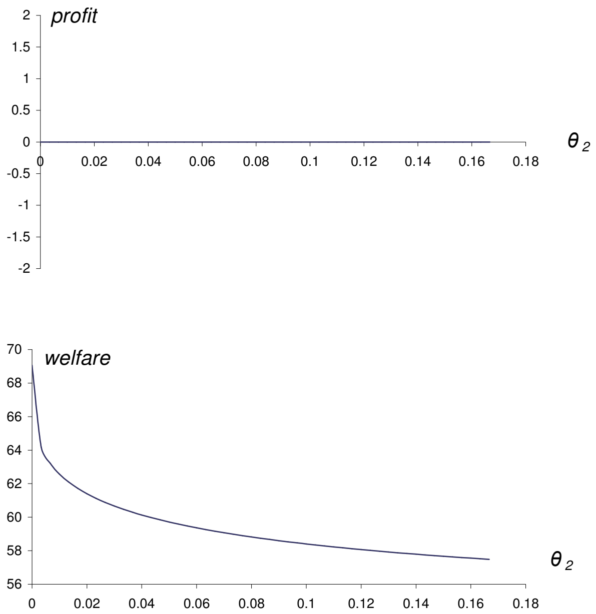

FIGURE 3.5 – *Evolution of the profit and social welfare over θ_2 (arbitrary unit). $p_1 = 6$, $p_2 = 2$ (arbitrary units), $\theta_1 = 0$, welfare-maximizing agent.*

Figure 3.5 gives the evolution of Ruhrgas's profit $\bar{\Pi}$ and the social welfare W_{FRG} over the Russian disruption probability θ_2. Here, we notice that the retailer's profit is always equal to 0 and social welfare decreases with the disruption probability. Therefore, since it is known that Ruhrgas earned a significant profit in the 1980s ((22)), it is more reasonable to model its behavior as a profit-maximizing firm, as we did thanks to our study.
Figure 3.6 gives the evolutions of the equilibrium quantities x_1^{eq} and x_2^{eq} over the disruption probability θ_2 in the social welfare maximizer framework. The main difference one can notice in comparison to the profit-maximizing situation is that there is no threshold effect. Indeed, there is always some risky gas which is imported even if the disruption probability is high. However x_2^{eq} decreases with θ_2.

An interesting lesson can be derived from this analysis : the import behavior of a tightly regulated monopoly significantly differs from the one chosen by a profit-maximizing one.

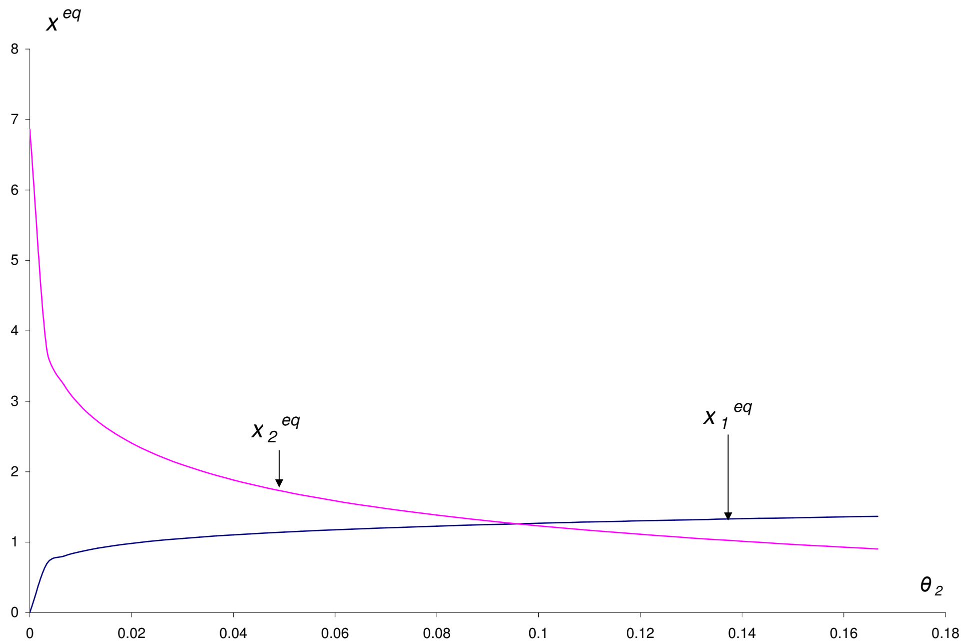

FIGURE 3.6 – *Evolution of the x_1^{eq} and x_2^{eq} over θ_2 (arbitrary unit). $p_1 = 6$, $p_2 = 2$ (arbitrary units), $\theta_1 = 0$, welfare-maximizing agent.*

§ 3.3 Case 2 : The Bulgarian situation

During the Russo-Ukrainian gas dispute of January 2009, the transit of Russian gas to Europe was cut for nearly two weeks. By far the most serious consequences were observed in the Balkans where some countries experienced an emergency situation, with parts of the population unable to heat their homes. On top of the intense emotion created by this quasi humanitarian crisis, this event reactivated a debate on the regulatory reforms needed for those countries.

In the Balkans, the regulatory framework of the natural gas industry is undergoing radical reforms with the aim of implementing the EU legislation on energy and competition. [5] A separation

5. This is the explicit goal of the Southeast Europe Energy Community Treaty that came into force on July 1, 2006.

between regulated infrastructure-related activities and retail activities similar to the one currently at work in Western Europe is expected.

Some pertinent insights for the natural gas market can be obtained from our model. Until now, the Bulgarian gas industry has been dominated by Bulgargaz Plc, the state-owned gas company, which holds a monopoly on the transmission and distribution of natural gas throughout the country.

There is currently increasing concern about potential threats to the security of gas supply for this country in the coming decade. In fact, Bulgaria is characterized by a huge dependence upon imports from a single large supplier (Russia) and the country's gas demand is expected to grow strongly alongside its economic transition. As a result, there is a sound debate about the possibility of creating new import infrastructures that would connect Bulgaria and other Southeast European countries to new sources of gas located either in the Caspian area or in Western Europe. Given the huge uncertainties attached to these projects, it is worthwhile to consider a benchmark scenario based on a continuing total dependence on Russian imports.

Thanks to the previous model, this case is relatively easy to analyze as follows. Here, we assume that n retailers are competing to serve the Bulgarian gas market. These firms have a reduced choice and can only purchase their gas from a unique producer : Gazprom, the Russian gas company. Hence, with our notations, the sets I and J are $I = \{1, 2, ..., n\}$ and $J = \{1\}$. Let's denote by x_i the amount of natural gas bought by the firm i. x^0 denotes also the total quantity sold by the producer $x^0 = \sum_{i=1}^{n} x_i$ and θ the probability that Russia cuts its production, either for technical, economical or political reasons. The price charged by the producer is p, the elasticity values for the short- and long-run demands are respectively $\epsilon_1 = 0.3$ and $\epsilon_0 = 1.2$.[6] Besides, we give arbitrary values for the other exogeneous parameters : $a = 1$ and $p = 1$ in arbitrary units. We assume that in case of disruption, there are some *force majeure* provisions that allow the import of gas from neighboring countries. We will denote by c this minimum gas quantity in Bulgaria in the event of disruption. The maximization problem can thus be written for each firm i :

$$\begin{aligned} \text{Max} \quad & (f(x_0) - p)x_i - \theta \tfrac{x_i}{x_0} \int_c^{x_0} g(x_0, t)dt + \theta p x_i \\ & x_i \geq 0 \end{aligned} \tag{3.13}$$

We denote by Π each firm's profit : $\Pi(x_i) = (f(x_0) - p)x_i - \theta \frac{x_i}{x_0} \int_c^{x_0} g(x_0, t)dt + \theta p x_i$. Assuming that the natural gas demand takes an iso-elastic functional form, we have

$$\Pi(x_i) = a x_0^{-\frac{1}{\epsilon_0}} x_i \left(1 - \theta \frac{\epsilon_1}{\epsilon_1 - 1}\right) + \theta a \frac{\epsilon_1}{\epsilon_1 - 1} c^{-\frac{1}{\epsilon_1}+1} x_i x_0^{\frac{1}{\epsilon}-1} - p(1-\theta)x_i \tag{3.14}$$

6. The review of empirical studies presented in (9) supports this assumption of an elasticity value greater than one for the long-run price elasticity of the natural gas demand in a European country.

To simplify our expressions, we call

$$\alpha = a\left(1-\theta\frac{\epsilon_1}{\epsilon_1-1}\right) \tag{3.15}$$

$$\beta = \theta a\frac{\epsilon_1}{\epsilon_1-1}c^{-\frac{1}{\epsilon_1}+1} \tag{3.16}$$

The first-order conditions calculation gives :

$$\frac{\partial\Pi}{\partial x_i}(x_i) = \alpha x_0^{-\frac{1}{\epsilon_0}-1}\left(x_0-\frac{x_i}{\epsilon_0}\right)+\beta x_0^{\frac{1}{\epsilon}-2}\left(x_0+\left(\frac{1}{\epsilon}-1\right)x_i\right)-p(1-\theta) \tag{3.17}$$

We can show that the function $\Pi(x_i, x_j, j \neq i)$ where the variable is x_i and x_j, $j \neq i$ are considered constant is concave. The uniqueness of an optimum for each firm is thus guaranteed. The demonstration of the profit's concavity is given in Appendix 6.

Appendix 2 gives the technical study of the dependence of the gas volume and price in the Bulgarian market over the problem's parameters.

Figure 3.7 gives the evolution of the natural gas price in the market, over the number of retailers n, for $\theta = 0.15$ and $c = 0.4$ in arbitrary units.

As expected, the price decreases with the number of retailers as stringent competition leads to cheaper products and smaller profits. We notice that the price converges towards a finite value p_∞, that can be calculated. For this purpose, we need to study the convergence of the sequence $nx_{eq}(n)$ when $n \longrightarrow \infty$. This study is carried out in Appendix 3.

Figure 3.8 shows how p_∞ evolves with θ for $c = 0.4$ (arbitrary unit). We already know that in the case of completely secure supply (i.e. $\theta = 0$), the standard pure and perfect competition study allows us to assert that the market price converges towards the producer's price p when n is large enough. Our model arrives at the same conclusion : indeed, when $\theta = 0$, we can easily calculate $p_\infty(0) = p$.

The conclusion we can draw from the pure and perfect competition situation is quite interesting : if the alternative imports capacity is low enough (which is quite realistic for the current Bulgarian situation) and the number of trading firms is large, insecure supplies make the gas retail price higher than the import price, which obviously decreases the consumers utility, even if consumers are compensated if disruption occurs. This indicates that, added to the "oligopolistic margin", there exists a "security margin" charged by the retailers to compensate the disruption costs they have to support in the event of supply failure. This "security margin" increases with the disruption risk θ. This study illustrates how the disruption costs are passed along to consumers : the consumer surplus is thus a decreasing function of the disruption risk.

As far as retailers' profit is concerned, we can prove that the industry's total expected profit is nought and does not vary with θ, in the pure and perfect competition situation. A formal proof

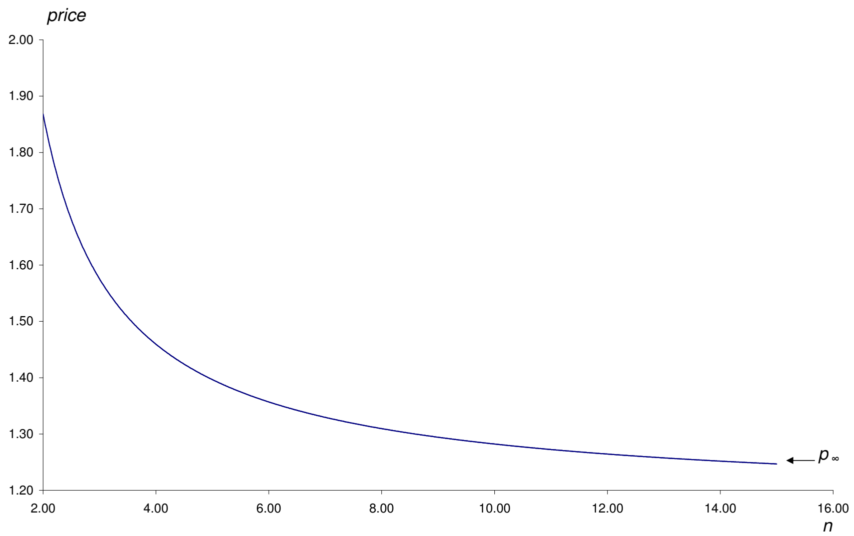

FIGURE 3.7 – *Evolution of the gas price in the market over n. $\theta = 0.15$, and $c = 0.4$ in an arbitrary unit.*

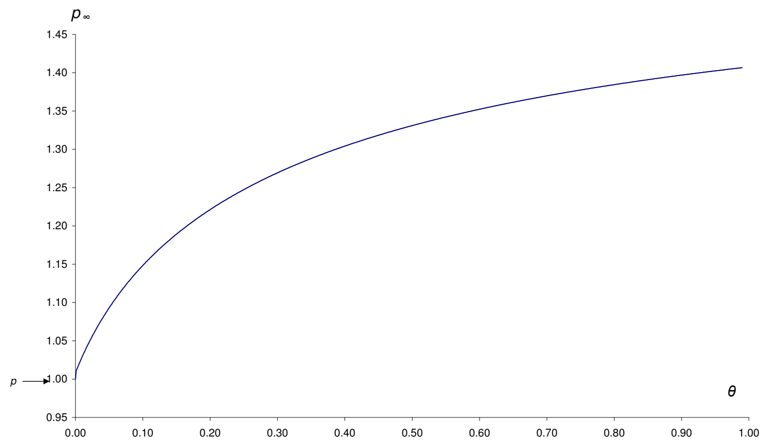

FIGURE 3.8 – *Evolution of p_∞ (arbitrary unit) over θ. $c = 0.4$ in an arbitrary unit.*

of this result is given in Appendix 5. Therefore, the total revenue derived from the nonnegative difference between import and retail price is exactly equal to the expected total disruption cost. As a result, the national welfare of this importing country is a decreasing function of the disruption probability θ.

Figure 3.9 gives the evolution of the price over the disruption probability θ for $n = 6$ and $c = 0.4$ in arbitrary units.

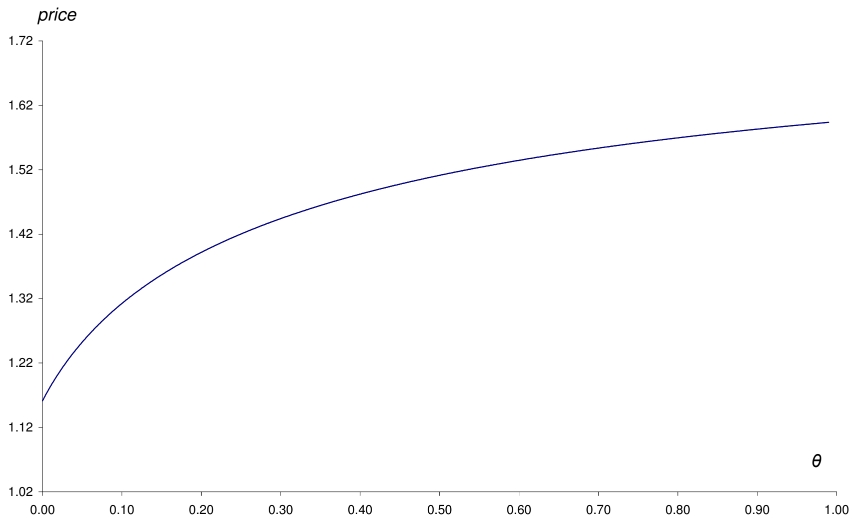

FIGURE 3.9 – *Evolution of the gas price in the market (arbitrary unit) over θ. $n = 6$ and $c = 0.4$ in an arbitrary unit.*

The price increases with the probability θ because if the supplier is not secure, the retailers need to charge a high natural gas price in order to ensure their long-run profit, so that they can compensate the loss due to any disruption, which can occur quite frequently.

Let's study now the impact of any disruptive behavior on the gas amount imported to the Bulgarian market. We also study the possibility of controlling the market by a national gas regulator. In this study, we assume the existence of an efficient social welfare maximizing regulator that has a perfect information on contract prices, disruption probabilities, and disruption costs. [7] Among the large set of possible regulatory instruments (e.g., imposing the firms to hold some

7. Thus, we do not model the principal-agent interactions between the regulator and the regulated firms.

precautionary storage), we focus on a possible regulatory intervention on the firms' contracting decisions. More specifically, let's assume that a possible regulation fixes a maximum amount X bought by each retailer i, in order to optimize the expected social welfare (shared between the retailers and the consumers).

We denote by W the total social welfare :

$$W = W_{consumers} + W_{retailers}$$

where

$$\begin{array}{lll} W_{consumers} = \int_0^{x_0} f(t)dt - f(x_0)x_0 & & \text{Consumer surplus} \\ W_{retailers} = \sum_{i=1}^{n} \left((f(x_0) - p)x_i - \theta \frac{x_i}{x_0} \displaystyle\int_c^{x_0} g(x_0, t)dt + p\theta x_i \right) & & \text{Retailers' profits} \end{array} \tag{3.18}$$

Under the iso-elasticity assumptions, we can calculate analytically welfare W if the quantity of gas bought by each retailer x_i is x (the equilibrium is symmetric, see Appendix 2) :

$$W(x) = \tau n^{-\frac{1}{\epsilon_0}+1} x^{-\frac{1}{\epsilon_0}+1} + \beta n^{\frac{1}{\epsilon}} x^{\frac{1}{\epsilon}} - np(1-\theta)x \tag{3.19}$$

where

$$\tau = a \left(\frac{\epsilon_0}{\epsilon_0 - 1} - \theta \frac{\epsilon_1}{\epsilon_1 - 1} \right) \tag{3.20}$$

$$\beta = \theta a \frac{\epsilon_1}{\epsilon_1 - 1} c^{-\frac{1}{\epsilon_1}+1}. \tag{3.21}$$

Figure 3.10 represents the evolution of the welfare over the quantity bought by each retailer x for $\theta = 0.15$, $n = 6$, and $c = 0.4$ in arbitrary units.

We notice that there is an optimal amount x_{max} to be bought by each retailer to ensure a maximum welfare. We will now compare this quantity to the one imported by the retailers if they were to interact freely without any regulation. Figure 3.11 gives the evolution of x_{max} and x_{eq} over θ for $n = 6$ and $c = 0.4$ in arbitrary units. We notice that there is a specific disruption probability θ_{lim}, that depends only on the inner-market characteristics (i.e. ϵ_0, ϵ_1, n, c, a and p) such as :

$$\begin{array}{ll} \text{if } \theta \leq \theta_{lim} & x_{eq} \leq x_{max} \\ \text{if } \theta > \theta_{lim} & x_{eq} > x_{max} \end{array} \tag{3.22}$$

The main conclusion to draw from this study is the following : to optimize the social welfare, a regulator should fix a maximum amount X sold by Gazprom to the Bulgarian retailers only if the risk of disruption is high : $\theta > \theta_{lim}$. In that case, the maximum amount X must be $x_{max}(\theta)$. No regulation should be imposed if the producer is not too risky (i.e. $\theta \leq \theta_{lim}$) for any restriction on the gas amount would decrease the social welfare.

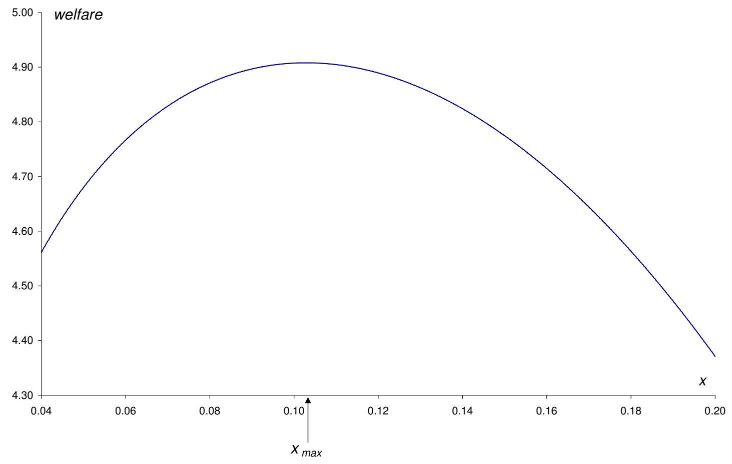

FIGURE 3.10 – *Evolution of the social welfare over x (arbitrary units). $\theta = 0.15$, $n = 6$, and $c = 0.4$ (arbitrary units).*

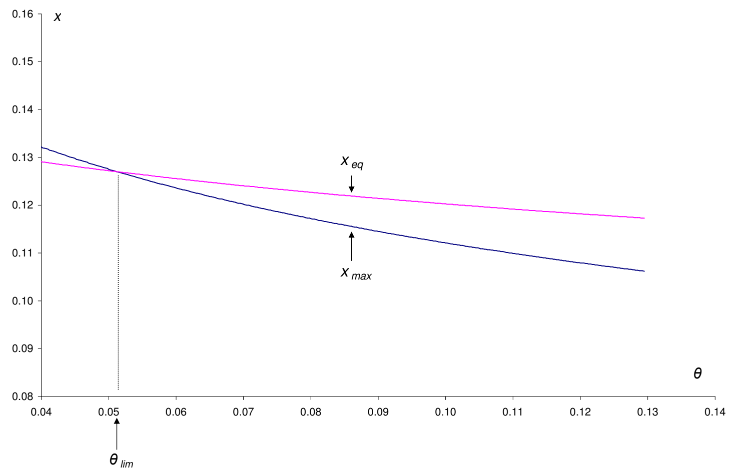

FIGURE 3.11 – *Evolution of x_{eq} and x_{max} (arbitrary units) over θ. $n = 6$ and $c = 0.4$ (arbitrary unit).*

At this stage of our model, it is interesting to study the evolution of the probability θ_{lim}, that is the regulation determining factor, over the alternative import capacity amount c. Economically speaking, it is easy to predict that this probability increases with c. Indeed, if the alternative gas import capacity is high in the event of an emergency, it is possible to tolerate frequent disruptions, without any regulation. Figure 3.12 represents the evolution of θ_{lim} over the capacity c, for $n = 6$, $p = 1$ and $a = 1$ in arbitrary units.

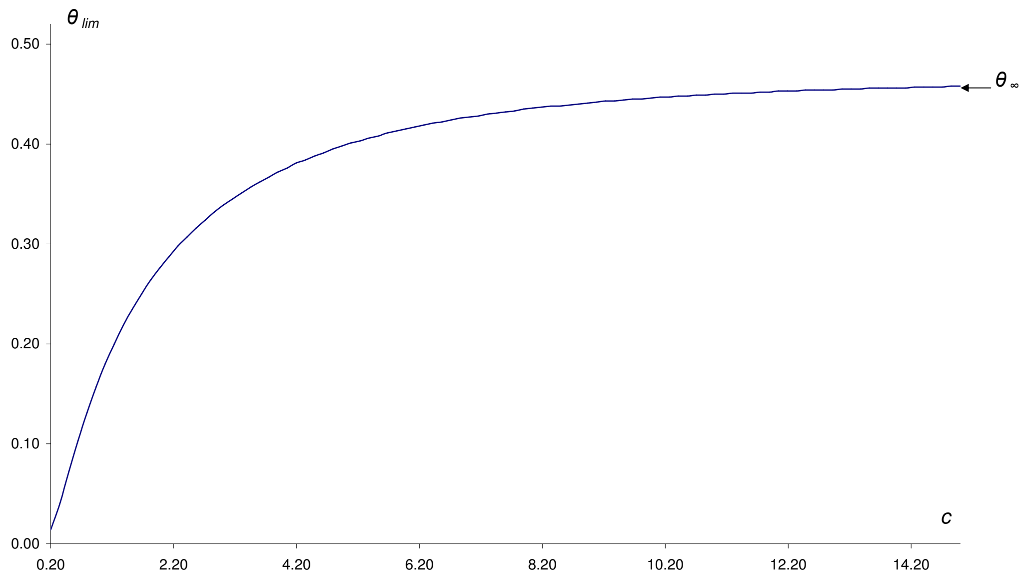

FIGURE 3.12 – *Evolution of θ_{lim} over c (arbitrary unit). $n = 6$.*

We notice that the probability θ_{lim} converges, for large capacities towards a finite value θ_∞ that depends only on ϵ_0, ϵ_1, a and p. In our example, $\theta_\infty \approx 0.5$. The main conclusion to draw is that for very risky producers ($\theta > \theta_\infty$), a regulation of import volumes must always be imposed in order to optimize the social welfare regardless of the alternative import amount c.

§ 3.4 Case 3 : The Spanish situation

In order to diversify the supply origins of gas and hence limit the disruption risks in the market, in 1998, the Spanish government passed the *Hydrocarbon law*, that is, a quota regulation that imposes on each Spanish gas trader not to buy from any producer more than 60% of the total amount purchased. To our knowledge, Spain, with the aim to reduce its energetic dependence on the Algerian imports, was the first European country (since the 90s) to vote such a regulation. As a consequence, this regulation lead to a supply diversification during the 90s and Spain signed many LNG contracts with different producers : Lybia, Nigeria, Quatar etc. As a result, the Algerian gas share in the Spanish consumption reached 43% in 2006 (11), which makes, *a priori*, the 60% rate useless. However, with the Medgaz project, a pipeline linking Algeria to Spain which is in construction, the Algerian gas exports towards Spain are expected to increase significantly in the forthcoming decades. To our knowledge, there are very few studies that have proposed theoretical works in order to analyze the Spanish regulation and predict its consequences on the consumers. Therefore, it can be meaningful to propose a model-based analysis able to describe the Spanish gas market trade regulation, taking into account the dominance of the Algerian supplies. The goal of this section is to predict, in the case of an important increase of the Algerian share market the evolution of the Spanish natural gas market under its specific regulation and to see whether different regulations could have a better impact on social welfare.

3.4.1 The models

In this section, we propose an extension of the model developed earlier. Supply interruptions are taken into consideration thanks to a subjective probability of disruption, which is appreciated by the traders while choosing their strategic amount imports. In case of any interruption, a disruption cost is imposed on the traders in order to compensate their customers, who are technologically dependent on gas supplies for their usual consumption. In this study, we intend to compare two sorts of market regulations. The first one imposes a disruption cost on the traders in any crisis situation, whereas the second one imposes for each trader a maximum importing rate from any producer, which is the Spanish regulatory regime.

We will divide the gas Spanish imports into two categories : standard natural gas, mainly imported from Algeria (index 2) and LNG brought from western Africa, Libya, Quatar and Egypt (index 1). Historically, there have been very few supply disruptions of LNG imports to Spain, principally because of the diversity of their origins. Therefore, we will assume in first approximation for our model that there is no risk of disruption in the LNG supplies. On the contrary, we will suppose that the Spanish will to become less dependent on the Algerian gas imports corresponds, for the Spanish strategic choices, to the existence of a non negligible disruption probability θ from Algeria, either for political, economical or technical reasons. Actually, the historical Algerian supply interruptions have been rare and concerned only the gas imports towards Italy. To further simplify our model, we will assume the existence of one big Spanish trader-firm (Gaz Natural) that owns a high share market, controls the transport and distribution networks and that is very slightly affected by the actions of the other traders. We denote by x_1 the LNG gas amount bought by Gaz Natural and x_2 the natural gas amount bought from Algeria. $x_0 = x_1 + x_2$ is the total gas quantity brought by Gaz Natural to the market. We will assume an iso-elastic form for both the long- and short-run (inverse) demand functions : ϵ_0 (resp. ϵ_1) is the long (resp. short) run elasticity. We will assume also that $\epsilon_0 > \epsilon_1$ ((15)). The long-run inverse demand function gives the evolution of the charged price in the Spanish gas market over the quantities provided :

$$price(x_0) = f(x_0) = a x_0^{-\frac{1}{\epsilon_0}}$$

The constant a must be calibrated on the Spanish gas market. The short-run demand function $g(x_0, t)$ is the consumers' willingness to buy quantity t in case of disruption if the long-run regime corresponds to the consumption of an amount x_0, taking into consideration the addiction to the gas technology, which happens to be very heavy to replace (especially when one notices that supply disruptions usually last just a few weeks).

$$g(x_0, t) = a x_0^{\frac{1}{\epsilon}} t^{-\frac{1}{\epsilon_1}}$$

We assume that the Spanish gas demand is characterised as follows : $\epsilon_0 = 1.2$ and $\epsilon_1 = 0.3$. In the upstream part of the gas trade, it is natural to assume that the prices differ depending on the sources, taking into acount the production costs and transports. This situation is quite realistic since gas trade remains dominated by bilateral long term contracts. In these contracts, the price of gas is settled thanks to predetermined indexation formulas that establish a direct linkage with the oil products prices (Netback value). We assume that these gas prices remain unaffected by short disruptions. We denote by p_1 (resp. p_2) the LNG mean price (resp. the Algerian natural gas). We assume that the Spanish inner gas production is negligible compared to its imports.

– The first regulation we intend to describe imposes for the traders a disruption cost to be provided to the customers, in the event of delivery interruption. As in our model described in chapter 2, to estimate this cost, we will use the short-run demand function and calculate the consumer's surplus variation in case of disruption : if the producer i interrupts its supply, the disruption cost associated is $\int_{x_0 - x_i}^{x_0} g(x_0, t)dt$. The spanish natural gas trader decides the amounts bought x_1 and x_2 in order to optimize its expected profit Π :

$$\Pi(x_1, x_2) = f(x_0)x_0 - p_1 x_1 - (1 - \theta)p_2 x_2 - \theta \int_{x_1}^{x_0} g(x_0, t)dt$$

We will call this regulation the *compensation regulation*.

– The second regulation fixes a rate α (the Spanish case corresponds to $\alpha = 0.6$) such as $x_{1,2} \leq \alpha(x_1 + x_2)$. The rate α is determined so that the total social welfare is maximum, giving the prices p_1 and p_2 and the disruption probability θ.
This is the *Spanish regulation.*

The main difference between the two models corresponds to a conceptual divergence between the customers' compensation in case of disruption. The *compensation regulation* imposes a disruption cost compensation and leaves Gaz Natural free to choose its imported amounts to maximize its expected profit. The obvious consequence is that the trader will naturally prefer dealing with secure producers, even if they sell at a high price. The *Spanish regulation* initially constrains the amounts x_i and the trader no longer compensates the customers in case of disruption. The consumers' supply security relies directly on the imports' origins diversity. Both models are static : they study the market over a typical period of one year, where the parameters $p_{1,2},\ \theta,\ \epsilon_{0,1}$ and a

are considered constants and exogeneous to the models. In particular, we have used the following values : $\epsilon_0 = 1.2$ and $\epsilon_1 = 0.3$.

The *compensation regulation* has been studied in detail in chapter 2. Hence, we only give the results obtained and the conclusions drawn about the social welfare in order to compare it with the current *Spanish regulation*.

3.4.2 The Spanish regulation

In the Spanish market, the trader's optimization program is the following :

$$\begin{array}{ll} \text{Max} & \Pi(x_1, x_2) \\ x_1 \geq 0 & \\ x_2 \geq 0 & \\ x_1 \leq \alpha(x_1 + x_2) & \\ x_2 \leq \alpha(x_1 + x_2) & \end{array}$$

where the profit is :

$$\Pi(x_1, x_2) = f(x_0)x_0 - p_1x_1 - p_2x_2 = ax_0^{-\frac{1}{\epsilon_0}+1} - p_1x_1 - p_2x_2$$

Since $\epsilon^0 \geq 1$, the profit is a concave function of the variables $x_{1,2}$. Hence, ***the existence and uniqueness of an optimal amount*** (x_1^{eq}, x_2^{eq}) ***is guaranteed***.

The parameter α belongs to [0.5,1]. A situation with no regulation corresponds to $\alpha = 1$. Since the Spanish government assumes Algeria to be a risky producer (i.e. $\theta > 0$), it is natural to suppose that $p_1 > p_2$ (the Algerian gas is cheaper), which is a realistic hypothesis. In the absence of regulation, the trader will buy all his gas from Algeria. Therefore, we deduce that the optimal amount x_2^{eq} is such as the constraint $x_2 \leq \alpha(x_1 + x_2)$ is saturated (KKT Theorem). Therefore, we can search the optimum amount on the line $x_1^{eq} = \frac{1-\alpha}{\alpha}x_2^{eq}$:

$$\begin{aligned} x_2^{eq} &= \left(\frac{p_1\left(\frac{1-\alpha}{\alpha}\right)+p_2}{a\left(1-\frac{1}{\epsilon_0}\right)}\right)^{-\epsilon_0} \alpha^{1-\epsilon_0} \\ x_1^{eq} &= \left(\frac{p_1\left(\frac{1-\alpha}{\alpha}\right)+p_2}{a\left(1-\frac{1}{\epsilon_0}\right)}\right)^{-\epsilon_0} \alpha^{-\epsilon_0}(1-\alpha) \end{aligned} \tag{3.23}$$

The regulator fixes the rate α to optimize the social welfare. On the one hand, a market situation where $\alpha \approx 0.5$ constrains the trader too much and reduces its profit. On the other hand, $\alpha \approx 1$ allows Gaz Natural to buy its gas exclusively from Algeria, which reduces the customers' welfare because of the possible supply interruptions. We conclude that, economically speaking, there is an optimal value for α that ensures a maximum social welfare, shared between the trader and the consumers. The social welfare W can be written as follows :

$$W = \Pi_{trader} + S_{consumers}$$

where Π_{trader} is the trader's profit and $S_{consumers}$ is the expected consumers' welfare. The consumers' welfare values $\int_0^{x_0^{eq}} f(t)dt - f(x_0^{eq})x_0^{eq}$ if the supply is provided and $\int_0^{x_0^{eq}} f(t)dt - \int_{x_1^{eq}}^{x_0^{eq}} g(x_0^{eq}, t)dt - f(x_0^{eq})x_0^{eq}$ otherwise. As a result, the expected customer welfare is given by :

$$S_{consumers} = \int_0^{x_0^{eq}} f(t)dt - \theta \int_{x_1^{eq}}^{x_0^{eq}} g(x_0^{eq}, t)dt - f(x_0^{eq})x_0^{eq}$$

and the total social welfare is :

$$W = -p_1 x_1^{eq} - p_2 x_2^{eq} + a x_0^{eq - \frac{1}{\epsilon_0}+1} \left(\frac{\epsilon_0}{\epsilon_0 - 1} - \theta \frac{\epsilon_1}{\epsilon_1 - 1} \right) + \theta a \frac{\epsilon_1}{\epsilon_1 - 1} x_0^{eq \frac{1}{\epsilon}} x_1^{eq - \frac{1}{\epsilon_1}+1} \tag{3.24}$$

where x_1^{eq} and x_2^{eq} has been calculated in equation (3.23).

For our numerical applications, we give arbitrary values for the parameters : $p_1 = 6$, $p_2 = 2$ and $a = 1$ in arbitrary units.

Figure 3.13 gives the evolution of the social welfare W, in an arbitrary unit, over the regulation rate α, for $\theta = 0.05$. As expected, we notice the existence of an optimum rate α_{opt} that ensures a maximum welfare. Hence, a regulator should fix a rate α such as $\alpha = \alpha_{opt}(\theta)$.

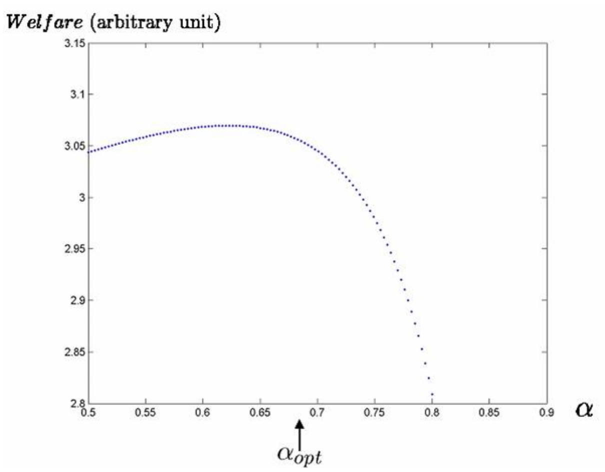

FIGURE 3.13 – *Evolution of the social welfare over α (arbitrary unit). $\theta = 0.05$.*

Figure 3.14 gives the evolution of the optimal rate α_{opt} over the disruption probability θ. We can easily demonstrate that if Algeria is assumed to be as secure an exporter as LNG suppliers, the optimum welfare is reached when $\alpha = 1$, or when no quota control is imposed, which is quite intuitive.

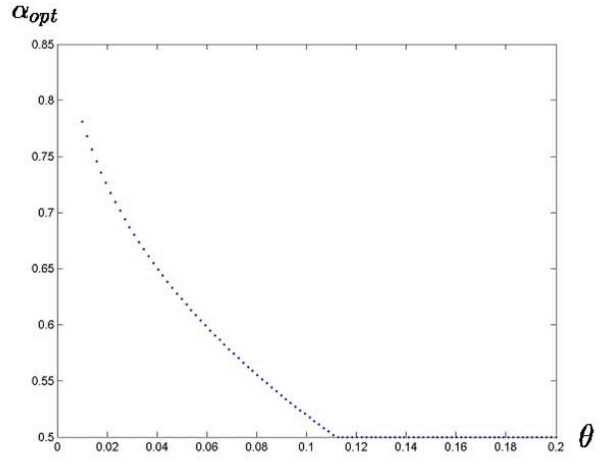

Figure 3.14 – *Evolution of the optimum rate over θ.*

The interpretation of figure 3.14 is as follows : If Algeria has a risky behavior, the regulation should be very stringent ($\alpha \longrightarrow 0.5$) to prevent consumers from the impacts of disruption. Hence, α_{opt} decreases with θ.

At this stage of our study, it is interesting to compare between the two kinds of regulations we introduced in section 3.4.1. The only criterium of differenciation we can use that is common to both models is the total social welfare. Figure 3.15 gives the evolution of the social welfare in both situations. The red curve represents the welefare when the traders have to pay a disruption cost (the *compensation regulation*) whereas the blue one represents the welfare, when the regulator fixes a rate α_{opt} in the market (the *Spanish regulation*).

We notice the existence of a probability limit θ_{lim} such as : if Algeria is not very risky $\theta \leq \theta_{lim}$, the *Spanish regulation* is better than the *compensation regulation*, whereas if Algeria has a risky behavior $\theta \geq \theta_{lim}$, the *compensation regulation* becomes better for the social welfare than the *Spanish regulation*. In our example, for $p_1 = 6$, $p_2 = 1$ and $a = 1$ in arbitrary units, the probability limit θ_{lim} is 0.26.

In conclusion, we will say that the main goal of this section is to study the Spanish gas trade under the *Hydrocarbon law*, which fixes a quota control on the foreign imports brought to the market. Indeed, in order to reduce the Spanish energetic dependence upon the Algerian gas supplies, the Spanish government imposes, for each retailer, not to buy from any gas producer more than 60% of the total amount purchased. With the Medgaz project and the possible increase of the Algerian share in the Spanish imports, this quota regulation is expected to play an important role in the Spanish retailers' strategic choices of their supply sources and to have major impacts on the social welfare. These consequences are studied in this section.

The current Spanish quota restriction is compared to another kind of regulation. The latter

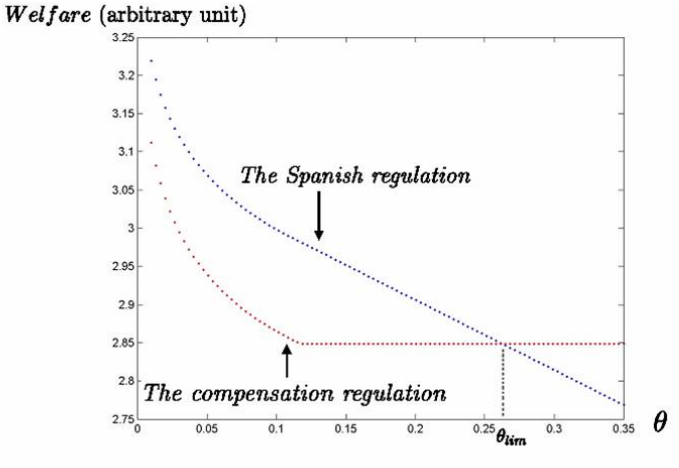

FIGURE 3.15 – *Evolution of the social welfare over θ, for both types of regulation.*

leaves the traders free to choose their sources of supply but imposes a compensation cost to be paid to the customers in case of supply interruption. Thanks to our model, we are able to deduce conditions on the Algerian disruption probability θ that give a preference, for social welfare optimization matters, to one of the regulations over the other.

§ 3.5 Conclusion

The main goal of this chapter is to apply our model to realistic past and current situations of the western European gas markets.

The German gas market of the 1980s, which is represented by the interaction between one major retailer, Ruhrgas AG, who brings gas to the end-user market and two major producers, Russia and Norway, has been accurately described by our model. We have shown in particular that if the Russian gas becomes too expensive or too risky, (compared to the Norwegian gas, which is supposed to be safe) with bounds that can be precisely determinated and that depend only on the inner market characteristics, no Russian gas would be brought to Germany by Ruhrgas AG as this would decrease its profit. We also show that the price charged by Ruhrgas in the German market would increase with the disruption probability.

The Bulgarian gas market is also a case analyzed thanks to our model. We assume the existence of a certain number of retailers that buy gas mostly from one risky producer : Gazprom. The main conclusions we can draw from our study are the following : Firstly, the gas price in the market, in case of pure and perfect competition, is higher than the producer's price, which is the pure and perfect competition gas price in the market if Russia is considered to be a safe supplier. This indicates that, added to the "oligopolistic margin", there exists a "security margin" charged by the retailers to compensate the disruption costs they have to support in the event of supply failure. This "security margin" increases with the disruption risk. Secondly, we show that, under some specific assumptions on the local *force majeure* supplies, the pure and perfect competition price increases with the Russian disruption probability. Finally, we show the existence of a threshold probability such as if the disruption probability is greater than the threshold, it is better for the overall social welfare to regulate the market (by means of quantities control) and not leave the actors to interact freely.

In order to reduce the Spanish energetic dependence upon the Algerian gas supplies, the Spanish government imposes, for each retailer, not to buy from any gas producer more than 60% of the total amount purchased. With the Medgaz project and the possible increase of the Algerian share in the Spanish imports, this quota regulation is expected to play an important role in the Spanish retailers' strategic choices of their supply sources and to have major impacts on social welfare. These consequences are studied in this chapter. The current Spanish quota restriction is compared to another kind of regulation. The latter leaves the traders free to choose their sources of supply but imposes a compensation cost to be paid to the customers in case of supply interruption. Thanks to our model, we are able to deduce conditions on the Algerian disruption probability θ that give a preference, for social welfare optimization matters, to one of the regulations over the other.

The results of this part are obtained by assuming the predominance of disruption costs in a firms' decisions, thereby a negligible role is thus given to the alternative crisis management techniques : strategic withdrawal from existing natural gas storages, alternative short-term imports, re-routing of existing gas flows, increased production from other suppliers that may compensate the shortfall of others. Following the impressive disruptions that occurred in Eastern Europe, concerns about the security of supply are now back at the top of the policy makers' agenda. The identification of the optimal measures to be implemented in the short-run to cope with a disruption is still an on-going issue. As a result, future research could expand the framework discussed in this part in order to identify optimal crisis management policies.

Bibliographie

[1] I. Abada, O. Massol, 2011. *Security of supply and retail competition in the European gas market. Some model-based insights.* Energy Policy 39 (2011), 4077-4088.

[2] D.W. Bunn, M. M. Mustafaoglu, 1978. *Forecasting Political Risk.* Management Science 24.15 (November) : 1557-67.

[3] T.J. Considine, 2006. *Is the strategic petroleum reserve our ace in the hole ?* The Energy Journal, 27, 3, (2006), 91-112.

[4] V. Costantini, F. Gracceva, A. Markandya, G. Vicinia, 2007. *Security of energy supply : Comparing scenarios from a European perspective.* Energy Policy, Vol. 35, No. 1, (January), 210-226.

[5] DG COMP, 2007. *DG Competition report on energy sector inquiry.* SEC(2006)1724, European Commission, Competition DG, 10 January, 2007, Brussels.

[6] R. Golombek, J. Vislie, M. Hoel, 1987 (Eds.). *Natural gas markets and contracts.* North-Holland.

[7] D. Helm, 2002. *Energy policy : security of supply, sustainability and competition.* Energy Policy 30 173-184.

[8] C. von Hirschhausen, A. Neumann, 2008. *Long-Term Contracts and Asset Specificity Revisited : An Empirical Analysis of Producer-Importer Relations in the Natural Gas Industry.* Review of Industrial Organization, Vol. 32, No. 2, pp. 131-143.

[9] M. Hoel, S. Strom, 1987. *Supply security and Import diversification of Natural Gas.* In R. Golombek, J. Vislie and M. Hoel (Eds.) : Natural gas markets and contracts, North-Holland.

[10] International Energy Agency, 2002. *Flexibility in natural gas supply and demand.* OECD/IEA, Paris.

[11] International Energy Agency, 2007. *Natural Gas Market Review.* OECD/IEA, Paris.

[12] International Energy Agency, 2008. *World Energy Outlook 2008.* OECD/IEA, Paris.

[13] B. Kruyt, D. P van Vuuren, H. J. M. de Vries, H. Groenenberg, 2010. *Indicators for energy security.* Energy Policy, Vol. 38, No. 4.

[14] N. Lefèvre, 2010. *Measuring the energy security implications of fossil fuel resource concentration.* Energy Policy, Vol. 38, No. 4 (April), pp. 1635-1644.

[15] A. S. Manne, K. Roland, G. Stephan, 1986. *Security of supply in the Western European market for natural gas.* Energy Policy, Vol. 14, No. 1 (February), Pages 52-64.

[16] A. Markandya, M. Pemberton, 2010. *Energy security, energy modeling and uncertainty.* Energy Policy, Vol. 38, No. 4 (April), pp. 1609-1613.

[17] O. Massol, S. Tchung-Ming, 2010. *Cooperation among liquefied natural gas suppliers : is rationalization the sole objective ?* Energy Economics, 32(4) 933-947.

[18] F. H. Murphy, M. A. Toman, H. J. Weiss, 1987. *A Stochastic Dynamic Nash Game Analysis of Policies for Managing the Strategic Petroleum Reserves of Consuming Nations.* Management Science, Vol. 33, No. 4 (April), pp. 484-499

[19] D. M. Newbery, 2000. *Privatization, Restructuring and Regulation of Network Utilities*, (The Walras-Pareto Lectures, 1995). MIT Press.

[20] A. Nichols, R. Zeckhauser, 1977. *Stockpiling Strategies and Cartel Prices.* Bell Journal of Economics 8(1), Spring, 66-96.

[21] J. Percebois, 2006. *Dépendance et vulnérabilite : Deux fas connexes mais différentes d'aborder les risques énergétiques.* Cahier N 06.03.64, Centre de Recherche en Economie et Droit de l'énergie (CREDEN), Université de Montpellier, France.

[22] M. Radetzki, 1992. *Pricing of Natural Gas in the West European Market.* Energy Studies Review, Vol. 4, No. 2.

[23] J. Vickers, G. Yarrow, 1988. *Regulation of privatised firms in Britain.* European Economic Review, Vol.32, No. 2-3 (March), pp. 465-472.

[24] N. Victor, D. G. Victor, 2006. *Bypassing Ukraine : exporting Russian gas to Poland and Germany.* In Victor, D.G., Jaffe, A.M., Hayes, M. H. (Eds), Natural Gas and Geopolitics : From 1970 to 2040, Cambridge University Press.

§ 3.6 Appendix 1

In this appendix, we calculate the partial derivative of $\bar{\Pi}_i$ with respect to the decision variable x^0_{ik}. This derivative is the sum of three terms : $\frac{\partial A}{\partial x^0_{ik}}, \frac{\partial B}{\partial x^0_{ik}}$ and $\frac{\partial C}{\partial x^0_{ik}}$ with :

$$\frac{\partial A}{\partial x^0_{ik}} = f'(x^0) \sum_{j \in J} x^0_{ij} + f(x^0) - p_k \tag{3.25}$$

$$\frac{\partial C}{\partial x^0_{ik}} = p_k \sum_{\{\omega \in \Omega \backslash \{0\}, k \in S_\omega\}} \theta(\omega) \tag{3.26}$$

The partial derivative of B with respect to x^0_{ik}, is a little bit more subtle to calculate. In fact, the collection of events ω has to be separated in two subsets depending on whether the particular producer k cuts its supplies under the state ω or not. We can write :

$$\frac{\partial B}{\partial x^0_{ik}} = - \sum_{\{\omega \in \Omega \backslash \{0\}, k \notin S_\omega\}} \theta(\omega) \frac{\partial DC_i(x^0, \omega)}{\partial x^0_{ik}} - \sum_{\{\omega \in \Omega \backslash \{0\}, k \in S_\omega\}} \theta(\omega) \frac{\partial DC_i(x^0, \omega)}{\partial x^0_{ik}} \tag{3.27}$$

Let's consider a particular producer k and buyer i. The distinction among the two cases is important since the partial derivative of $DC_i(x^0, \omega)$ with respect to x^0_{ik} takes a different literal expression in the two cases. If under a given state $\omega \in \Omega \backslash \{0\}$, the particular producer k cuts its supplies (i.e. $k \in S_\omega$), then the amount x^0_{ik} is both present in the overall disrupted volumes $(x^0 - x^\omega)$ as well as in i's disrupted purchases $\sum_{j \in J}(x^0_{ij} - x^\omega_{ij})$. In the other case (when k does not cut its production), both the overall disrupted quantities $(x^0 - x^\omega)$ and $\sum_{j \in J}(x^0_{ij} - x^\omega_{ij})$ become independent on the variable x^0_{ik}. Moreover, in the latter case, the integral boundaries can be manipulated so as to avoid any dependence on x^0_{ik}.
If $k \in S_\omega$,

$$\frac{\partial DC_i(x^0, \omega)}{\partial x^0_{ik}} = \frac{\sum_{\substack{(l,j) \in I \times J \\ l \neq i}} \left(x^0_{lj} - x^\omega_{lj} \right)}{(x^0 - x^\omega)^2} \int_{x^\omega}^{x^0} g(x^0, t) dt + \frac{\sum_{j \in J} (x^0_{ij} - x^\omega_{ij})}{x^0 - x^\omega} \left(\int_{x^\omega}^{x^0} \frac{\partial g}{\partial k}(x^0, t) dt + f(x^0) \right) \tag{3.28}$$

Whereas if $k \notin S_\omega$, we have a simpler expression :

$$\frac{\partial DC_i(x^0, \omega)}{\partial x^0_{ik}} = \frac{\sum_{j \in J} (x^0_{ij} - x^\omega_{ij})}{x^0 - x^\omega} \left(\int_{x^\omega - x^0}^{0} \frac{\partial g}{\partial k}(x^0, t + x^0) dt \right). \tag{3.29}$$

§ 3.7 Appendix 2

In this appendix, we theoretically solve the retailers' optimization problems given in formulation 3.13 of section 3.4.
Market transparency is an inherent assumption to our model (i.e. we assume that the n retailers have the same knowledge of the market in terms of prices and probability of disruption). Furthermore, mathematically speaking, we notice that the optimization problem 3.13 is symmetric for all the retailers. Consequently, we can already predict that the Nash-Cournot equilibrium is reached when all the amounts x_i are equal. Hence, let's call x_{eq} the equilibrium quantity bought by each retailer and use the first-order condition to find it. We can deduce an implicit function that gives x_{eq} (i.e. a relation between x_{eq} and the problem's parameters) from expression 3.17 :

$$\alpha n^{-1-\frac{1}{\epsilon_0}}\left(n-\frac{1}{\epsilon_0}\right)x_{eq}^{-\frac{1}{\epsilon_0}}+\beta n^{\frac{1}{\epsilon}-2}\left(\frac{1}{\epsilon}-1+n\right)x_{eq}^{\frac{1}{\epsilon}-1}-(1-\theta)p=0 \tag{3.30}$$

Actually, it is not possible to find general analytical expressions of the solution for (3.30). We will use numerical means to solve it. However, we can already predict that equation (3.30) has a unique solution. Indeed, $\forall n \in \mathbb{N}^*$ the function

$$g_n : x \longrightarrow \alpha n^{-1-\frac{1}{\epsilon_0}}\left(n-\frac{1}{\epsilon_0}\right)x^{-\frac{1}{\epsilon_0}}+\beta n^{\frac{1}{\epsilon}-2}\left(\frac{1}{\epsilon}-1+n\right)x^{\frac{1}{\epsilon}-1}-(1-\theta)p$$

is strictly decreasing on $\mathbb{R}_+^*$ and realizes a bijection from $\mathbb{R}_+^*$ to $\mathbb{R}$.
If we assume that an equilibrium is possible, we can calculate the price of the product in the market and study its dependence on the disruption probability θ and the number of retailers n.

$$price = a(nx_{eq})^{-\frac{1}{\epsilon_0}} \tag{3.31}$$

§ 3.8 Appendix 3

This appendix studies the price bahavior in the pure and perfect competition context p_∞.
Let's denote $\rho_n = nx_{eq}(n)$. Using equation 3.30 we deduce that ρ_n is the unique solution of

$$f_n(\rho_n)=\alpha\left(1-\frac{1}{n\epsilon_0}\right)\rho_n^{-\frac{1}{\epsilon_0}}+\beta\left(\frac{\frac{1}{\epsilon}-1}{n}+1\right)\rho_n^{\frac{1}{\epsilon}-1}-(1-\theta)p=0$$

Let's call f the function : $\mathbb{R}_+^* \longrightarrow \mathbb{R}$

$$f : x \longrightarrow \alpha x^{-\frac{1}{\epsilon_0}}+\beta x^{\frac{1}{\epsilon}-1}-(1-\theta)p$$

f is a decreasing function and realizes a bijection from $\mathbb{R}_+^*$ to $\mathbb{R}$. Let's call $\rho = f^{-1}(0)$ the unique solution of the equation $f(x)=0$ and let's show that $\rho_n \longrightarrow \rho$. Indeed, we have $f_n(\rho_n)-f(\rho)=0$.

Hence $\forall n \in \mathbb{N}^*$:

$$\alpha \left({\rho_n}^{-\frac{1}{\epsilon_0}} - \rho^{-\frac{1}{\epsilon_0}} \right) + \beta \left(\rho_n^{\frac{1}{\epsilon}-1} - \rho^{\frac{1}{\epsilon}-1} \right) = \frac{1}{n} \left(\frac{\alpha}{\epsilon_0} {\rho_n}^{-\frac{1}{\epsilon_0}} + \beta \left(\frac{1}{\epsilon} - 1 \right) \rho_n^{\frac{1}{\epsilon}-1} \right) \tag{3.32}$$

We can show easily that $\exists M \in \mathbb{R}_+^*$ such as $\forall n \in \mathbb{N}^*$ $|\rho_n| < M$ (that is to say the sequence ρ_n is boundned). Using equation 3.32, we conclude that $\alpha \left({\rho_n}^{-\frac{1}{\epsilon_0}} - \rho^{-\frac{1}{\epsilon_0}} \right) + \beta \left(\rho_n^{\frac{1}{\epsilon}-1} - \rho^{\frac{1}{\epsilon}-1} \right) \longrightarrow 0$ when $n \longrightarrow \infty$. Hence :

$$f(\rho_n) \longrightarrow f(\rho)$$

f being a continuous bijective function, f^{-1} is also a bijective continuous function and we conclude that $\rho_n = f^{-1}(f(\rho_n)) \longrightarrow f^{-1}(f(\rho)) = \rho$.
Finally, we can write the price limit p_∞ :

$$p_\infty = a\rho^{-\frac{1}{\epsilon_0}} \tag{3.33}$$

Using relation $\alpha\rho^{-\frac{1}{\epsilon_0}} + \beta\rho^{\frac{1}{\epsilon}-1} = (1-\theta)p$, we can calculate

$$\frac{\mathrm{d}\rho}{\mathrm{d}\theta}(\theta) = \frac{-p + \frac{\epsilon_1}{\epsilon_1-1} a\rho^{-\frac{1}{\epsilon_0}} - \frac{\epsilon_1}{\epsilon_1-1} a c^{1-\frac{1}{\epsilon_1}} \rho^{-\frac{1}{\epsilon}-1}}{-\frac{1}{\epsilon_0}\alpha\rho^{-\frac{1}{\epsilon_0}-1} + \beta\left(\frac{1}{\epsilon}-1\right)\rho^{\frac{1}{\epsilon}-1}} = \frac{-1}{\theta} \frac{1}{-\frac{1}{\epsilon_0}\alpha\rho^{-\frac{1}{\epsilon_0}-1} + \beta\left(\frac{1}{\epsilon}-1\right)\rho^{\frac{1}{\epsilon}-1}} \left(p - a\rho^{-\frac{1}{\epsilon_0}} \right). \tag{3.34}$$

If we assume that the force majeure imports capacity c is low enough, such as $c < \left(\frac{p}{a}\right)^{-\epsilon_0} \epsilon_1^{\frac{\epsilon_1}{1-\epsilon_1}} = 1.67 \left(\frac{p}{a}\right)^{-\epsilon_0}$, we can show (see Appendix 4) that

$$\forall \theta \in [0,1] \quad p \leq a\rho(\theta)^{-\frac{1}{\epsilon_0}}. \tag{3.35}$$

Hence, in this situation we conclude that $\forall \theta \in [0,1]$ $\frac{\mathrm{d}\rho}{\mathrm{d}\theta}(\theta) \leq 0$, or

$$\forall \theta \in [0,1] \quad \frac{\mathrm{d}p_\infty}{\mathrm{d}\theta}(\theta) \geq 0. \tag{3.36}$$

On the contrary, if $c > \left(\frac{p}{a}\right)^{-\epsilon_0} \epsilon_1^{\frac{\epsilon_1}{1-\epsilon_1}}$, we show that (see Appendix 4) :

$$\forall \theta \in [0,1] \quad \frac{\mathrm{d}p_\infty}{\mathrm{d}\theta}(\theta) \leq 0. \tag{3.37}$$

§ 3.9 Appendix 4

In this appendix, we show the properties stated in Appendix 3 :

$$\text{if} \quad c < \left(\frac{p}{a}\right)^{-\epsilon_0} \epsilon_1^{\frac{\epsilon_1}{1-\epsilon_1}} \quad \text{then} \quad \forall \theta \in [0,1] \quad p \leq a\rho(\theta)^{-\frac{1}{\epsilon_0}}$$

$$\text{and if} \quad c > \left(\frac{p}{a}\right)^{-\epsilon_0} \epsilon_1^{\frac{\epsilon_1}{1-\epsilon_1}} \quad \text{then} \quad \forall \theta \in [0,1] \quad p \geq a\rho(\theta)^{-\frac{1}{\epsilon_0}}$$

- We assume $c < \left(\frac{p}{a}\right)^{-\epsilon_0} \epsilon_1^{\frac{\epsilon_1}{1-\epsilon_1}}$
  Let's suppose that $\exists \theta_0 \in [0,1[$ such as $p \geq a\rho(\theta_0)^{-\frac{1}{\epsilon_0}}$. Using equation 3.34, we have $\frac{\mathrm{d}\rho}{\mathrm{d}\theta}(\theta_0) > 0$. We define θ_1 as follows :

$$\theta_1 = \sup \left\{ \theta \in [\theta_0, 1[\ / \ \frac{\mathrm{d}\rho}{\mathrm{d}\theta}(\theta) > 0 \right\}$$

  and let's show that $\theta_1 = 1$. If $\theta_1 < 1$, since the function $\theta \longrightarrow \rho(\theta)$ is continuously derivable, we can conclude that $\frac{\mathrm{d}\rho}{\mathrm{d}\theta}(\theta_1) = 0$. Using equation 3.34 we find that $p = a\rho(\theta_0)^{-\frac{1}{\epsilon_0}}$. However, we know that $\forall \theta \in [\theta_0, \theta_1[\quad \frac{\mathrm{d}\rho}{\mathrm{d}\theta}(\theta) > 0$. Hence, the function $\theta \longrightarrow \rho(\theta)$ is strictly increasing on the set $[\theta_0, \theta_1[$ and $\rho(\theta_1) > \rho(\theta_0)$. We already have $p \geq a\rho(\theta)^{-\frac{1}{\epsilon_0}}$. Thus we find

$$p \geq a\rho(\theta_0)^{-\frac{1}{\epsilon_0}} > a\rho(\theta_1)^{-\frac{1}{\epsilon_0}} = p \tag{3.38}$$

  which is absurd. Then $\theta_1 = 1$ and we conclude that $\frac{\mathrm{d}\rho}{\mathrm{d}\theta}(1) > 0$ or

$$p > a\rho(1)^{-\frac{1}{\epsilon_0}}. \tag{3.39}$$

  We can quite easily calculate $\rho(1)$:

$$\rho(1) = c\epsilon_1^{\frac{\epsilon_1}{\epsilon_1 - 1}} \tag{3.40}$$

  and using the condition $c < \left(\frac{p}{a}\right)^{-\epsilon_0} \epsilon_1^{\frac{\epsilon_1}{1-\epsilon_1}}$, we find that :

$$a\rho(1)^{-\frac{1}{\epsilon_0}} > p \tag{3.41}$$

  which is absurd, regarding equation 3.39.

  Hence :

$$\forall \theta \in [0,1] \quad p \leq a\rho(\theta)^{-\frac{1}{\epsilon_0}} \tag{3.42}$$

- We assume $c > \left(\frac{p}{a}\right)^{-\epsilon_0} \epsilon_1^{\frac{\epsilon_1}{1-\epsilon_1}}$
  Hence, $a\rho(1)^{-\frac{1}{\epsilon_0}} < p$ and $\frac{\mathrm{d}\rho}{\mathrm{d}\theta}(1) > 0$. We intend to show that $\forall \theta \in [0,1] \quad a\rho(\theta)^{-\frac{1}{\epsilon_0}} < p$. If we assume that $\exists \theta_0 \in [0,1[$ such as $a\rho(\theta_0)^{-\frac{1}{\epsilon_0}} \geq p$, we call θ_1 the probability :

$$\theta_1 = \inf \left\{ \theta \in [\theta_0, 1[\ / \ \frac{\mathrm{d}\rho}{\mathrm{d}\theta}(\theta) \geq 0 \right\}. \tag{3.43}$$

Here again, since the function $\theta \longrightarrow \rho(\theta)$ is continuously derivable, we have $\frac{\mathrm{d}\rho}{\mathrm{d}\theta}(\theta_1) = 0$. However, we know that $\forall\theta \in [\theta_0, \theta_1[$, $\frac{\mathrm{d}\rho}{\mathrm{d}\theta}(\theta) \leq 0$. Hence, $\rho(\theta_1) < \rho(\theta_0)$. However, we already have :

$$p = a\rho(\theta_1)^{-\frac{1}{\epsilon_0}} > a\rho(\theta_0)^{-\frac{1}{\epsilon_0}} \geq p \tag{3.44}$$

which is absurd. Thus our conclusion.

§ 3.10 Appendix 5

In this appendix, we show that the retailers' profit in the Bulgarian market is equal to 0, in the situation of pure and perfect competition. We will use the notation of Appendix 3.
The retailer's total profit is :

$$\Pi_{tot} = \sum_i \Pi_i = n\Pi(x_i) \tag{3.45}$$

where the individual profit $\Pi(x_i)$ is given in equation 3.14. Hence :

$$\Pi_{tot} = \alpha\rho^{-\frac{1}{\epsilon_0}+1} + \beta\rho^{\frac{1}{\epsilon}} - (1-\theta)p\rho \tag{3.46}$$

where α and β have been defined in section 3.4 and the variable ρ in Appendix 3.
We already know (Appendix 3) that ρ is such that $f(\rho) = 0$, where the function f is defined in Appendix 3. It is easy to notice that :

$$\Pi_{tot} = \rho f(\rho) \tag{3.47}$$

Therefore :

$$\Pi_{tot} = 0 \tag{3.48}$$

Thus our conclusion.

§ 3.11 Appendix 6

In this appendix, we show that the retailers' profit are concave functions of their decision variables. The demonstration uses the following values : $\epsilon_1 = 0.3$ and $\epsilon_0 = 1.2$.
Retailer i's profit is given in equation 3.14. We have to demonstrate that the profit Π_i is concave regarding the volume x_i and considering the other volumes $x_j, j \neq i$ as exogenous.
Π_i is twice differentiable and :

$$\frac{\partial \Pi_i}{\partial x_i}(x_i) = -\frac{\alpha}{\epsilon_0} x_0^{-\frac{1}{\epsilon_0}-1} x_i + \alpha x_0^{-\frac{1}{\epsilon_0}} + \beta\left(\frac{1}{\epsilon} - 1\right) x_0^{\frac{1}{\epsilon}-2} x_i + \beta x_0^{\frac{1}{\epsilon}-1} - (1-\theta)p$$

and

$$\begin{aligned}\frac{\partial^2\Pi_i}{\partial x_i^2}(x_i) = \quad & \frac{\alpha}{\epsilon_0}(1+\frac{1}{\epsilon_0})x_0^{-\frac{1}{\epsilon_0}-2}x_i + \beta\left(\frac{1}{\epsilon}-1\right)\left(\frac{1}{\epsilon}-2\right)x_0^{\frac{1}{\epsilon}-3}x_i \\ & -2\frac{\alpha}{\epsilon_0}x_0^{-\frac{1}{\epsilon_0}-1} + 2\beta\left(\frac{1}{\epsilon}-1\right)x_0^{\frac{1}{\epsilon}-2}\end{aligned}$$

The last expression can be rewritten as follows :

$$\frac{\partial^2\Pi}{\partial x_i^2}(x_i) = \frac{\alpha}{\epsilon_0}x_0^{-\frac{1}{\epsilon_0}-2}\left((1+\frac{1}{\epsilon_0})x_i - 2x_0\right) + \beta\left(\frac{1}{\epsilon}-1\right)x_0^{\frac{1}{\epsilon}-3}\left(\left(\frac{1}{\epsilon}-2\right)x_i + 2x_0\right)$$

We know that $\alpha \geq 0$, $\beta \leq 0$, $x_i \geq 0$, $x_0 \geq 0$, $1+\frac{1}{\epsilon_0} \leq 2$ and $\frac{1}{\epsilon} \geq 2$. Therefore : since $x_i \leq x_0$, then $\left(1+\frac{1}{\epsilon_0}\right)x_i \leq 2x_i \leq 2x_0$. Besides, $\left(\frac{1}{\epsilon}-2\right)x_i + 2x_0 \geq 0$. Finally, we have :

$$\forall x_i \geq 0,\ \forall x_j \geq 0, j \neq i,\ \frac{\partial^2\Pi_i}{\partial x_i^2}(x_i) \leq 0$$

Thus our conclusion.

Quatrième partie

A System Dynamics model for interfuel substitution

- Chapitre 4 -

Modeling interfuel substitution with a system dynamics approach.

§ 4.1 Introduction

Concerns about the natural gas sector are now back on top of policy makers' agenda. The reasons for this renewed interest are numerous and include : a rapid globalization of the natural gas trade (43), the rising share of gas technologies in the power generation sector inducing, in both Europe and Asia, an increased dependence on foreign sources (16), and the recent emergence of a Gas Exporting Countries Forum that is often depicted as an embryonic cartel (21).

Unsurprisingly, this context has triggered a renewed interest for energy economics models aimed at analyzing this industry. In particular, several partial equilibrium models in the vein of those pioneered in (22) have recently been proposed to represent the imperfect competition among gas producers ((4), (8), (15)). Besides the policy-oriented analyses provided in these articles, these detailed numerical models can be very useful for corporate planning purposes. For example, a firm that considers an investment in a large and lumpy gas transmission infrastructure may take advantage of these powerful tools to assess the relative appeal of various alternative routes by comparing the long-run impacts of the proposed infrastructure on the markets' outcome. Despite their great merits, [1] these models are not exempt from flaws. One of their most striking problems is connected with the relatively rudimentary treatment of the demand side, which is usually oversimplified to an affine inverse demand function. Indeed, a recent meticulous assessment of these models underlines that these contributions rely either on somewhat obsolete information on gas demand function or on more or less arbitrary calibration ((35), p.12). As the outcomes of a market equilibrium model based on the Cournot oligopoly theory are indubitably impacted by the price elasticity of the demand (41), some further investigations are clearly required to obtain a more satisfactory functional specification of the demand for natural gas. Such a functional form will have to take into account both the substitutability of natural gas by other fuels and the adjustment dynamics of consumption in reaction to fuel prices. This statement provides the motivation for this thesis' part.

Energy demand modeling has become a very productive activity since the 1970s. Indeed, a very large literature has approached the question using econometric analyses. As far as interfuel substitution is concerned, we can distinguish between early empirical specifications based on, for example, discrete choice models as in (19), and models predicted in theory where a flexible functional form is aimed at being estimated in coherence with standard microeconomic assumptions (profit or utility maximizing behavior, 0-degree homogeneity of the demand function, symmetry, law of demand, etc.) as in, for example, (12), (31), (6), or (42). Notwithstanding the immense value of these statistical approaches, it must be acknowledged that putting theory to work to model energy demand can turn out to be far from a sinecure because of numerous practical considerations (cf. the informed list reported in (44)). For example, practitioners may be compelled to adopt a more simplified dynamic specification to model short- and long-run effects than those recommended by theoretical arguments based on an assumed dynamic optimization behavior (44). This type of consideration may turn out to be problematical if the obtained demand model is aimed at being embedded within a decision-support tool designed to serve the needs of users (corporate planners

1. For example, they capture a very detailed representation of the supply side of the natural gas industry including : transmission network, production constraints, etc.

and executives) who may have forgotten some of their statistical education and could feel uncomfortable with a modeled dynamics that hardly mimics their *a priori* mental representation of a putty-clay type of dynamics. To increase their confidence with the model's validity, modelers can look for an approach that endeavors to build on their detailed understanding of the gas industry. Given the strong record of applications of system dynamics for strategic modeling purposes ((38), (23)), this technique constitutes an appealing methodology.

Numerous system dynamics-based models have been developed for energy planning purposes. A non-exhaustive list includes : *(i)* the models originating from research initiated at Dartmouth College in the late 1970s and then refined during nearly two decades to support energy policy analyses conducted by the US federal administration ((26), (27), (28), (45)), *(ii)* the broad approach of (36) that analyzed the US energy transition with an integrated energy-economy model and the extended climate-economy model of (10), *(iii)* the numerous models surveyed in (11) that are aimed at informing electric utility policies and *(iv)* the models dedicated to the oil and/or gas industries such as (7), (29), (5) and (32).

Since the 1980s, an impressive stream of research that encompasses all the facets of natural resources (economic, management, policy) has been conducted in Norway. As far as natural gas is concerned, the affluence and diversity of this "Norwegian school" is well exemplified in (14). Amusingly, this book that contains the Cournot equilibrium model of (22) mentioned above also includes (47), a putty-clay model of OECD-European industrial energy demand that presented a very good explanation of historical fuel substitution during the period 1960-1983. In a subsequent study (25), it has been shown that this framework can also provide a very good fit to an historical time-series of fuel choices in OECD-European electricity production. In this contribution, we propose to take advantage of this system dynamics methodology to select and estimate a more satisfactory demand function aimed at being implemented in an imperfect competition model (1). To do so, an updated version of this system dynamics-based model is first presented and recalibrated to check the capability of this approach to explain the substitutions between the three main fuels : oil, coal and natural gas. The model is then simulated to generate data that depict the dependence of fuel consumption over fuel prices. Based on these "pseudo data", an interesting functional form is proposed to model the demand function for natural gas, that can be generalized to the three fuels.

This part is organized as follows : chapter 4 provides a brief review of the methodology presented in (47) and (25). The results obtained with a calibrated model are also given, for different countries. In chapter 5, the system dynamics model is put to work to construct an adapted demand function. This demand function will be used in natural gas markets modeling, presented later in this manuscript.

The same notation will be used in chapters 4 and 5.

§ 4.2 The model

In this section, we briefly review the model detailed in (47). This model aims at predicting the consumption of coal, oil and natural gas observed at time t using both the historical and current values of fuel prices, and the history and current value of the overall demand for hydrocarbon fuels. In this model, the dynamics of interfuel substitution involves a distinction between the flow of freshly installed equipment, and the stocks of existing equipment that is represented by two vintages of capital. The model is based on a putty-clay framework and assumes that the choice of fuels can be freely adjusted *ex ante*, whereas no substitution is possible *ex post*. Thanks to this decomposition, the model captures the irreversibility associated with the decision to install and operate a durable burning equipment.

To begin with, table 4.1 clarifies the model boundaries and divides the variables and parameters into those endogenous and those exogenous to the model :

Table 4.1 – *An overview of the model boundaries.*

Endogenous	**Exogenous**
Investment in new burning equipment Fuel shares in newly installed equipment Installed burning capacity per fuel option Capacity utilization factors of installed equipment Consumption of the various fuels	Total energy consumption Fuel market prices

To simplify, the fuel options are indexed by an integer i and the fuel option coal (respectively oil, and natural gas) is labeled 1 (respectively 2, and 3). The fuel shares in the new burning equipment installed at time t are assumed to be determined by the relative cost of the three fuel options. The total cost C_i of fuel option i is given by the following formula :

$$C_i = \frac{CC_i}{PBT_i} + OO_i + \frac{P_i + Q_{CO_2 i}.P_{CO_2}}{E_i} - PR_i \ , \qquad (4.1)$$

where CC_i is the capital cost, PBT_i is the associated payback time, OO_i denotes the operating cost (fuel and carbon cost excluded), P_i is the fuel price, P_{CO_2} is the price of CO_2 if any, $Q_{CO_2 i}$ is the CO_2 emission factor of fuel i, E_i is the burner efficiency, and PR_i is a premium, that is, a parameter that reflects the miscellaneous unmodeled features of fuel i such as flexibility, availability, consumption inertia, etc. In (47) and (25), the price of CO_2 has not been taken into account. Thus, our approach is, to some extent, more general.

The share s_i of fuel option i in the new burning equipment is determined by the relative cost of the three fuel options. The following multinomial logit model is used :

$$s_i = \frac{e^{-\alpha C_i}}{\sum_i e^{-\alpha C_i}} \ , \tag{4.2}$$

where α is a (nonnegative) parameter, and C_i are the total costs defined in (4.1). By construction, the obtained shares satisfy $\forall i, \ s_i \in [0,1]$ and $\sum_i s_i = 1$. In addition, the share s_i is, *ceteris paribus*, a decreasing function of the fuel price P_i. Besides, one may notice that shares are determined on the basis of differences in total costs and thus differences in the values of the premiums. As these are adjustable parameters, it may be easier to determine a reference point : hereafter, the premium for coal PR_1 has thus been set equal to 0. It is also interesting to underline that the presence of exponents in the logit formula tends to accentuate the differences in total costs as they are converted into fuel choices. A small value of α translates into equal shares for all fuels, whereas a large value of α indicates that minor differences in total cost lead to major differences in the resulting fuel shares. [2] Actually, the validity of this logit model conceptually presupposes a "macroscopic" perspective, meaning that the energy system under scrutiny must contain a large enough number of individual decision-makers.

In this model, capital is measured in units of capacity to burn fuels (that is, in energy unit per unit of time). Thus, the total investment I represents the overall capacity of new burning equipment. The total investment in new equipment associated with the fuel option i is denoted I_i and satisfies :

$$I_i = s_i I. \tag{4.3}$$

We can now detail the dynamics of fuel substitution. As mentioned above, a vintaging structure is used to portray the aging process of installed equipment. Here, two vintages of capital are kept track of. A more precise description of the aging process should consider more vintages, or continuous aging. However, (47) justifies this choice of a 2-vintages representation by the lack of precise data, and the fact that the model's behavior seems insensitive to the number of modeled vintages. Accordingly, two stock variables are defined for each fuel option i : the capacity of recently installed equipment, the "new" ones KN_i, and those of the older ones KO_i. Investment in new burners I_i increases the capacity of the new equipment. New equipment becomes old after a use of half the lifetime T_i. If, as in (47) (p. 99), a "*fairly wide distribution of lifetimes*" can be assumed, the flow variable associated with the transformation of new equipment into old ones can be assumed to be equal to $\frac{1}{T_i/2}^{\text{th}}$ of the overall capacity of new-burners KN_i. Similarly, an old equipment is scrapped after a use of $\frac{T_i}{2}$ and the flow of scrapped old equipment DO_i is assumed

2. In (25), an informed interpretation is given for α : if the total costs follow a Weibull distribution, α is inversely proportional to the standard deviation of this distribution.

to be equal to $\frac{KO_i}{T_i/2}$. With these assumptions, the dynamics can be formulated as follows :

$$\frac{dKN_i}{dt} = I_i - \frac{KN_i}{\frac{T_i}{2}} , \tag{4.4}$$

$$\frac{dKO_i}{dt} = \frac{KN_i}{\frac{T_i}{2}} - \frac{KO_i}{\frac{T_i}{2}} . \tag{4.5}$$

A simple interpretation of these equations can be provided. For each fuel i at time t, the change in the overall stock of new equipment with respect to time is given by the inflow of new equipment associated with investment I_i, and the outflow caused by aging (that is, the equipment that is no longer new and has to be reallocated into the old category). Similarly, the temporal variation of the stock of old burners results from : the inflow of these previously new equipment, and the outflow corresponding to the scrapping of old equipment.

The next step is to model the dependence between the flow of total investment I and the overall stock of existing equipment. We can first define $K_i = (KN_i + KO_i)$ the total capacity of installed burning equipment with fuel option i, and K the total capacity of installed burning equipment : $K = \sum_i K_i$.

At time t, the overall capacity of scrapped equipment is :

$$DO = \sum_i DO_i = \sum_i \frac{KO_i}{\frac{T_i}{2}} . \tag{4.6}$$

Let's call ED the overall demand for the three fuels at time t, which is an exogenous parameter in this model. Common sense suggests that investment in new equipment should be related in some way to the observed discrepancy between demand and the installed capacity of existing equipment. As this adjustment is usually not instantaneous, (47) introduces the parameter TI, the time to adjust investments that *"determines how fast investments adjust simulated capacity toward exogenous demand."* Accordingly, the total investment has to be modeled as an increasing function of $\frac{ED-K}{TI}$. In addition, investment has to be connected to the total scrapping of old equipment DO to allow a regeneration of the stock of equipment. To model these interactions, (47) postulates the following formula that defines the total investment as a function of these parameters :

$$I = DO.f\left(\frac{ED-K}{TI.DO}\right) , \tag{4.7}$$

where f is a piecewise continuous function that has the following expression :

$$\begin{aligned} f(x) &= x+1 \quad \text{if } x \geq 0 , \\ f(x) &= e^{a.x} \quad\;\; \text{if } x < 0 , \end{aligned} \tag{4.8}$$

where a is a nonnegative parameter. We can observe that, if the total demand ED is equal to the installed capacity K (that is, $ED = K$), the investment will be large enough to compensate for the scrapped equipment DO ($f(0) = 1$). If $ED > K$, investments cause a net rise in the stock of installed equipment ($f(x) > 1$ if $x > 0$). If $ED < K$, some positive investment values can be obtained. However, since $I < DO$, they will cause a net drop in the installed capacity ($f(x) < 1$ if $x < 0$). In the case $ED < K$, the chosen functional specification differs slightly from the affine one used in the original model ((47), fig. 2). This change is guided by the desire to implement a robust formulation to extreme condition testing (30). With an affine specification, a very large drop in demand ED could result in a negative investment value, that is, the premature scrapping of "new" equipment (especially those with the most desirable fuel option). To remedy this, an exponential specification is implemented to insure a nonnegative investment value.

One then has to determine the capacity utilization to allow the model to track exogenous energy demand in case of large downward variations (compared to total scrapping DO). Capacity utilization U is simply defined as :

$$U = \frac{ED}{K} \ . \tag{4.9}$$

Here, capacity utilization is assumed not to be fuel specific as the same capacity utilization figure is posited for the three fuels :

$$\forall i, \quad U_i = U \ . \tag{4.10}$$

As a result, the simulated demand for fuel i, denoted $\hat{D}_i$, is :

$$\hat{D}_i = U_i K_i = ED \frac{K_i}{K} \ . \tag{4.11}$$

Contrary to (47) and (25), we do not model installed plants with multi-firing capability that, in the short-run, are able to switch from one fuel to another and back again in response to price signals. In this model, all the installed equipment is thus supposed to be inflexible with respect to fuel choice. The decision to abandon this part of the original model was guided by market observations that suggest a phase-out of fuel switching capability in industrial plants after the 1980s. (39) and (40) provides an informed discussion on the causes of this phase-out based on the extra-cost and inconvenience associated with the maintenance of a multi-firing capability, and the progressive tightening of emission limits on the burning of fossil fuels.

To summarize, the model (equations (4.1)-(4.11)) corresponds to a system of non-linear differential equations. The associated initial conditions will be detailed in the next section. Because of its complexity, this system has to be simulated with numerical techniques (Euler's method).

§ 4.3 Numerical resolution

This section presents the numerical scheme we have used to solve the model. The time scale has been divided into $N + 1$ time-steps $t_0, t_1 ... t_N$, where $t_{j+1} - t_j$ is constant and equal to $h = \frac{T}{N}$,

$[0, T]$ being the study's horizon. If X is a variable that depends on time, we denote by X^j the value of X at t_j : $X^j = X(t_j)$. If N is big enough (or h small enough), the term

$$\frac{X^{j+1} - X^j}{h}$$

is a good approximation of X's derivative at t_j : $\frac{\partial X}{\partial t}(t_j)$.

Equations (4.4) and (4.5) can therefore be approximated by the following :

$$\frac{KN_i^{j+1} - KN_i^j}{h} = I_i^j - \frac{KN_i^j}{\frac{T_i}{2}} , \tag{4.12}$$

$$\frac{KO_i^{j+1} - KO_i^j}{h} = \frac{KN_i^j}{\frac{T_i}{2}} - \frac{KO_i^j}{\frac{T_i}{2}} . \tag{4.13}$$

Equations (4.12) and (4.15) constitute a numerical (Euler's) scheme to solve partial differential equations. The investment I_i^j is obtained using relations (4.3) and (4.7) :

$$I_i^j = s_i^j \; DO^j \; f\left(\frac{ED^j - K^j}{TI \; DO^j}\right) \tag{4.14}$$

and thanks to (4.3) and (4.6), we can write I_i^j's explicit dependence on KN_i^j and KO_i^j and plug it in (4.12) and (4.13) :

$$\frac{KN_i^{j+1} - KN_i^j}{h} = \left(\frac{e^{-\alpha C_i^j}}{\sum_i e^{-\alpha C_i^j}}\right)\left(\sum_i \frac{KO_i}{\frac{T_i}{2}}\right) f\left(\frac{ED^j - \sum_i \left(KO_i^j + KN_i^j\right)}{TI \sum_i \frac{KO_i}{\frac{T_i}{2}}}\right) - \frac{KN_i^j}{\frac{T_i}{2}} , \tag{4.15}$$

$$\frac{KO_i^{j+1} - KO_i^j}{h} = \frac{KN_i^j}{\frac{T_i}{2}} - \frac{KO_i^j}{\frac{T_i}{2}} . \tag{4.16}$$

or

$$KN_i^{j+1} = KN_i^j + h\left(\left(\frac{e^{-\alpha C_i^j}}{\sum_i e^{-\alpha C_i^j}}\right)\left(\sum_i \frac{KO_i}{\frac{T_i}{2}}\right) f\left(\frac{ED^j - \sum_i \left(KO_i^j + KN_i^j\right)}{TI \sum_i \frac{KO_i}{\frac{T_i}{2}}}\right) - \frac{KN_i^j}{\frac{T_i}{2}}\right) , \tag{4.17}$$

$$KO_i^{j+1} = KO_i^j + h\left(\frac{KN_i^j}{\frac{T_i}{2}} - \frac{KO_i^j}{\frac{T_i}{2}}\right) . \tag{4.18}$$

If the differential equations' initial conditions are known, *i.e.*, $KN_i^0 = KN_i(t = 0)$ and $KO_i^0 = KO_i(t = 0)$, then it is possible to solve, recursively, equations (4.17) and (4.18) and calculate KN_i^j and KO_i^j, $\forall j \in \{0, 1...N + 1\}$.We can prove that the numerical solution converges towards the exact solution when the time-step h is small enough. Mathematically, this can be written by :

$$\forall i,\ \forall j,\quad KN_i^j \longrightarrow KN_i(t_j) \quad \text{when } h \longrightarrow 0 \tag{4.19}$$

$$\forall i,\ \forall j,\quad KO_i^j \longrightarrow KO_i(t_j) \quad \text{when } h \longrightarrow 0 \tag{4.20}$$

§ 4.4 Calibration and results

In this section, we present the data used in our simulations and detail the calibration of the model before discussing the obtained results.

4.4.1 Context

The national energy contexts (domestic resource endowment, composition of the industrial sector, energy policies and energy taxation regimes, etc.) vary greatly from one industrial country to another and national specificities play a non-negligible role in the fuel consumption patterns observed in the industrial sector. Accordingly, a country-level perspective has been adopted to analyze the cases of eight industrial countries that are members of the International Energy Agency (IEA) : Canada, France, Germany, Italy, Japan, South Korea, the UK and the USA. The hydrocarbon fuel consumption of their industral sectors are the largest among IEA members and, in 2008, collectively represented 79.6% of the overall industrial fuel consumption of all the IEA member countries (17).

In this study, we aim at analyzing the adjustment to the relative fuel prices that occurred during the period 1978-2008.[3] This sample period covers the second oil shock, the oversupply-based counter shock associated with the collapse of oil prices that started in 1986 and, more recently, the high-oil price regime that began in late 2003 and unfolded until 2008. During these last 30 years, there has been a net decline in the energy intensity in these eight economies. With exception of South Korea where a net rise in the fuel demand of the industrial sector has been observed, the overall amount of fuel consumption in the energy sector has either diminished (Europe and USA) or has been maintained (Canada, Japan). In terms of fuel substitution, the share of coal remained steady whereas gas consumption increased sharply at the expense of oil (17).

4.4.2 Data and model calibration

The data employed in this study consists of time series gathered from the IEA. The fossil fuel consumption data - measured in toe - are those listed in the "Total Industry" category in the IEA World energy balances under the headings "Coal and coal products", "Oil products" and "Gas" (17). Similarly, the price data refer to the national end-use prices in US dollar reported in (18) under the headings "Steam coal", "High sulfur fuel oil" and "Natural gas".[4] All prices are given

3. The non-inclusion of the earlier period has been imposed by practical considerations on data availability. Indeed, the IEA no longer provides time series on end-user prices for the period 1960-1977.

4. For periods with missing price data, end-use prices have been reconstructed using the indices of energy prices by sector reported by the IEA.

in 2008 US$/toe. In South Korea, natural gas consumption began just after the commencement of gas imports in 1987. In that country, an infinite price of natural gas was hence assumed for the period 1978-1986.

We can now detail the model's calibration. In (47) and (25), a Bayesian approach is used where most of the parameters' values are derived from direct observations (costs, efficiency of the burners, etc.) and educated guesses (the coefficient for the logit specification α). Some parameters (especially those pertaining to preferences and the initial values of the stocks), are then revised thanks to an iterative procedure aimed at improving the fit between simulated and historical behavior. Arguably, such an iterative procedure may somehow involve some subjectivity (30). To remedy this, an automatic calibration (AC) procedure is applied to minimize the deviation between a simulated outcome and historical data. According to (30), a parsimonious approach should guide the practical implementation of AC. Accordingly, the use of AC has been restricted to "the smallest possible calibration problems." In line with Moxnes' Bayesian approach, *a priori* information has thus been used for the observable parameters (costs, efficiency of the burners, etc.) and the AC procedure has been applied to solely adjust the value of the most uncertain parameters (initial values of the stocks, α, etc.).

Our assumptions are based on (47) (table 2) and are summarized in table 4.2 (Note : the CO_2 emission factors are drawn from (17) :

TABLE 4.2 – *Cost assumptions for the industrial sector.*

	Year	Coal	Oil	Gas
Capital costs CC_i (\$/utoe [a] per year)	all	410	200	200
Payback time PBT_i (years)	all	5	5	5
Other operating costs OO_i (\$/utoe[a] per year)	all	70	40	40
Burner efficiencies E_i	1978-1982	70	75	75
(% useful)	1983-2008	71	76	76
Lifetime of burners T_i (years)	all	25	25	25
CO_2 emission factor $Q_{CO_2 i}$ (tCO_2/toe)	all	3.881	3.207	2.337

a. Hereafter, utoe is used to denote useful toe.

In addition, the parameters associated with the investment function have to be defined. The coefficient a in the function f defined in equation (4.8) has been set equal to 0.231, a value interpolated from the exteme left point in (47) (Fig. 2). Besides, the time to adjust total investments TI is assumed to be equal to 1 year because, contrary to the 1960s (Moxnes used 0.25 year), we can reasonably posit that industrial investment during the period 1978-2008 was not primarily guided by "building ahead of demand" motives.

We can now detail the main features of the AC procedure. Here, we rely on the model reference optimization (MRO) method described in (30). We first specify an error function capable of measuring the distance between the observed and simulated behavior as a function of the model's parameter values. For each fuel i, a fuel-specific distance can be evaluated with the absolute error, that is, the sum of the absolute discrepancies between historical D_i^t and estimated $\hat{D}_i^t$ fuel consumption. The model's error function is thus defined as the sum of these three fuel-specific distances :

$$e = \sum_i \sum_t |D_i^t - \hat{D}_i^t| \ . \tag{4.21}$$

We note that this function gives an equal weight to each fuel and each observation no matter when it was recorded. Using the model's equations above, it is possible to specify the error e as a multivariate function of the parameters to be estimated, namely the nonnegative values of the initial stocks $\left(KO_i^0\right)_i$ and $\left(KN_i^0\right)_i$, the nonnegative coefficient for the logit specification α, and the premiums for both oil PR_2 and natural gas PR_3 (PR_1 is set equal to 0\$/toe).

Following (30), the AC procedure is then specified as an optimization problem : finding the parameter values that minimize this distance subject to feasibility constraints (the non-linear equations presented in the preceding section). The optimization problem at hand is a nonconvex, nonlinear mathematical program that can be successfully attacked by modern global solvers. [5] Table 4.3 reports the parameters' values obtained thanks to the AC procedure, for the countries studied.

Table 4.3 – *Calibrated values of the parameters.*

	Initial capacities (Mtoe/year)						alpha (utoe/\$)	Premiums (\$/utoe)	
	KN^0_{coal}	KO^0_{coal}	KN^0_{oil}	KO^0_{oil}	KN^0_{gas}	KO^0_{gas}	α	PR_{oil}	PR_{gas}
Canada	3.63	0.58	-	13.56	13.95	-	0.0073	55.1	140.2
France	8.35	-	-	20.33	6.17	-	0.0220	0.0	119.0
Germany	13.35	34.90	-	30.83	-	23.16	0.0112	-	317.7
Italy	3.52	-	-	19.56	8.14	-	0.0107	105.5	229.0
Japan	14.95	22.01	-	98.02	5.57	-	0.0047	-5.8	124.1
Korea	2.74	-	10.31	0.00	-	-	0.0128	-80.7	48.0
UK	9.13	-	-	16.68	13.92	-	0.0087	352.2	400.3
USA	59.35	-	11.56	116.39	-	176.90	0.0304	-25.2	99.0

From these calibration results, several facts stand out. First, the initial stocks of new burners in 1978 suggest that, with the exception of Korea, the installed oil burning capacities mainly

5. Here, the LINDOGlobal optimization procedure is applied.

consist of old burners, revealing a limited investment in oil burning appliances in the previous years. This finding looks coherent with the oil diversification policies initiated after the first oil shock. Then, (47) and (25) underlines that the multinomial logit model used for the investment shares involves an implicit assumption : that the total costs follow a Weibull distribution. Thus, α the coefficient for the logit specification is inversely proportional to σ the standard deviation of the cost distribution : ($\sigma = \frac{\pi}{\alpha\sqrt{6}}$). According to the obtained values, the standard deviations of total costs range from \$42.2 per useful toe in the USA to \$270.0 per useful toe in Japan. Finally, the relatively large values of the natural gas premiums (compared to the oil ones) reveal a strong preference regarding that fuel in investments. Several features of natural gas can justify this preference, such as the wish to diversify energy sources in oil-importing economies after the two oil shocks, and the cleanliness of natural gas at a time of raising environmental concerns.

4.4.3 Results and validation

The validation of a system dynamics model usually involves two dimensions : *(i)* structural validity, and *(ii)* behavioral validity. The purpose of the former is to check whether the implemented structure constitutes, or not, an adequate representation of the phenomenon to be modeled, whereas the aim of the latter is to compare the model generated behavior to the observed behavior ((3), (33)).

In this study, the modeling framework is derived from a classical approach and is thus firmly grounded in previous knowledge. Nevertheless, a meticulous check of its structural validity is carried out. The model at hand has a moderate complexity which considerably eases these verifications (logical coherence of the set of modeled equations, the dimensional consistency of each equation, the robustness against extreme parameter values, etc.). Following a recommendation in (33), this model was also submitted to the judgment of a group of practitioners (corporate planners, executives) and academics whose research is focused on energy issues. All these assessments confirmed the logical soundness of this model built to capture the main drivers of the fuel substitutions dynamics. Accordingly, we can feel confident in the model's ability to "generate the right behavior for the right reasons."

Concerning behavioral validity, figure 4.1 and figure 4.2 show both the historical and simulated demand behavior for the eight countries. A visual inspection of these plots suggests that the calibrated models satisfactorily capture the history of fuel consumption in these countries.

In addition, some quantitative tools for the analysis of fit are reported in table 4.4. The root mean square errors (RMSE) measure the magnitude of the errors. To ease comparisons across series/countries, a normalized measure of these errors is also presented : the mean absolute percent error (MAPE). According to these findings, the fit to historical behavior is quite good, particularly for Canada, Italy and Japan. The large MAPE figure obtained for South Korea's industrial gas demand can be explained by the formulation chosen for the AC procedure. Indeed, our ob-

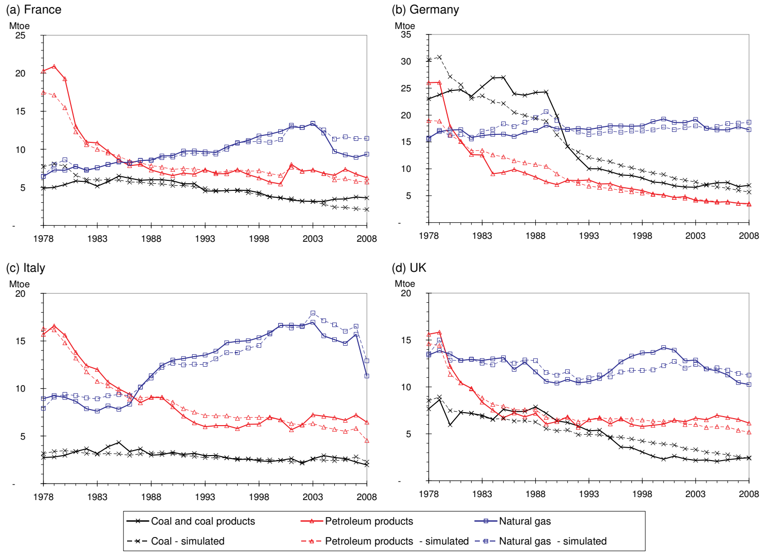

FIGURE 4.1 – *Historical and simulated consumption behavior of industrial annual fuel demand in France, Germany, Italy and the UK.*

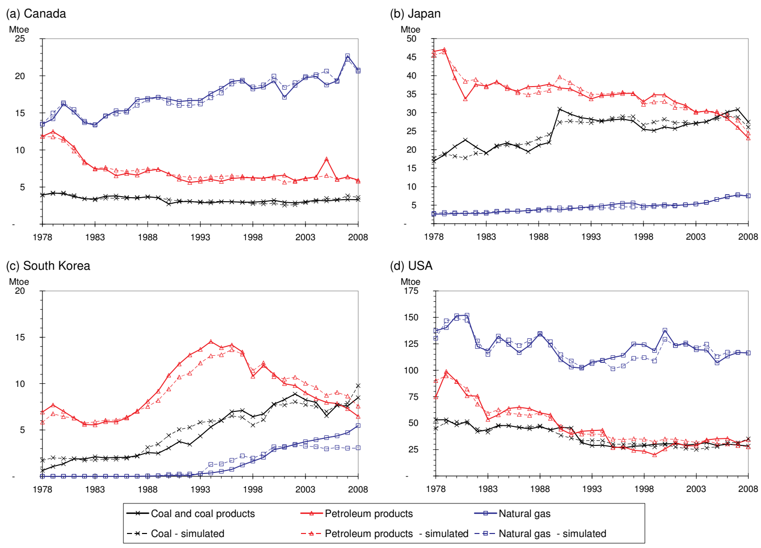

FIGURE 4.2 – *Historical and simulated consumption behavior of industrial annual fuel demand in Canada, Japan, South Korea and USA.*

Table 4.4 – *Error analysis of the model.*

		MAPE (%)	RMSE (ktoe)	U_M (%)	U_S (%)	U_C (%)
Canada	Coal	5.4%	232	0.1%	3.5%	96.4%
	Oil	4.9%	557	0.0%	9.4%	90.6%
	Gas	2.4%	574	0.1%	1.2%	98.8%
France	Coal	14.5%	1045	0.1%	30.1%	69.7%
	Oil	8.4%	1244	3.8%	68.7%	27.5%
	Gas	5.7%	891	5.3%	0.1%	94.6%
Germany	Coal	15.5%	3002	0.0%	1.9%	98.1%
	Oil	12.0%	2230	0.1%	23.8%	76.1%
	Gas	6.2%	1250	0.3%	2.5%	97.2%
Italy	Coal	9.5%	385	3.9%	16.7%	79.4%
	Oil	9.9%	865	1.1%	2.0%	97.0%
	Gas	6.9%	924	3.2%	1.9%	94.9%
Japan	Coal	5.4%	1700	0.8%	0.1%	99.0%
	Oil	3.5%	1553	2.6%	0.6%	96.8%
	Gas	6.8%	400	5.7%	4.3%	90.0%
Korea	Coal	22.4%	781	7.6%	6.6%	85.8%
	Oil	8.2%	927	1.9%	12.7%	85.3%
	Gas	63.2% [a]	721	1.5%	31.3%	67.2%
UK	Coal	21.4%	950	2.6%	24.9%	72.5%
	Oil	7.8%	693	0.5%	12.9%	86.6%
	Gas	6.0%	916	1.3%	15.7%	82.9%
USA	Coal	8.3%	3783	1.6%	3.4%	95.0%
	Oil	15.3%	6519	5.5%	3.3%	91.2%
	Gas	4.0%	6066	3.0%	0.0%	97.0%

a. In Korea, gas consumption began in 1987. To avoid a division by 0, this figure corresponds to the period 1987-2008.

jective function pays attention to absolute differences between historical and simulated values, whereas the MAPE is a relative average normalized measure. For Korea, the MAPE is heavily twisted by the presence of large relative errors during the decade 1987-1997. During that period, gas consumption was ramping up in South Korea and the resulting consumed volumes remained small relative to the generated error. For the UK, most of the discrepancies are observed on the gas and coal series between 1996 and 2003, a period of very low gas prices underpinned by increased competition and upstream developments in the North Sea. During that period, many market observers documented a "dash for gas" causing the premature scrapping of coal burning equipment replaced by gas-fired ones, a behavior that has not been modeled here. Arguably, the observed discrepancies between the simulated and historical series for both gas and coal provide an order of magnitude of the amplitude of this unmodeled phenomenon. For Germany, the model poorly explains the oil and coal consumption monitored in the 1980s. For Germany, the model hardly explains oil and coal consumption in the 1980s but performs significantly better in the subsequent period. Several explanations can be proposed for this poor performance including *(i)* the possibility of under-optimal fuel choices in GDR industries prior to German reunification, *(ii)* the unmodeled subsequent modernization of these industries, *(iii)* the possibly debatable quality of the "reconstructed" energy statistics for the aggregate country in the 1980s (especially those on energy prices), and *(iv)* the unmodeled coal-friendly policy conducted in West Germany that resulted in a net rise in coal consumption between 1979 and 1983 ((34)).

In addition, the Theil inequality statistics detailed in (37) provide a useful decomposition of the mean square errors in terms of bias (U_M), unequal variation (U_S), and unequal co-variations (U_C). In most cases, the largest share of the MSE can be ascribed to U_C the imperfect covariation component of the Theil inequality statistics. The low bias and variation components of these statistics indicate that the errors are unsystematic, meaning that the models can replicate the observed behaviors.

These results together with the graphical representations suggest that the model does a good job of tracking the observed interfuel substitutions.

§ 4.5 Conclusion

The extent to which alternative fuels can substitute for natural gas in the industrial sector is an issue of substantial interest to both energy policy analysts and corporate planners alike. It has recently been underlined that most of the large-scale representations of the natural gas industry embed a rudimentary representation of the demand side.

To remedy this, a revisited version of the system dynamics model proposed by (47) is put to work to analyze fuel choices in the industrial sector. This model emphasizes the role of prices in analyzing interfuel substitutions and captures the dynamic adjustment of demand to relative fuel prices using a vintaging structure. Using data on eight of the OECD countries for the period 1978-2008, we found that this model can satisfactorily replicate past patterns of fuel consumption. These performances make the model an appealing tool to examine fuel substitution possibilities in industrial energy demand.

This approach has been used in order to calculate the demand function for natural gas, taking into account the possible fuel substitution. The results are presented in the next chapter.

- Chapitre 5 -

Construction of a fuel demand function portraying interfuel substitution.

§ 5.1 Introduction

The system dynamics model presented in chapter 4 offers great appeal for the prospective analysis of industrial energy demands. Given the poor representation of the demand side included in most natural gas market models, one could thus wish to embed this system dynamics-based model within a partial equilibrium model of the natural gas markets. Unfortunately, all these models require the formal specification of a single-equation function of the demand for natural gas. In this section, this system dynamics-based representation is put to work to construct such a single-equation demand function.

Hereafter, the reference year is assumed to be $t_0 = 2008$ (the calibration presented in chapter 4 focuses on the period [1978,2008]) and we analyze the future annual consumption of a given fuel, in one of the countries listed above, in year $t > 2008$. For the sake of clarity, we detail the case of natural gas consumption in Canada but this approach is general and can be used to model the industrial demand of any two other fuels in any country. In addition, we assume the availability of an exogenous scenario that details the evolution of future total final fossil energy consumption and both coal and oil domestic prices in any future year $t > 2008$.

In this chapter, we will use chapter 4's notation.

5.1.1 Modeling next year's demand

To begin with, we focus on the first future year (that is, $t_0+1 = 2009$) and detail the construction of a single-equation demand function for that year. To do so, a series of simulations of the system dynamics model are conducted with, *ceteris paribus*, various values of the 2009 price of natural gas. Using a large sample (1000 values) of possible 2009 gas prices regularly drawn over a wide range, we can generate a large data set that depicts the instantaneous change in the quantity of natural gas demanded in 2009 as a function of the 2009 price of that fuel.

As an illustration, figure 5.1 depicts the results of a series of numerical simulations conducted for the case of Canada with the following exogenous parameters : $FP_{coal}(2009) = 165\$/\text{toe}$ (coal price in 2009), $FP_{oil}(2009) = 1030\$/\text{toe}$ (oil price in 2009), and $ED(2009) = 30\text{Mtoe}$ (total final energy consumption in 2009).

These "pseudo data" can in turn be used to estimate the parameters of a single-variable, single-equation, demand function for the year 2009. This empirical demand function aims at providing an easy-to-handle representation of the quantity of fuel consumed in 2009 (the response variable) as a function of the own fuel price that year (the explanatory variable). Our simulation results (given in figure 5.1) suggest that the quantity of fuel demanded should be modeled as a smooth and monotonically decreasing function of that fuel's price. For a very low price level, the fuel under scrutiny nearly captures all the new investments whereas the quantity demanded saturates at large

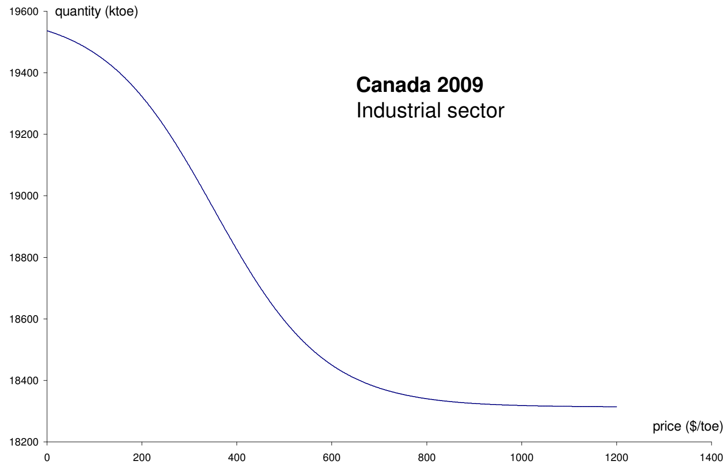

FIGURE 5.1 – *The numerical demand function. Canada, industrial sector, natural gas, year 2009.*

values of this fuel prices, and this saturation level is set by the capacity of previously installed burning equipment. As all our simulations suggested the presence of an "S" shaped pattern, we explored the possibility of modeling these simulation results with an empirically determined sigmoid curve. Among the set of mathematical functions with an S-shaped curve (*e.g.*, logistic function, Gompertz function, etc.), our experiments lead us to consider the hyperbolic tangent. For each year t, we thus propose to fit the relation between simulated demand and price with the following functional form :

$$\hat{q}(p) = \beta + \delta.\left(1 - \tanh\left(\gamma.(p - pc)\right)\right) \tag{5.1}$$

where $\hat{q}$ is the approximated quantity of fuel demanded in 2009, p is the 2009 fuel price (the explanatory variable), t is the time, and β, δ, γ, and pc are nonnegative parameters. The function $tanh$ is the hyperbolic tangent :

$$\forall x \in \mathbb{R}, \quad \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \tag{5.2}$$

According to Formula (5.1), the proposed approximated demand function is monotonicaly decreasing. This specified demand does not rise to $+\infty$ when the price is very low. This is principally due to the fact that the total final energy demand is exogenous to our model. Hence, the demand for natural gas remains upper-bounded. When the price is very high, we can notice that the quantity demanded converges towards a finite positive value β, that captures the "clay" effect, that is, the remaining demand originating from all the previous investments done in that fuel.

An interesting interpretation can be associated with the proposed approximation. This specification can be decomposed in two components, a constant term β that captures the rigidity associated with past decisions, and a price-variable term that measures the instantaneous reaction of demand to the current price (that is, $\delta.(1 - \tanh(\gamma.(p - pc)))$). Concerning the latter term, the parameter pc, which is the inflexion point of the curve (cf. figure 5.2), can be interpreted as a measure of the price of an alternative composite energy utilizing both coal and oil. Thus, the value of this parameter is influenced by the prices of both coal and oil products. The curvature parameter γ represents how fast the natural gas usage drops within a year, if the gas price rises. It is directly linked to the derivative of the demand function at the competing energy price pc. The amplitude parameter δ is connected with the share of the total annual fuel demand that is subject to interfuel substitutions.

If we denote by $q(p)$ the simulated demand provided by the system dynamics model and $\hat{q}(p)$ the one given in equation (5.1), the *error* (distance between q and $\hat{q}$) can be defined as follows :

$$error = \frac{< |q(p) - \hat{q}(p)| >}{< q(p) >} \tag{5.3}$$

The $< . >$ is the mean value. The mean value of a one-variable function f is defined as follows :

$$< f >= lim_{a \to +\infty} \frac{\int_{-a}^{a} f(x)\mathrm{d}x}{2a} \tag{5.4}$$

The values of the parameters β, δ, γ and pc are derived from a minimization of the *error* function.

Table 5.1 – *Optimal parameters, Canada, industrial sector, natural gas, year 2009.*

β	(ktoe)	$1.84\ 10^4$
δ	(ktoe)	$0.65\ \ 10^3$
γ	(\$/toe^{-1})	0.0043
pc	(\$/toe)	352

As an illustration, table 5.1 details the optimal values of the parameters β, δ, γ and pc found for the case of natural gas industrial consumption in Canada for the year 2009. Figure 5.2 illustrates the quality of the numerical fit for that case. Apparently, the proposed formulation (5.1) does an excellent job of tracking the simulated gas consumption as it is almost impossible to distinguish between the simulated pseudo-data and the proposed S-shaped approximation. This finding is also confirmed by the numerical value of the associated *error* which is low : 10^{-3}.

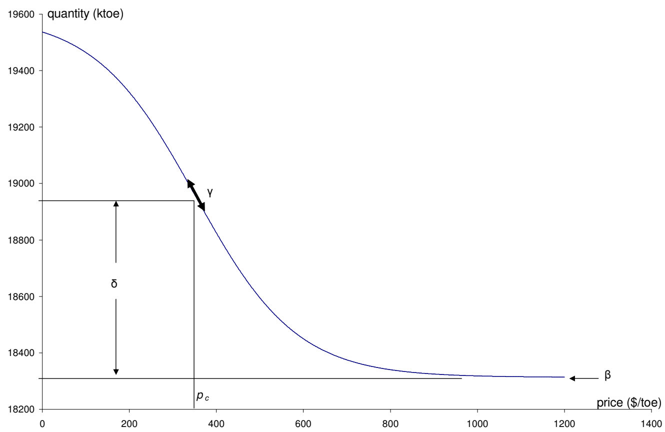

FIGURE 5.2 – *The numerical fit. Canada, industrial sector, natural gas, 2009.*

Given the good fit offered by this specification, we can discuss the implied short-run price elasticity of natural gas demand. This elasticity is given by the following function (issued from equation (5.1)) :

$$\epsilon(p) = -\frac{p\gamma\delta.\left(1 - \tanh^2(\gamma(p - pc))\right)}{\beta + \delta.\left(1 - \tanh(\gamma(p - pc))\right)} \tag{5.5}$$

Ceteris paribus, this short-run elasticity is a decreasing function of the addiction parameter β, which is quite intuitive.

With usual numerical values, the graph of this function has the shape depicted in figure 5.3. From the example of Canada in 2009, the magnitude of the short-run price elasticity of natural gas demand remains low. Our experiments conducted with the other seven countries systematically confirmed the fact that, in the short-run, industrial consumers appear to be very little responsive to natural gas price increases. Of course, such a low price-response can have far-reaching consequences when analyzing security of supply issues (2) or the possibility to exert market power in the short-run with the help of an imperfect competition model.

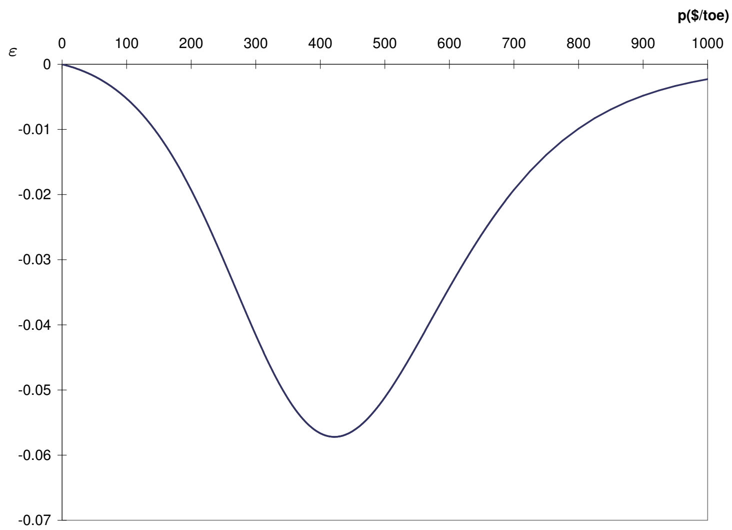

FIGURE 5.3 – *The short-run price elasticity of industrial demand for natural gas. Canada, 2009.*

Of course, the values of the parameters β, δ, γ and pc are conditioned by the chosen scenario (that is. $ED(2009)$, $FP_{oil}(2009)$, and $FP_{coal}(2009)$). Some sensitivity analysis can thus be conducted to analyze the influence of the assumptions embedded in the scenario. As an example, we can study how the value of pc varies with the assumed coal and oil prices. Figure 5.4 gives the evolution of pc over the oil price $FP_{oil}(2009)$, in Canada in 2009. The coal price $FP_{coal}(2009)$ is fixed at 163\$/toe. Our findings show that the price of the alternative energy is an increasing function of the oil price. The saturation effect observed is due to the coal price that remains constant.

Similarly, we can analyze the influence of the global energy demand $ED(2009)$ on the natural gas addiction quantified by the parameter β. Hence, figure 5.5 gives the evolution of β for Canada over the assumed global energy demand $ED(2009)$ for the year 2009. One can notice that there is always a remaining addiction $\beta \neq 0$ even if there is no global energy demand $ED = 0$. This is due to the previous (to 2009) investments in natural gas.

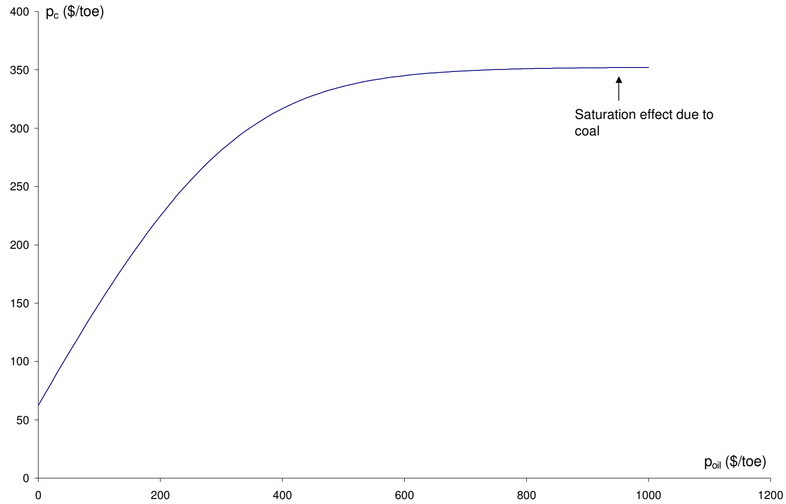

FIGURE 5.4 – *The evolution of pc over $FP_{oil}(2009)$. Canada, industrial sector, natural gas, year 2009.*

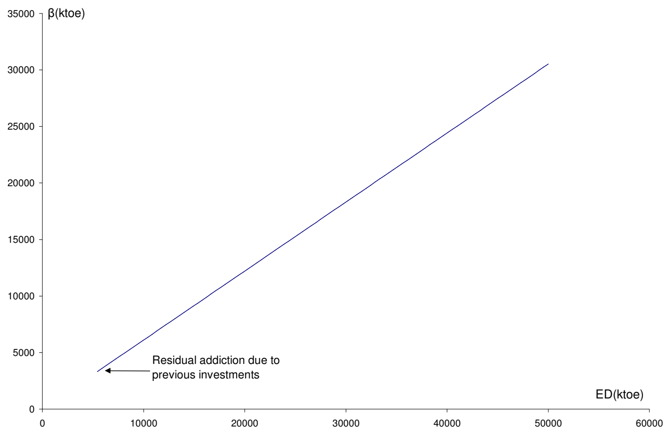

FIGURE 5.5 – *The evolution of β over $ED(2009)$. Canada, industrial sector, natural gas, year 2009.*

According to these findings, the empirical approach at hand provides an acceptable approximation of the relation between simulated fuel demand and the fuel own price in year $t_0 + 1$ for a wide range of possible scenarios. Such a specification could usefully be put to work to refine the demand-side treatment embedded in most static oligopolistic models of the natural gas industry. For example, the popular models detailed in (13), (4) and (15) systematically postulate a simple, downward sloping, affine function to represent the connection between gas price and the volume demanded during the base year. Nevertheless, one could rightly feel uncomfortable with a model solely based on a static vision of the natural gas industry. At least two types of arguments can be advanced to consider a dynamic specification. Firstly, on the supply side, natural gas is an exhaustible resource and gas producers typically have to decide an intertemporal policy (investment, extraction path, etc.). Secondly, on the demand side, the magnitude of the long-run price elasticity of fuel demand is notoriously larger than its short-run counterpart. Any sudden rise in the price of a given fuel can, *ceteris paribus*, have far reaching negative consequences on both the volumes of fuel demanded during actual and future time periods. Concerning the supply-side, some progress has been made as a couple of recent imperfect competition models propose a dynamic treatment of the supply-side ((20), (9)). On the contrary, the dynamic adjustment of volumes demanded to prices has, to our knowledge, never been taken into consideration within an imperfect competition model of the natural gas industry. In most cases, demand behaviour is simplified to an affine demand function depicting an instantaneous relationship between current prices and volumes demanded without any reference to past prices. Such a statement obviously calls for some investigation.

5.1.2 Modeling future demands

By construction, the system dynamics approach presented above is coherent with the fact that the occurrence of a large gas price rise at a given future time t' will result in a lower demand for that fuel during the subsequent periods. In this subsection, we aim at putting this model to work to specify a single-equation demand function that captures such a dynamic adjustment. To begin with, we report how a meticulous analysis of a large number of simulated demand outcomes has guided us in the construction of such a dynamic specification. Then, a numerical example is detailed to illustrate the performances of the proposed specification.

5.1.2.1 Simulations : paving the way to a multivariate specification

Now, we focus on the demand for natural gas at a given future time period t. We assume that an exogenously defined scenario gives, for each time period $t' \leq t$, the overall energy demand $ED(t')$ and the prices of the two alternative fuels $FP_{coal}(t')$ and $FP_{oil}(t')$. Our approach can be decomposed into three successive steps.

1. A large number (10,000) of scenarios have been generated for the future prices of natural gas at any future time t' with $t' < t$. Hereafter, J is used to denote the set of scenarios. If j is used to index the generated scenarios, a gas price scenario can thus be written as $(p^j_{t'})_{t'<t}$

a vector with $t-1$ components. From a practical perspective, these future prices have been randomly generated assuming that future gas prices are i.i.d. random variables that follow an uniform distribution on the interval $[0, 700]\$/\text{toe}$. These assumptions allow us to explore a large domain of possible future price scenarios [1].

2. Then, we propose to analyze, for each scenario j, the instantaneous relationship between the current price of natural gas p_t and q_t^j the volume of natural gas demanded at time t. To do so, the current price p_t is varied so as to generate by simulation, for each scenario j, a data set of 1000 observations of the volume demanded. Unsurprisingly, these observations suggested the presence of a downward sloping, "S" shaped relation between the price p_t and q_t^j.

3. Each of these data sets has in turn been used to fit the following "S-shaped" specification. As a result, we have estimated, for each scenario j, the parameters β_t^j, δ_t^j, γ_t^j and pc_t^j (according to equation (5.1)) :

$$\hat{q}_t^j(p_t) = \beta_t^j + \delta_t^j.\left(1 - \tanh\left(\gamma_t^j.(p_t - pc_t^j)\right)\right) \ . \tag{5.6}$$

For each parameter, we can gather the values $(\beta_t^j)_{j\in J}$, $(\delta_t^j)_{j\in J}$, $(\gamma_t^j)_{j\in J}$ and $(pc_t^j)_{j\in J}$ obtained for the various scenarios j and analyze their distributional properties. Two interesting findings emerged from this analysis. Firstly, the "dispersion", measured either in absolute terms (with the sample standard deviation) or in relative terms (with the coefficient of variation) was extremely low for the series $(\delta_t^j)_{j\in J}$, $(\gamma_t^j)_{j\in J}$ and $(pc_t^j)_{j\in J}$. Accordingly, the values of these three parameters are not influenced by previous gas prices. Secondly, on the contrary, the values $(\beta_t^j)_{j\in J}$ are intimately connected with those of previous gas prices. Moreover, we systematically observed that, with two scenarios j_1 and j_2 that are such that $p_{t'}^{j_1} \leq p_{t'}^{j_2}$ for all $t' < t$, a comparison of the values $\beta_t^{j_1}$ and $\beta_t^{j_2}$ provided $\beta_t^{j_1} \geq \beta_t^{j_2}$, $\forall t' < t$. This latter observation is coherent with the interpretation given for β_t^j in the previous subsection, *i.e.*, a parameter that captures the "clay" effect associated with past investment decisions.

This three-step approach has been replicated for several time horizons t (in the range $t_0 + 2$ and $t_0 + 30$ years), for various countries, various alternative scenarios for both the overall energy demand and the prices of the two alternative fuels (coal and oil). Our empirical findings systematically confirmed the fact that : *(i)* the parameters δ_t^j, γ_t^j and pc_t^j do not depend on previous gas prices, whereas *(ii)* β_t^j exhibits a clear dependence on past values of the natural gas prices. From these investigations, it appears that : the index j can be dropped on the parameters δ_t, γ_t and pc_t, and that β_t^j can be viewed as the value taken by β_t a multivariate function of past gas prices evaluated at the particular point $(p_{t'}^j)_{t'<t}$, *i.e.*, :

$$\hat{q}_t^j(p_t) = \beta_t\left((p_{t'}^j)_{t'<t}\right) + \delta_t.\left(1 - \tanh\left(\gamma_t.(p_t - pc_t)\right)\right) \ . \tag{5.7}$$

1. More subtle probabilistic models fitted on historical time series, including alternative distributions and autocorrelation, have also been considered. Given that the obtained results did not differ from those detailed here, we have decided to maintain these rough assumptions.

In addition, one may wish to elaborate on the path-dependency that is at work for the parameter β_t. As this parameter reflects the rigidity associated with past decisions, it is tempting to relate it to q^j_{t-1} the volumes demanded at time $t-1$ in the scenario j (modulo some aging/scrapping of installed equipment). These latter volumes can, in turn, be approximated by the "S" shaped function $\hat{q}^j_{t-1}$ evaluated at the particular price p^j_{t-1} :

$$\hat{q}^j_{t-1}(p^j_{t-1}) = \beta^j_{t-1} + \delta_{t-1}.\left(1 - \tanh\left(\gamma_{t-1}.(p^j_{t-1} - pc_{t-1})\right)\right) \ . \tag{5.8}$$

Here, the overall volume $\hat{q}^j_{t-1}(p^j_{t-1})$ can also be decomposed into : those precisely decided at date $t-1$, and those encapsulated within the term β^j_{t-1} that reflects earlier decisions. This latter term can in turn be related, modulo some aging/scrapping of installed equipment, to q^j_{t-2}, a volume that can be approximated by $\hat{q}^j_{t-2}(p^j_{t-2})$ and so on...

Because of this nested scheme, one could wish to model the function $\beta_t\left((p^j_{t'})_{t'<t}\right)$, given in (5.7), with an additive specification that explicitly tracks the contributions of earlier vintages :

$$\beta_t\left((p^j_{t'})_{t'<t}\right) = \beta_{0,t} + \sum_{t'<t} h_{t'\to t}\left(\delta_{t'}.\left(1 - \tanh\left(\gamma_{t',t}.(p^j_{t'} - pc_{t',t})\right)\right)\right) \ , \tag{5.9}$$

where $\beta_{0,t}$ denotes the contribution of burners initially present at time t_0, and $h_{t'\to t}$ is a function that models the aging of burning appliances installed at date t'. Rather than specifying these aging processes, we consider that the aging function only alters the amplitude parameters $\delta_{t'}$ so that equation (5.9) can be rewritten as follows :

$$\beta_t\left((p^j_{t'})_{t'<t}\right) = \beta_{0,t} + \sum_{t'<t} \delta_{t',t}.\left(1 - \tanh\left(\gamma_{t',t}.(p^j_{t'} - pc_{t',t})\right)\right) \ . \tag{5.10}$$

Since this formula, which will be confirmed in the following section, holds for a huge number of possible values of $(p^j_{t'})_{t'<t}$, we can drop the scenario index j, and write :

$$\forall (p_{t'})_{t'<t},\ \beta_t\left((p_{t'})_{t'<t}\right) = \beta_{0,t} + \sum_{t'<t} \delta_{t',t}.\left(1 - \tanh\left(\gamma_{t',t}.(p_{t'} - pc_{t',t})\right)\right) \ . \tag{5.11}$$

If we denote $\delta_{t,t} = \delta_t$, both equations (5.10) and (5.7) suggest to model the volumes demanded at date t thanks to the following multivariable specification :

$$\hat{q}_t\left((p_{t'})_{t'\leq t}\right) = \beta_{0,t} + \sum_{t'\leq t} \delta_{t',t}.\left(1 - \tanh\left(\gamma_{t',t}.(p_{t'} - pc_{t',t})\right)\right) \ , \tag{5.12}$$

where $\beta_{0,t}$, $(\delta_{t',t})_{t'\leq t}$, $(\gamma_{t',t})_{t'\leq t}$ and $(pc_{t',t})_{t'\leq t}$ are unknown parameters to be determined numerically.

5.1.2.2 Estimation and performance

We can now clarify the calibration procedure used to fit the approximation specified in equation (5.12). As in subsection 5.1.1, we need to define a distance between the simulated demand function $q(t)$ and the theoretically proposed one $\hat{q}(t)$. Let us re-write the functions while showing the main variables : $q(t,(p_{t'})_{t'\leq t})$ and $\hat{q}(t,(p_{t'})_{t'\leq t})$. It is difficult to define a distance because of the multivariable aspect of the functions, the variables being $(p_{t'})_{t'\leq t}$ and t. Therefore, we define the time-depending error as follows :

$$error(t) = \lim_{n\longrightarrow\infty}\frac{1}{n}\sum_{j=1}^{n}\frac{|q(t,(p^j_{t'})_{t'\leq t}) - \hat{q}(t,(p^j_{t'})_{t'\leq t})|}{q(t,(p^j_{t'})_{t'\leq t})} \tag{5.13}$$

where the variables p^j_1, p^j_2... p^j_t are randomly selected between 0 and 700\$/toe (uniform distribution), for all $j \in \mathbb{N}$. Thanks to the strong law of large numbers, we know that $\frac{1}{n}\sum_{j=1}^{n}\frac{|q(t,(p^j_{t'})_{t'\leq t})-\hat{q}(t,(p^j_{t'})_{t'\leq t})|}{q(t,(p^j_{t'})_{t'\leq t})}$ converges when $n \longrightarrow \infty$.

Here again, our method minimizes the (time-depending) errors in order to estimate the parameters $\beta_{0,t}$, $\delta_{t',t}$, $\gamma_{t',t}$ and $pc_{t',t}$. In the following, we report an illustration obtained for Canada, in year 2013. The following scenario has been used : constant fuel prices FP_{coal}=165 \$/toe, FP_{oil}=1030 \$/toe and a constant overall energy demand ED=30 Mtoe. Table 5.2 gives the values of the parameters $\beta_{0,2013}$, $\gamma_{t',2013}$, $pc_{t',2013}$ and $\delta_{t',2013}$ for $t' \in \{2009...2013\}$.

TABLE 5.2 – *Optimal parameters, Canada, industrial sector, natural gas, year 2013.*

time (t')	2009	2010	2011	2012	2013
$\delta_{t',2013}$ (ktoe)	595	611	625	636	644
$\gamma_{t',2013}$ (\$/toe^{-1})	0.0043	0.0043	0.0043	0.0043	0.0043
$pc_{t',2013}$ (\$/toe)	352	352	352	352	352

$\beta_{0,2013}$ (ktoe)	1.55 10^4

The error we find numerically is small : $\forall t,\ error(t) \leq 10^{-3}$. This is a validation of the use of the functional form provided in equation (5.12) to mathematically describe the demand function.

The new parameter $\beta_{0,t}$ decreases with time. Indeed, formula (5.12) indicates that $\beta_{0,t}$ is the residual demand in year t, when all the previous natural gas prices are very high. This residual consumption is expected to decrease with time if no investments are made in natural gas (which is the case when the natural gas prices are high, considering equation (4.2)).

At a fixed time t, the parameter $\delta_{t',t}$ increases with t', which is intuitive : the consumption dependence on natural gas price in year t' is less and less important in the future. If the global demand remains constant over time, the parameters $\delta_{t',t}$ behave like the following :

$$\delta_{t',t} = \delta_0 \kappa^{t-t'}$$

where δ_0 and κ are constants. We found out that the new parameter κ is roughly the same for all the countries we studied : $\kappa = 0.95$.

There are many advantages to using our model to make a demand forecast. First, we take into account the inertia present in energy consumption, which is due to all the past investments in coal, oil and natural gas. Second, the demand function estimated for gas naturally depends on the other fuel prices. Thus, a competition between fuels, thanks to the substitution aspect, appears in the demand function. Finally, this technique takes into consideration the intertemporal dependence between consumption and prices. Indeed, fuel prices in year t will influence the demand in future years $t' \geq t$. Obviously, if the natural gas price is high in 2010, for instance, compared to the other fuels, few investments will be made in that fuel and the corresponding demand will therefore be low in the future years. In (1), it is shown that this functional form can be used for building imperfect competition models of natural gas markets.

It has been stated before that the addiction parameter β_0 decreases with time. Figure 5.6 shows the evolution of β_0 between 2009 and 2023.

The decrease of β_0 is quasi exponential. We can numerically estimate the following dependence : $\beta_{0,t} = Ke^{-(t-2009)/\tau}$. The values of the constants K and τ in the case we studied (Canada) are $K = 1.83.10^4$ ktoe and $\tau = 19$ years, which is roughly the investments depreciation time factor $T_{natural\ gas}$.

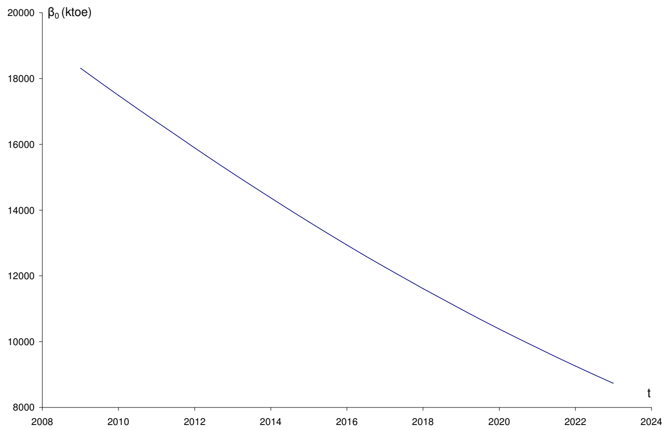

Figure 5.6 – *The evolution of β_0 over time. Canada, industrial sector, natural gas.*

§ 5.2 Conclusion

The previous chapter described a system dynamics approach in order to capture the fuel substitution in the overall fossil energy consumption, between the consumption of oil, coal, and natural gas. An application of the model has been carried out to construct a dynamic demand function for natural gas. More particularly, a large number of simulations have been conducted with the aim to propose an adapted single-equation specification for the demand for natural gas. From these investigations, it appears that a smooth, S-shaped, function can be used to represent the instantaneous reaction of fuel demand to price. In addition, this approach provides the ingredients necessary to capture the dynamic influence of past fuel prices on current consumption level. An extended multivariable specification has thus been derived and successfully tested.

As a result, a multivariable demand function that makes the gas consumption depend on all the previous prices has been estimated and calibrated. Such a function, captures at the same time, fuel substitution and the dynamics of the natural gas consumption. After a linearization process, this demand function was used in large-scale natural gas markets modeling, as shown in the following chapters.

This chapter demonstrates the potential of that system dynamics-based method for deriving demand curves and thus offers a promising approach to further enhance the relevance of existing large-scale models of the natural gas industry.

Bibliographie

[1] I. Abada, V. Briat, S. A. Gabriel, O. Massol, 2010. *A Generalized Nash-Cournot Model for the North-Western European Natural Gas markets with a Fuel Substitution Demand Function : The GaMMES Model.* Economix Working Paper, available at http ://economix.fr/fr/dt/2011.php.

[2] I. Abada, O. Massol, 2011. *Security of supply and retail competition in the European gas market. Some model-based insights.* Energy Policy 39 (2011), 4077-4088.

[3] Y. Barlas, 1989. *Multiple tests for validation of system dynamics type of simulation models.* European Journal of Operational Research 42 (1), 59-87.

[4] M.G. Boots, F. A. M. Rijkers, B. F. Hobbs, 2004. *Trading in the downstream European gas market : a successive oligopoly approach.* The Energy Journal 25 (3) 74-102.

[5] K. C. Chi, W. J Nuttall, D. M Reiner, 2009. *Dynamics of the UK natural gas industry : System dynamics modeling and long-term energy policy analysis.* Technological Forecasting and Social Change 76 (3), 339-357.

[6] T. J. Considine, T. D. Mount, 1984. *The Use of Linear Logit Models for Dynamic Input Demand Systems.* The Review of Economics and Statistics 66 (3), 434-443.

[7] P. I. Davidsen, J. D. Sterman, G. P Richardson, 1990. *A Petroleum life cycle model for the U.S. with endogenous technology, exploration, recovery, and demand.* System Dynamics Review 6 (1), 66-93.

[8] R. G. Egging, S. A. Gabriel, 2006. *Examining market power in the European natural gas market.* Energy Policy 34 (17), 2762-2778.

[9] R. G. Egging, F. Holz, S. A. Gabriel, 2010. *The World Gas Model : A multi-period mixed complementarity model for the global natural gas market.* Energy 35 (10), 4016-4029.

[10] T. S. Fiddaman, 1997. *Feedback complexity in integrated climate-economy models.* Doctoral Thesis Submitted to the Alfred P. Sloan School of Management. Massachusetts Institute of Technology, Cambridge, MA.

[11] A. Ford, 1997. *System Dynamics and the Electric Power Industry.* System Dynamics Review 13 (1), 57-85.

[12] M. A. Fuss, 1977. *The Demand for Energy in Canadian Manufacturing.* Journal of Econometrics 5 (1), 89-116.

[13] R. Golombek, E. Gjelsvik, K. E. Rosendahl, 1995. *Effects of liberalizing the natural gas market in Western Europe.* The Energy Journal, 16 (1), 85-111.

[14] R. Golombek, M. Hoel, J. Vislie, 1987. *Natural gas markets and contracts.* North-Holland, Amsterdam.

[15] F. Holz, C. van Hirschhausen, C. Kemfert, 2008. *A strategic model of European gas supply (GASMOD).* Energy Economics, 30 (3), 766-788.

[16] A. Honoré, 2010. *European Natural Gas. Demand, Supply and Pricing.* Oxford Institute for Energy Studies. Oxford University Press, Oxford.

[17] International Energy Agency, 2010a. World energy balances. IEA World Energy Statistics and Balances. OECD/IEA, Paris.

[18] International Energy Agency, 2010b. Energy Prices and Taxes. OECD/IEA, Paris.

[19] P. L. Joskow, F. S. Mishkin, 1977. *Electric Utility Fuel Choice Behavior in the United States.* International Economic Review 18 (3), 719-736.

[20] W. Lise, B. F. Hobbs, 2008. *Future evolution of the liberalised European gas market : Simulation results with a dynamic model.* Energy 33 (7), 989-1004.

[21] O. Massol, S. Tchung-Ming, 2010. *Cooperation among liquefied natural gas suppliers : Is rationalization the sole objective ?* Energy Economics 32 (4), 933-947.

[22] L. Mathiesen, K. Roland, K. Thonstad, 1987. *The European network gas market : Degrees of market power on the selling side.* In : Golombek, R., Hoel, M., Vislie J. (Eds.), Natural Gas Markets and Contracts. North-Holland, Amsterdam.

[23] J. D. W. Morecroft, 2007. *Strategic Modelling and Business Dynamics.* John Wiley and Sons, Chichester.

[24] E. Moxnes, 1987. *The dynamics of Interfuel Substitution in the OECD-Europe Industrial Sector.* In : Golombek, R., Hoel, M., Vislie J. (Eds.), Natural Gas Markets and Contracts. North-Holland, Amsterdam.

[25] E. Moxnes, 1990. *Interfuel substitution in OECD-European electricity production.* System Dynamics Review 6 (1), 44-65.

[26] R. F. Naill, 1977. *Managing the Energy Transition : A System Dynamics Search for Alternatives to Oil and Gas.* Ballinger Publishing Company, Cambridge, MA.

[27] R. F. Naill, 1992. *A system dynamics model for national energy policy planning.* System Dynamics Review 8 (1), 1-19.

[28] R. F. Naill, S. Belanger, A. Klinger, E. Petersen, E., 1992. *An analysis of the cost effectiveness of US energy policies to mitigate global warming.*, System Dynamics Review 8 (2), 111-128.

[29] Y. Olaya, I. Dyner, 2005. *Modelling for Policy Assessment in the Natural Gas Industry.* The Journal of the Operational Research Society, 56 (10), 1122-1131.

[30] R. Oliva, 2003. *Model calibration as a testing strategy for system dynamics models.* European Journal of Operational Research, 151 (3), 552-568.

[31] R. S. Pindyck, 1979. *The Structure of World Energy Demand.* M.I.T. Press, Cambridge, MA.

[32] R. Ponzo, A. Dyner, S. Arango, E. R. Larsen, 2011. *Regulation and development of the Argentinean gas market.* Energy Policy 39 (3), 1070-1079.

[33] H. Qudrat-Ullah, B. S. Seong, 2010. *How to do structural validity of a system dynamics type simulation model : The case of an energy policy model.* Energy Policy 38 (5), 2216-2224.

[34] P. Renou-Maissant, 1999. *Interfuel competition in the industrial sector of seven OECD countries.* Energy Policy 27 (2), 99-110.

[35] Y. Smeers, 2008. *Gas models and three difficult objectives.* ECORE Discussion papers 2008/13. Louvain-la-Neuve.

[36] J. D. Sterman, 1981. *The Energy Transition and the Economy : A System Dynamics Approach.* Doctoral Thesis Submitted to the Alfred P. Sloan School of Management. Massachusetts Institute of Technology, Cambridge, MA.

[37] J. D. Sterman, 1984. *Appropriate summary statistics for evaluating the historical fit of system dynamics models.* Dynamica 10 (Winter), 51-66.

[38] J.D Sterman, 2000. *Business Dynamics : Systems Thinking and Modeling for a Complex World.* Irwin McGraw-Hill, Boston.

[39] J. Stern, 2007. *Is There A Rationale for the Continuing Link to Oil Product Prices in Continental European Long-Term Gas Contracts ?* Oxford Institute for Energy Studies NG 19. Oxford.

[40] J. Stern, 2009. *Continental European Long-Term Gas Contracts : is a transition away from oil product-linked pricing inevitable and imminent ?* Oxford Institute for Energy Studies NG 34. Oxford.

[41] J. Tirole, 1988. *The Theory of Industrial Organization.* MIT Press, Cambridge, MA.

[42] G. Urga, C. Walters, 2003. *Dynamic translog and linear logit models : a factor demand analysis of interfuel substitution in US industrial energy demand.* Energy Economics 25 (1), 1-21.

[43] D. G. Victor, A. M. Jaffe, M. H. Hayes, 2006. *Natural Gas and Geopolitics : From 1970 to 2040*. Cambridge University Press, Cambridge, UK.

[44] G. C. Watkins, 1992. *The econometric analysis of energy demand : perspectives of a practitioner*. In Energy Demand : Evidence and Expectations, Hawdon, D. (ed.), Surrey University Press and Academic Press Limited : London.

[45] F. P Wood, J. C. Geinzer, 1997. *The Ideas Model and Its Use in Developing the US Climate Change Action Plan*. In : Bunn, D.W., Larsen, E.R. (Eds.), In Systems Modelling for Energy Policy. John Wiley and Sons, Chichester.

Cinquième partie

Modeling natural gas markets in Europe : the GaMMES model

- Chapitre 6 -

A Generalized Nash-Cournot model for natural gas markets with a fuel substitution demand function : The GaMMES model.

§ 6.1 Introduction

Quantitative studies and mathematical models are necessary to understand the economic and strategic issues that define energy markets in the world. In that vein, the study of natural gas markets is particularly interesting because most of them, particularly in Europe, show a high dependence on a small number of producers' exports. According to Mathiesen et al. (44), this market structure can be analyzed with strategic interactions and market power. This market power can be exerted at the different stages of the gas chain : by the producers in the upstream market or the local intermediate traders in the downstream market. The European markets are also characterized by long-term contracts established between the producers and the intermediate local independent traders. These long-term contracts were initially designed as a risk-sharing measure between producers and local traders. They are usually analyzed, in particular, as a tool to mitigate the producers' market power. The combination of strategic interactions and long-term contracts makes the study of the natural gas markets evolution particularly subtle and rich.

The economic literature provides an important panel of numerical models whose objective is to describe the natural gas trade structure. As an example, we can cite the "World Gas Trade Model" (Baker Institute) (49), the "EUGAS" model (Cologne University) (48), the "GASTALE" model (Energy Research Centre of the Netherlands) (41) or the "World Gas Model" (University of Maryland) ((14), an extension of the work developed in (21) and (22)). However, most of these models present some necessary simplifying assumptions concerning either the description of the market economic structure or the demand function. For instance, the "EUGAS" model assumes pure and perfect competition between the players and thus neglects market power to allow a detailed description of the infrastructure. The "GASTALE" and "World Gas Model" depict strategic interactions between the players via a Nash-Cournot competition and the latter model also uses exogenous long-term contracts. However, the former model does not include investments in production or in pipeline and storage infrastructure. Besides, the demand representation for all these previous models does not explicitly take into account the possible substitution between different types of fuels (natural gas, oil, and coal, for instance). All these drawbacks have been analyzed in detail in (51)

The partial equilibrium model we develop, named GaMMES, Gas Market Modeling with Energy Substitution, tries to address some of the limitations proposed in (51). It is also based on an oligopolistic approach of the natural gas markets. The interaction between all the players is a Generalized Nash-Cournot competition and we explicitly take into consideration, in an endogenous way, the long-term contractual aspects (prices and volumes) of the markets. Our representation of the demand is new and rich because it includes the possible substitution, within the overall primary energy consumption, between different types of fuels. Hence, in our work, we mitigate market power exerted by the strategic players : they cannot force the natural gas price up freely because some consumers would switch to other fuels.

We study both the upstream and downstream stages of the gas chain, while modeling the possible strategic interactions between all the players, through all the stages. The production side is detailed at the production node level and we choose a functional form derived from Golombek (27) for the production costs. We assume, in our representation that the producers sell their gas through long-term contracts to a set of independent traders who sell it back to end-users, where the Nash-Cournot competition is exerted. Storage and transportation aspects are taken care of by global regulated storage and transportation operators. Producers also have the possibility to directly target end-users for their sales. Both producers and independent traders share market power. The long-term contracts are endogenous to our model and this property (among others) makes our formulation a Generalized Nash-Cournot game. The introduction of non-symmetric independent traders that can exert market power in the spot markets and contract in the long-term with the producers, and are in an oligopolistic competition with them in the downstream induces a rich, double layer economic structure. This is a new feature of the description of the natural gas trade. It allows us to represent long-term contracts and mitigate the producers' market power.

The demand side is also detailed. We use a system dynamics approach (3) in order to model possible fuel substitutions within the fossil primary energy demand of a consuming country, between the consumption of coal, oil, and natural gas. This approach allows us to derive a new and interesting mathematical functional form for the demand function that includes naturally the competition between these. This particular new feature of the gas markets description that we have introduced in our model induces a flexibility in the gas demand representation. It allows us, for example, to study the sensitivity of gas consumption and prices over the oil and coal prices.

We include all the possible investments in the gas chain (production, infrastructure, etc.) and make the long-term contract prices and quantities endogenous to the model using an MCP (mixed complementarity problem) formulation.

This part is divided as follows : chapter 6 presents a theoretical framework for GaMMES. It gives a general description of the chosen economic structure representation. All the players are presented and are divided into two categories : the strategic and the non-strategic ones. The strategic interaction is also detailed in this chapter. A brief description of a system dynamics approach to model the consumers' behavior investment in coal, oil or natural gas so that their utility is optimized is provided. This chapter also presents the mathematical representation of the markets : the optimization programs associated with all the strategic and non-strategic players are presented and discussed. We also explain in particular how we make the long-term contract prices and volumes endogenous to our model. Chapter 7 is an application of our model to the European natural gas trade where the calibration process and the results are discussed. A comparison between our model, a more standard one where the demand does not take into consideration fuel substitution and the European Commission natural gas forecast is carried out in order to compare between the results. In particular, forecasts of the consumption, prices, production, and gas dependence are provided and discussed. Besides, long-term contracts aspects (prices and volumes) are also given, analyzed, and discussed. Chapter 8 gives the theoretical framework of Generalized Nash-Cournot games. It presents in particular the VI/QVI formulations of Standard/Generalized Nash-Cournot games. A formal discussion about VI/QVI solutions is provided, as well as conditions that characterize the VI solutions.

The same notation will be used in chapters 6 and 7.

§ 6.2 The model

6.2.1 Economic description

Our description of the natural gas markets divides them into two stages.

The upstream market is represented by gas producers, each with a dedicated trader (export division) to sell gas to other traders or directly to end-users. An example would be Gazexport for Gazprom. The set of producers and dedicated traders is denoted as P.

Besides the market players just mentioned, there are a number of independent traders whose activity is to buy gas from the big producers (or their traders) and to sell it to the final users in the downstream market. This type of traders includes all the firms whose production is small, compared to their sales (e.g., EDF and GDF-SUEZ [1]). The associated index for these players is I.

The different target markets (the consumers) are divided into three sectors : power generation, industrial, and residential, represented respectively as D_1, D_2 and D_3. However, it is easy to demonstrate that if the sectors do not interact with each other (i.e., the different demand curves are independent), the study of only one sector can easily be generalized to the three. We will make the assumption that the different demand curves do not interact (as an example, the gas price in the industrial sector does not depend *a priori* on the residential price), which may not be realistic for some situations. Hence, to simplify our notation and modeling, we will consider only one consumption set D to represent each country's gross natural gas consumption.

We assume that each dedicated trader can either establish long-term contracts with the independent traders or sell his gas to the spot markets.

The first situation corresponds to a gas trade under a fixed, contracted price, not dependent on the quantities sold (in a first approximation). These quantities are also fixed by the contract. The second situation is characterized by the fact that the spot price is a consequence of the competition between all the traders in the downstream markets, via a specified inverse demand function.

The long-term contracts we consider are modeled as follows : each pair of producer-independent trader have to contract, if needed, on a fixed volume that must be exchanged each year, at a fixed price. We allow for seasonal flexibility within a year, for the low-consumption regimes. This description takes into account the basis of the long-term contracts' Take-Or-Pay clauses (34). For computational reasons and to keep the model's formulation simple, we do not allow for annual

1. GDF-SUEZ produces 4.4% of its natual gas supplies (26)

flexibility of the long-term contract volumes.

All the traders compete via a Nash-Cournot interaction, during a finite number of years Num. Time will be indexed by $t \in T$ (five-year time steps) and we will take into account seasonality by distinguishing, for each year t, between the off-peak and peak seasons. The seasons will be indexed by M. They basically correspond to different demand regimes.

More precisely, the strategic interaction between the players is modeled as the following : the producers can sell their gas directly to the end-users in the spot markets, or to the independent traders via long-term contracts. The independent traders buy gas from the producers only via these long-term contracts and they can sell gas to all the possible spot markets. All the producers and the independent traders are strategic players. They are in competition in the spot markets where they exert market power. This situation is modeled using a Nash-Cournot competition. All the strategic players (producers and independent traders) see the same inverse demand function. All the markets are liberalized. Therefore, each producer can make contracts with all the possible independent traders and sell gas to all the possible spot markets. Similarly, an independent trader can make contracts with all the possible producers and sell gas to all the possible spot markets. Each trader can also store gas in all the possible storage nodes, if the storage capacity is sufficient. The competition in the upstream is not represented as an oligopoly (unlike some models like (41)). Indeed, we do not model the possible traders' demand functions that can be considered, *a priori*, by the producers in their optimization programs. The upstream activity, which is dominated by long-term contracts, is modeled with a supply/demand equilibrium in the long-term between the producers and the independent traders. The corresponding long-term contract price is issued from the supply/demand equality constraints' dual variables.

Since the model is dynamic, we need to take care of possible capacity investment. For infrastructure-related capacity, this corresponds to additional installed capacities. Regarding the production, we do not explicitly model exploration activities, because of a lack of geological data. Therefore, we assume that investments only increase the extraction capacity. We also make the model conservative as we do not endogenously consider possible additional reserves due to exploration activities. Therefore, a gas-producing firm may want to increase its production capacity by investing if this would lead to an increase of its revenue.
We take into consideration the depreciation of the production capacity in the upstream side of the market by introducing a depreciation factor per time unit at each production node : dep_f. To simplify the model (and because of a lack of data concerns), we decided not to take into account the transport or storage capacity depreciations.

The main advantage of the GaMMES model is that it takes into account, in an endogenous way, long-term contracts between the independent traders and the producers. Obviously, this representation is quite realistic for the natural gas trade since the latter is still dominated by long-term selling/purchase prices and volumes. In 2004 the long-term contracts' imports represented more than 46% of the European natural gas consumption and 80% of the total European imports (17) and (35). Another advantage inherent to our description is that the inverse demand function explicitly takes into consideration the possible substitution between consumption for natural gas and the competing fuels.

Considering the energy substitutions in the natural gas demand mitigates the market power that can be exerted by all the strategic players in the end-use markets. Indeed, this is due to the fact that the consumers have the ability to reduce the natural gas share in their energy mixes if the market price for natural gas is much higher than the substitution fuel's (such as oil and coal) price. Therefore, the producers may not have much incentive to reduce their natural gas production in order to force the price up. This model property allows us to take into account the natural gas price dependence on oil and coal prices. Indeed, the Nash-Cournot interaction will link the natural gas price to the coal and oil prices because of the demand function dependence on these parameters.

In order to take into consideration the intra and extra-European physical network of the transport and distribution networks, we need to introduce a pipeline operator whose role is to minimize the transmission costs over all the arcs of the topology. We denote by N the set of all the nodes including the production nodes, the consuming markets, and the storage nodes. Added to the transport cost minimization objective, the pipeline operator also has the possibility to make investments in order to increase the arc capacities, if necessary.

All the arc transport costs are exogenous to the model. The congestion prices are taken into consideration endogenously : they can be obtained by computing the dual variables corresponding to the infrastructure capacity constraint. The set of all these arcs is A. An arc can either be a pipeline or an LNG route.

In order to be able to meet high levels of consumption, we assume that the independent traders have access to a set of storage nodes to store natural gas in the off-peak season, and withdraw it in the peak one. Obviously, they have to support a capacity reservation, storage, withdrawal, and transport costs. All the storage nodes, indexed by the set S, are managed by a global storage operator player. This player can invest in order to increase the storage capacity of each storage node.

Both the pipeline and the storage operators are assumed not to have market power. The storage and transport costs are hence exogenous to the model. The strategic players are therefore the producers/dedicated traders and the independent traders. Obviously, this assumption is an important simplification of reality, where market power can also be exerted by the storage and pipeline operators. However, it is consistent with what can be found in the literature (14), (41).

The storage cost, which is assumed to be supported by the independent traders, is represented using capacity reservation and storage/withdrawal costs. We consider that the average time for the storage investments to be realized is $delay_s$ years (five years). The situation is similar for the infrastructure ($delay_i$) and production capacity investments ($delay_p$) costs supported by the pipeline operator and the producers.

6.2.2 Notation

The units chosen for the model are the following : quantities in toe (i.e., Ton Oil Equivalent) or Bcm and unit prices in \$/toe or \$/cm. The following table summarizes the notation chosen for the exogenous parameters and the endogenous variables.

Exogenous factors

P	set of producers-dedicated traders
I	set of independent traders
D	set of gas consuming countries in the downstream market (no distinction between the sectors) $D \subset N$
T	time $T = \{0, 1, 2, ..., Num\}$
M	set of seasons. Off-peak (low-consumption) and peak (high-consumption) regimes
F	set of all the gas production nodes. $F \subset N$
N	set of the nodes
S	set of the storage nodes $S \subset N$
A	set of the arcs (topology)
Rf_f	production node f's total gas resources (endowment)
Kf_f	production node f's initial capacity of production, year 0
Lf_f	production node f's maximum increase of the production capacity (in %)
Ic_s	injection marginal cost at storage node s (constant)
Wc_s	withdrawal marginal cost at storage node s (constant)
Rc_s	reservation marginal cost at storage node s (constant)
Ls_s	storage node s's maximum increase of the storage capacity (in %)
Pc_f	production cost function, production node f
Tc_a	transport marginal cost through arc a (constant)
Tk_a	pipeline initial capacity through arc a, year 0
Ks_s	initial storage capacity at node s, year 0
Is_s	investment marginal costs in storage (constant)
Ip_f	investment marginal costs in production (constant)
Ik_a	investment marginal costs in pipeline capacity through arc a (constant)
La_a	arc a's maximum increase of the transport capacity (in %)
O	incidence matrix $\in \mathrm{M}_{F\times P}$. $O_{fp} = 1$ if and only if producer p owns production node f
B	incidence matrix $\in \mathrm{M}_{I\times D}$. $B_{id} = 1$ if and only if trader i is located at the consumption node d
$M1$	incidence matrix $\in \mathrm{M}_{F\times N}$. $M1_{fn} = 1$ if and only if node n has production node f
$M2$	incidence matrix $\in \mathrm{M}_{I\times N}$. $M2_{in} = 1$ if and only if trader i is located at node n
$M3$	incidence matrix $\in \mathrm{M}_{D\times N}$. $M3_{dn} = 1$ if and only if node n has market d
$M4$	incidence matrix $\in \mathrm{M}_{S\times N}$. $M4_{sn} = 1$ if and only if node n has storage node s
$M5$	incidence matrix $\in \mathrm{M}_{A\times N}$. $M5_{an} = 1$ if and only if arc a starts at node n
$M6$	incidence matrix $\in \mathrm{M}_{A\times N}$. $M6_{an} = 1$ if and only if arc a ends at node n
H	maximum value for the quantities produced and consumed
δ^t_{md}	an inverse demand function parameter
β^t_{md}	an inverse demand function parameter
γ^t_{md}	an inverse demand function parameter
pc^t_{md}	an inverse demand function parameter

We could have used different upper bounds for the different variables. However, to simplify the notation, we will use the same value H.

fl_f	production node f's flexibility : the maximum modulation between production during off-peak and peak seasons
min_{pi}	percentage of the minimum quantity that has to be exchanged on the long-term contract trade between i and p
δ	discount factor
$delay_{s,i,p}$	period of time necessary to undertake the technical investments
$loss_a$	loss factor through transportation arc a
dep_f	depreciation factor of the production capacity at production node f

Endogenous variables

x^t_{mfpd}	quantity of gas produced by p from production node f for the end-use market d, year t, season m, in Bcm
zp^t_{mfpi}	quantity of gas produced by p from production node f dedicated to a long-term contract with trader i, year t, season m in Bcm
zi^t_{mpi}	quantity of gas bought by trader i from producer p with a long-term contract year t, season m in Bcm
up_{pi}	quantity of gas sold by producer p to trader i with a long-term contract, each year in Bcm
ui_{pi}	quantity of gas bought by trader i from producer p on a long-term contract, each year in Bcm
y^t_{mid}	quantity of gas sold by i to the market d, year t, season m in Bcm
ip^t_{fp}	producer p's increase of production node f's production capacity, due to investments in production, year t, in Bcm/time unit
q^t_{mfp}	production of producer p from production node f, year t, season m in Bcm
p^t_{md}	market d's gas price, result of the Cournot competition between all the traders, year t, season m, in \$/cm
η_{pi}	long-term contract price contracted between producer p and trader i in \$/cm
r^t_{is}	amount of storage capacity reserved by trader i at node s, year t in Bcm
in^t_{is}	volume injected by trader i at storage node s, year t in Bcm
is^t_s	increase of storage capacity at node s, year t due to the storage operator investments in Bcm/time unit
ik^t_a	increase of the pipeline capacity through arc a, year t, due to the TSO investments in Bcm/time unit

fp^t_{mpa}	gas quantity that flows through arc a from producer p year t, season m in Bcm
fi^t_{mia}	gas quantity that flows through arc a from trader i year t, season m in Bcm
τ^t_{ma}	the dual variable associated with arc a capacity constraint year t, season m in \$/cm. It represents the congestion transportation cost over arc a

The table is divided into two parts. The upper half represents the exogenous parameters or functions whereas the lower half represents the different decision variables and the inherent retail prices.
The indices p, d, i, f, n, s, a, m and t are such that $p \in P$, $d \in D$, $i \in I$ $f \in F$, $n \in N$, $s \in S$, $a \in A$, $m \in M$ and $t \in T$.
The long-term contract between producer p and trader i fixes both a unit selling price and an amount to be purchased by the independent trader i each year from producer p. Both price and quantity will be specified endogenously by the model.
Matrix O is such that $O_{fp} = 1$ if producer p owns production node f and $O_{fp} = 0$ otherwise.

Figure 6.1 represents a schematic overview of GaMMES.

6.2.3 The inverse demand function

We need to specify a functional form for the inverse demand function, which links the price p_d at market d to the quantity brought to the market. Most of the natural gas models (49), (48), (41), (14) do not take into account fuel substitution. Let h be the specific inverse demand function. We assume that the long-term contract quantities do not directly influence the market competition price, which is to say that $p^t_{md} = h(\sum_i y^t_{mid} + \sum_f \sum_p x^t_{mfpd})$. (Actually, this assumption is necessary to guarantee the concavity of the objective functions of each strategic player's maximization problem, regardless of the quantities decided by the other competitors. Otherwise, this assumption can be dropped if linear functions are used).
As mentioned in the introduction, we want to capture the inter-fuel substitution in the fossil primary energy consumption. To be able to do so, we used a system dynamics approach that models the behavior of the consumers who have to decide whether they invest in new technologies that use either oil, coal or natural gas. Our model, has been fully presented in Part 3 and (3).

We have run and calibrated the system dynamics model with the primary natural gas consumption and the industrial gas prices. The main result that will be used in this part is as follows : if we denote by Q^t_{md} the total quantity $\sum_i y^t_{mid} + \sum_f \sum_p x^t_{mfpd}$ that is brought to the spot market d at season m of year t, the system dynamics approach provides the following inverse demand function :

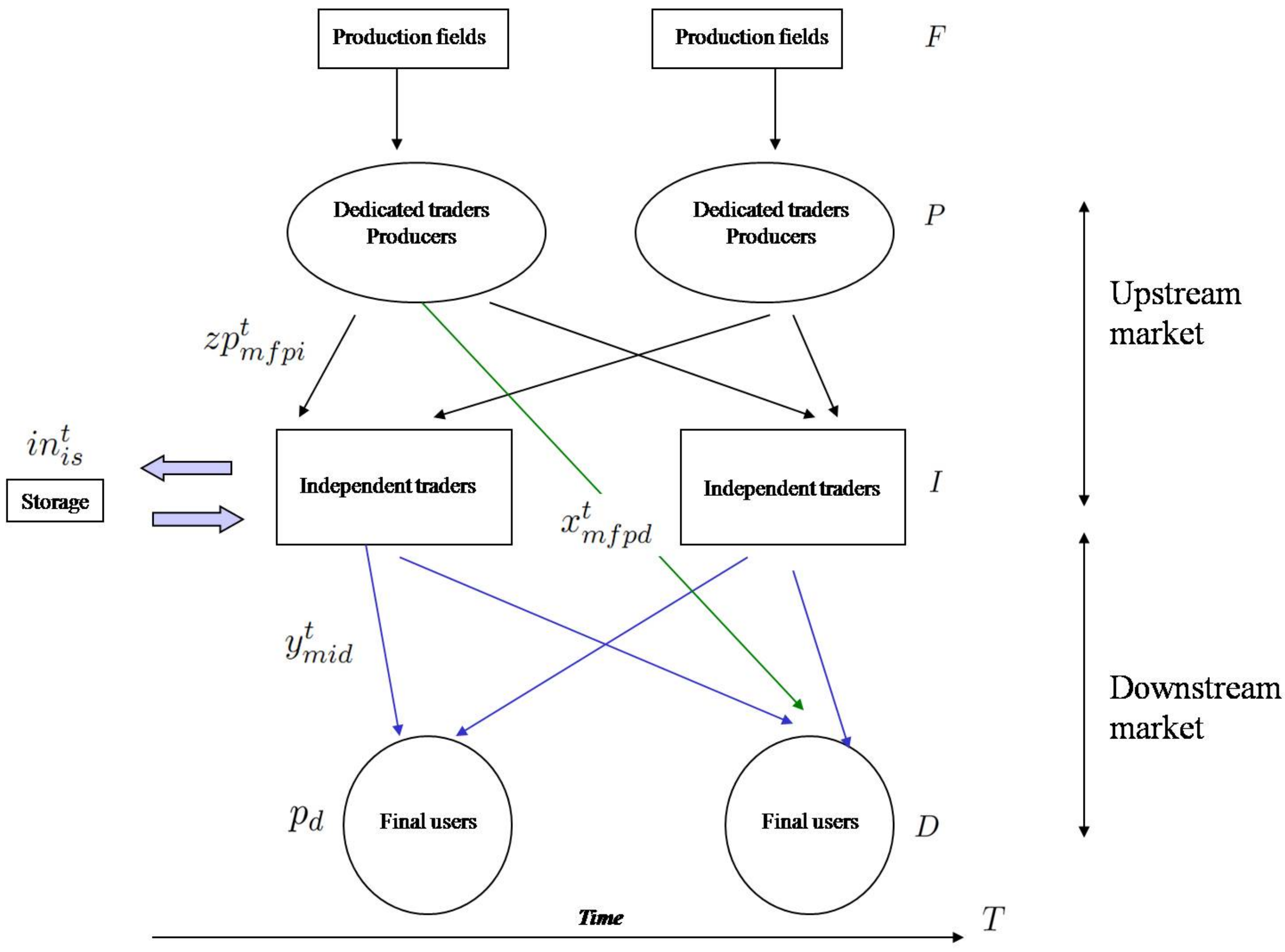

FIGURE 6.1 – *The market representation in GaMMES.*

$$p_{md}^t = \begin{cases} pc_{md}^t + \frac{1}{\gamma_{md}^t}\text{atanh}\left(\frac{\delta_{md}^t+\beta_{md}^t-Q_{md}^t}{\delta_{md}^t}\right) & \text{if } Q_{md}^t \geq \beta_{md}^t + \frac{\delta_{md}^t\beta_{md}^t}{\delta_{md}^t+\beta_{md}^t} \\ p'c_{md}^t + \frac{1}{\gamma_{md}'^t}\text{atanh}\left(\frac{\delta_{md}'^t+\beta_{md}'^t-Q_{md}^t}{\delta_{md}'^t}\right) & \text{if } Q_{md}^t \leq \beta_{md}^t + \frac{\delta_{md}^t\beta_{md}^t}{\delta_{md}^t+\beta_{md}^t} \end{cases} \tag{6.1}$$

where the parameters δ, β, γ and pc, which are time and season-dependent must be calibrated. Note that no confusion should be made between the discount rate δ and the inverse demand function's parameter δ_{md}^t because they are not defined with the same indices. Q_{md}^t is the gross gas consumtion in market d at year t and season m and p_{md}^t is the corresponding gas market price. Note that this function links the gas price and volume in the spot markets. It links the price to the consumed volume at the same year. In general, the system dynamics approach links the price at year t to all the volumes $Q_{md}^{t'}$, $t' \leq t$ previously consumed. However, the general multivariable demand function is not theoretically invertible. Therefore, when writing the inverse demand function, we preferred sticking to a monovariable formulation, by making the parameter β time-dependent.

The distinction between the domains $Q_{md}^t \geq \beta_{md}^t + \frac{\alpha_{md}^t\beta_{md}^t}{\alpha_{md}^t+\beta_{md}^t}$ and $Q_{md}^t \leq \beta_{md}^t + \frac{\alpha_{md}^t\beta_{md}^t}{\alpha_{md}^t+\beta_{md}^t}$ is needed to take into account the anticipated scrapping of burners [2] and avoid absurd situations where the price rises towards $+\infty$ (and also to guarantee the concavity of the objective functions). The parameters α', β', γ' and $p'c$ are calculated to ensure the continuity of h and its derivative h'.

The function atanh is such that :

$$\forall x \in (-1,1)\ \text{atanh}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

The following table gives the values of the inverse demand function parameters, for the primary natural gas consumption in year 2003 in France, Germany, Italy, the UK, Belgium and the Netherlands. The natural gas volumes in 2002 are exogenous.

Parameters	France	Germany	Italy	UK	Belgium	The Netherlands
$\beta(\times 10^3\text{ktoe})$	22.87	43.70	41.28	41.88	22.89	23.49
$\delta(\times 10^3\text{ktoe})$	2.76	4.00	3.60	2.80	2.76	1.05
$p_c(\$/\text{toe})$	172.5	242.9	268.3	175.8	230.4	217.5
$\gamma(\times 10^{-2}(\$/\text{toe})^{-1})$	0.72	0.98	0.96	1.00	1.48	0.88
$\beta'(\times 10^3\text{ktoe})$	0.00	0.00	0.00	0.00	0.00	0.00
$\alpha'(\times 10^3\text{ktoe})$	13.20	24.67	23.23	23.18	13.20	12.81
$p_c'(\$/\text{toe})$	350.8	404.1	441.2	379.5	316.6	549.1
$\gamma'(\times 10^{-2}(\$/\text{toe})^{-1})$	0.96	1.03	0.96	0.79	1.99	0.48

Figure 6.2 gives the demand function shape (i.e., the variation of the quantity Q_d over the price p_d in a given market). Note that we preferred showing the demand function rather than the inverse demand function for more clarity.

2. We will call burner a technology that can use either coal, oil or natural gas. Note that our apporach concerns the primary natural gas consumption (not only the electricity generation demand).

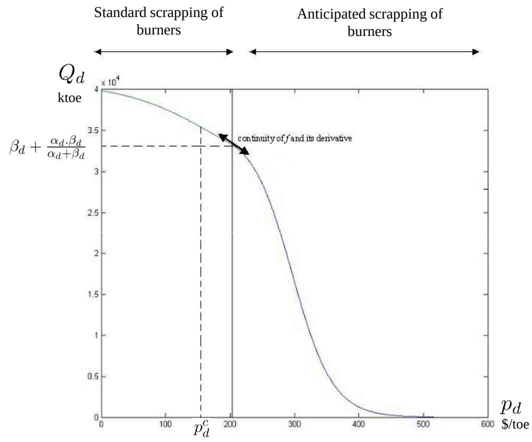

FIGURE 6.2 – *The demand function.*

We take account for the anticipated scrapping of burners to avoid situations where the quantity does not converge towards 0 when the price is very high. Obviously, such situations provide demand functions that cannot be used in Nash-Cournot competition modeling. Hence, we distinguish between two domains of the demand function, regarding whether we are in a standard scrapping regime or the anticipated scrapping one. This distinction is clearly shown in equation 9.1. Also, Figure 6.2 shows the difference between the domains : $Q^t_{md} \geq \beta^t_{md} + \frac{\alpha^t_{md}\beta^t_{md}}{\alpha^t_{md}+\beta^t_{md}}$ (standard scrapping of burners) and $Q^t_{md} \leq \beta^t_{md} + \frac{\alpha^t_{md}\beta^t_{md}}{\alpha^t_{md}+\beta^t_{md}}$ (anticipated scrapping of burners). The inflexion point of the demand function, which is shown in Figure 6.2, is the parameter pc^t_{md}. It represents a competition price, regarding the consumption of natural gas. It is an aggregation of the oil and coal prices and can be seen as threshold for the gas price that determines whether natural gas is a competitive fuel or not. This feature captures the possible fuel substitution in the natural gas inverse demand function. Besides, Fugure 6.2 shows that the domains distinction and the calibration of the (inverse) demand function ensures its continuity and differentiability.

As said in the economic description of the markets, we need to distinguish between the off-peak/peak season parameters of the inverse demand function. Besides, as explained in Part 3, to calibrate the demand function for the future, we need to specify a scenario for the fossil primary energy demand and the oil and coal market prices. Our system dynamics approach will allow us to understand how the fossil demand is going to be shared between the consumption of the three fuels.

6.2.4 The mathematical description

This section details the mathematical description of our model. It presents the optimization problems of all the supply chain players. Note that the dual variables are written in parentheses by their associated constraints.

Producer p's maximization program is given below. The corresponding decision variables are zp^t_{mfpi}, x^t_{mfpd}, ip^t_{fp}, q^t_{mfp} and up_{pi}. A producer can extract natural gas from all the possible production nodes he owns. He can sell gas to the independent traders via long-term contracts or directly target the spot markets, where a Nash-Cournot competition is exerted, between him, the other producers, and the independent traders. He pays the transportation costs necessary to bring gas to the independent traders' location (for the LTCs sales) or the spot markets (for the spot markets sales). Production investments are also considered.

$$
\begin{aligned}
\text{Max}\quad & \\
& \sum_{t,m,f,i} \delta^t \eta_{pi}(zp^t_{mfpi}) \\
& + \sum_{t,m,f,d} \delta^t \left(p^t_{md}(x^t_{mfpd} + \overline{x^t_{mfpd}}) \right) x^t_{mfpd} \\
& - \sum_{t,f} \delta^t Pc_f \left(\sum_{t' \le t} \sum_{m} q^{t'}_{mfp}, Rf_f \right) \\
& + \sum_{t,f} \delta^t Pc_f \left(\sum_{t' < t} \sum_{m} q^{t'}_{mfp}, Rf_f \right) \\
& - \sum_{t,f} \delta^t Ip_f ip^t_{fp} \\
& - \sum_{t,m,p,a} \delta^t ((Tc_a + \tau^t_{ma}) fp^t_{mpa})
\end{aligned}
$$

such that :

$$
\begin{aligned}
\forall t,\ f, \quad & \sum_{p} \sum_{t' \le t} \sum_{m} q^{t'}_{mfp} - Rf_f && \le 0 \quad (\phi^t_f) && (6.2a) \\
\forall t,\ f,\ m, \quad & \sum_{p} q^t_{mfp} - Kf_f(1 - dep_f)^t && && \\
& - \sum_{p} \sum_{t' \le t - delay_p} ip^{t'}_{fp}(1 - dep_f)^{t-t'} && \le 0 \quad (\chi^t_{mf}) && (6.2b) \\
\forall t,\ m,\ f, \quad & - q^t_{mfp} + \left(\sum_{i} zp^t_{mfpi} + \sum_{d} x^t_{mfpd} \right) && \le 0 \quad (\gamma^t_{mfp}) && (6.2c) \\
\forall t,\ f \quad & \sum_{m} \sum_{p} ((-1)^m q^t_{mfp}) - fl_f && \le 0 \quad (\vartheta 1^t_f) && (6.2d) \\
\forall t,\ f, \quad & - \sum_{m} \sum_{p} ((-1)^m q^t_{mfp}) - fl_f && \le 0 \quad (\vartheta 2^t_f) \quad \forall t,\ f,\ d,\ m, \quad x^t_{mfpd} - O_{fp}H && \\
& && && (6.2e) \\
\forall t,\ f,\ i,\ m, \quad & zp^t_{mfpi} - O_{fp}H && \le 0 \quad (\epsilon 2^t_{mfpi}) && (6.2f) \\
\forall t,\ f,\ m, \quad & q^t_{mfp} - O_{fp}H && \le 0 \quad (\epsilon 3^t_{mfp}) && (6.2g) \\
\forall t,\ f, \quad & ip^t_{fp} - O_{fp}H && \le 0 \quad (\epsilon 4^t_{fp}) && (6.2h)
\end{aligned}
$$

$$\begin{aligned}
\forall t,\ f, \quad & \sum_p ip^t_{fp} - Lf_f Kf_f (1-dep_f)^t \\
& - Lf_f \sum_p \sum_{t' \le t - delay_p} ip^{t'}_{fp} (1-dep_f)^{t-t'} && \le 0 \quad (\iota p^t_f) && (6.3a) \\
\forall t,\ m,\ n, \quad & \sum_a M6_{an} fp^t_{mpa}(1-loss_a) - \sum_a M5_{an} fp^t_{mpa} \\
& + \sum_f M1_{fn} q^t_{mpf} - \sum_d \sum_f M3_{dn} x^t_{mfpd} \\
& - \sum_i \sum_f M2_{in} zp^t_{mfpi} && = 0 \quad (\alpha p^t_{mpn}) && (6.3b) \\
\forall t,\ i, \quad & up_{pi} - \sum_{f,m} zp^t_{mfpi} && = 0 \quad (\eta p^t_{pi}) && (6.3c) \\
\forall\ i, \quad & ui_{pi} - up_{pi} && = 0 \quad (\eta_{pi}) && (6.3d) \\
\forall t,\ m,\ d,\ i,\ f, \quad & zp^t_{mfpi},\ x^t_{mfpd},\ ip^t_{fp},\ q^t_{mfp},\ up_{pi} && \ge 0
\end{aligned}$$

We denote by $\overline{x^t_{mfpd}}$ the total amount of gas brought in year t, season m to the market d by all the players different from producer p. Hence, the total quantity brought to the market $Q^t_{dm} = \sum_i y^t_{mid} + \sum_f \sum_p x^t_{mfpd}$ will be denoted $Q^t_{dm} = x^t_{mfpd} + \overline{x^t_{mfpd}}$ in order to clearly show the strategic interaction and the dependence of Q^t_{dm} over x^t_{mfpd} (producer p's decision variable). Using this notation, the KKT conditions will written more easily.
The term

$$\sum_{t,m,f,i} \delta^t \eta_{pi}(zp^t_{mfpi}) + \sum_{t,m,f,d} \delta^t \left(p^t_{md}(x^t_{mfpd} + \overline{x^t_{mfpd}}) \right) x^t_{mfpd}$$

is the revenue, which is obtained from the sales on the long-term contracts to the independent traders or directly from the retail markets.
The term

$$\sum_{t,m,p,a} \delta^t ((Tc_a + \tau^t_{m,a}) fp^t_{mpa})$$

is the transport and congestion costs charged by the pipeline operator to producer p. The dual variable τ^t_{ma} is associated with the pipeline capacity constraint through the arc a. It represents the congestion price on the corresponding pipeline (see the transport operator optimization problem for more explanation).
The term

$$\sum_{t,f} \delta^t Ip_f ip^t_{fp}$$

is the investment cost in production at the different production nodes.
The term

$$\sum_{t,f} \delta^t \left(Pc_f \left(\sum_{t' \le t} \sum_m q^{t'}_{mfp}, Rf_f \right) - Pc_f \left(\sum_{t' < t} \sum_m q^{t'}_{mfp}, Rf_f \right) \right)$$

is the actualized production cost. This term's explanation is as follows :
The production cost (at production node f) Pc_f depends on two variables, the total quantity produced, which will be denoted q and the natural gas resources Rf_f. The Golombek production cost function we used is as follows :

$$\forall q \in [0, Rf_f),\ Pc_f(q, Rf_f) = a_f q + b_f \frac{q^2}{2} - \ Rf_f c_f \left(\frac{Rf_f - q}{Rf_f} \ln \left(\frac{Rf_f - q}{Rf_f} \right) + \frac{q}{Rf_f} \right) \tag{6.4}$$

or if written for the marginal production cost

$$\forall q \in [0, Rf_f),\ \frac{\mathrm{d}Pc_f}{\mathrm{d}q} = a_f + b_f q \ + \ c_f \ln \left(\frac{Rf_f - q}{Rf_f} \right) \tag{6.5}$$

In our model, the production cost function is dynamic. The gas volume available to be extracted is dynamically reduced at each period, taking into account the exhaustivity of the resource.
If at year 1, the production is $q1$ and at year 2 $q2$, the total cost is thus :

$$cost = Pc_f(q1, RES_f) + \delta(Pc_f(q1 + q2, RES_f) - Pc_f(q1, RES_f))$$

Hence, to estimate that cost at year t, we need to calculate the production cost of the sum over all the extracted volumes until year t and subtract the cost we have at year $t - 1$.

The explanation of the constraints is straightforward :
The constraint (9.16a) bounds each production node's production by its reserves.
The constraint (9.16b) bounds the seasonal quantities produced by each production node's production capacity, explicitly taking into account the different dynamic investments. The total installed production capacity decreases with time because of the production depreciation factor dep_f.
The constraint (9.16c) states that the total production must be greater than the sales (to the long-term and spot markets). The constraints (9.16d) and (9.16e) can be rewritten as follows :

$$\forall t,\ f \ |\sum_m ((-1)^m \sum_p q^t_{mfp})| \leq fl_f$$

This fixes a maximum spread between the off-peak/peak production at each production node. $(-1)^m$ is equal to 1 in the off-peak season and -1 in the peak season.
The constraint (9.17a) is a market-clearing condition at each node, regarding the flows from producer p depending on whether this node is a production node, an independent trader location or a demand market.
The constraint (9.16j) bounds the capacity expansion of each production node f : each year, the investment decided to increase the production capacity is less than $100 \times Lf_f$ percent the installed capacity at that year. A historical study of the capacity expansion of some production nodes allowed us to calibrate the value of Lf_f : $Lf_f = 0.20$.
The constraint (9.17b) equates the sales of producer p for the long-term contracts to the contracted volume up_{pi}, each year.
The constraint (9.17c) describes the following : For each pair of producer/independent trader

(p, i), the gas quantity sold by p in the long-term contract market must be equal to the gas quantity purchased by i. Therefore, this is a supply/demand equation in the long-term contracts market. The associated dual variable η_{pi} is the corresponding contract unit selling/purchase price, because we do not assume the existence of market power in the long-term contract trade. Using this technique, it is possible to make the long-term contracts prices and volumes endogenous to the description so that they become an output of the model.
The constraint (and the similar other ones) (9.16f) allows producer p to use only the production nodes he owns (for production, investments, sales, etc.). We recall that the incidence matrix O is such as $O_{fp} = 1$ if and only if producer p owns production node f.

Independent trader i's maximization program is given below. The corresponding decision variables are zi^t_{mpi}, y^t_{mid}, r^t_{is}, in^t_{is} and ui_{pi}. The independent trader buys gas only from the producers via long-term contracts. The sales are dedicated to all the spot markets, where trader i is in an oligopolistic competition with the other independent traders and the producers. He can store his gas in all the different storage nodes while supporting capacity reservation, storage and withdrawal costs. He also has to support the transportation costs to bring gas to the spot markets or to store/withdraw it.

$$
\begin{aligned}
&\text{Max} \\
&\quad \sum_{t,m,d} \delta^t \left(p^t_{md}(y^t_{mid} + \overline{y^t_{mid}}) y^t_{mid} \right) \\
&\quad - \sum_{t,p,m} \delta^t \left(\eta_{pi} zi^t_{mpi} \right) \\
&\quad - \sum_{t,s} \delta^t \left(Rc_s r^t_{is} \right) \\
&\quad - \sum_{t,s} \delta^t \left((Ic_s + Wc_s) in^t_{is} \right) \\
&\quad - \sum_{t,m,i,a} \delta^t \left(Tc_a + \tau^t_{ma} \right) fi^t_{mia}
\end{aligned}
$$

such that :

$$
\begin{array}{rlrll}
\forall t,\ m, & \sum_p zi^t_{mfpi} - \left(\sum_d y^t_{mid} + (-1)^m \sum_s in^t_{is} \right) & = 0 & (\psi^t_{mi}) & (6.6a) \\
\forall t,\ s, & in^t_{is} - r^t_{is} & \leq 0 & (\mu^t_{is}) & (6.6b) \\
\forall t,\ m,\ n, & \sum_a M6_{an} fi^t_{mia}(1 - loss_a) - \sum_a M5_{an} fi^t_{mia} & & & \\
 & - \sum_d M3_{dn} y^t_{mid} + \sum_p M2_{in} zi^t_{mpi} & & & \\
 & - (-1)^m \sum_s M4_{sn} in^t_{is} & = 0 & (\alpha i^t_{min}) & (6.6c) \\
\forall t,\ p, & ui_{pi} - \sum_m zi^t_{mpi} & = 0 & (\eta i^t_{pi}) & (6.6d) \\
\forall\ p, & ui_{pi} - up_{pi} & = 0 & (\eta_{pi}) & (6.6e) \\
\forall t,\ m,\ p, & - zi^t_{mpi} + min_{pi} \sum_m zi^t_{mpi} & \leq 0 & (\upsilon^t_{mpi}) & (6.6f) \\
\forall t,\ s, & \sum_i r^t_{is} - Ks_s - \sum_{t' \leq t - delay_s} is^{t'}_s & \leq 0 & (\beta s^t_s) & (6.6g) \\
\forall t,\ m,\ s,\ d, & zi^t_{mpi},\ y^t_{mid},\ r^t_{is},\ in^t_{is},\ ui_{pi} & \geq 0 & &
\end{array}
$$

We denote by $\overline{y^t_{mid}}$ the total amount of gas brought in year t, season m to the market d by all the players different from trader i. Hence, the total quantity brought to the market $Q^t_{dm} = \sum_i y^t_{mid} + \sum_f \sum_p x^t_{mfpd}$ will be denoted $Q^t_{dm} = y^t_{mid} + \overline{y^t_{mid}}$ in order to clearly show the strategic interaction and the dependence of Q^t_{dm} over y^t_{mid} (trader i's decision variable). Using this notation, the KKT conditions will be written more easily. Note that the producers and independent traders see the same inverse demand function in the spot markets. The notation we have chosen implies that :

$$\forall p,\ i,\ d,\ t,\ m,\quad Q^t_{dm} = \sum_i y^t_{mid} + \sum_f \sum_p x^t_{mfpd} = y^t_{mid} + \overline{y^t_{mid}} = x^t_{mfpd} + \overline{x^t_{mfpd}} \tag{6.7}$$

The term

$$\sum_{t,m,d} \delta^t \left(p^t_{md}(y^t_{mid} + \overline{y^t_{mid}})y^t_{mid}\right) - \sum_{t,p,m} \delta^t \left(\eta_{pi} zi^t_{mpi}\right)$$

is the net profit.
The term

$$\sum_{t,s} \delta^t \left(Rc_s r^t_{is}\right)$$

is the storage capacity reservation cost.
The term

$$\sum_{t,s} \delta^t \left((Ic_s + Wc_s) in^t_{is}\right)$$

are the storage/withdrawal costs.[3]
The term

$$\sum_{t,m,i,a} \delta^t \left(Tc_a + \tau^t_{ma}\right) fi^t_{mia}$$

is the transport and congestion costs charged by the pipeline operator from the independent trader i.

As for the feasibility set, it is also easy to specify :
The constraint (9.20a) is a gas quantity balance for each trader. The term $(-1)^m$ is equal to 1 in the off-peak season and -1 otherwise. An implicit assumption we use in our description is that all the storage nodes must be "empty" (regardless of the working gas quantitities) at the end of each year.
The equation (9.20b) implies that each independent trader has to pay for a storage reservation quantity, each year and at each storage node s, to be able to store his gas.
The constraint (9.20d) forces each trader to purchase the same quantity, in long-term contracts from each producer and year.
The constraint (9.20e) is similar to the constraint (9.17c) of the producers' optimization program. For each pair of producer/independent trader (p, i), the gas quantity sold by p in the long-term contract market must be equal to the gas quantity purchased by i. Therefore, this is a supply/demand equation in the long-term contracts market. The associated dual variable η_{pi} is the corresponding contract unit selling/purchase price, because we do not assume the existence of market power in the long-term contract trade. Using this technique, it is possible to make the long-term contracts prices and volumes endogenous to the description so that they become an output of the model.

3. There are no storage losses in the model. They can easily be taken into account by increasing the transportation losses of the arcs that start at the storage nodes.

The constraint (9.20f) fixes a minimum percentage of the annual contracted volume min_{pi} that has to be exchanged between p and i each season of each year.
The constraint (6.6g) is a storage constraint expressed at each storage node, taking into account the investments decided by the storage operator.

On the transportation side of our model, we will assume that the producers pay the transport costs to bring natural gas from the production nodes to the independent traders' locations and the end-use markets. The traders support the transport costs to store/withdraw gas or bring it to the end-users for their sales.

The pipeline operator optimization (cost minimization) program is given below. The corresponding decision variables are fp^t_{mpa}, fi^t_{mia} and ik^t_a. The piepline operator minimizes the total transportation, congestion, and capacity investments costs.

$$\text{Min} \quad \sum_{t,m,a} \delta^t \left(Tc_a + \tau_{ma}^t\right) \sum_p fp_{mpa}^t + \sum_{t,m,a} \delta^t \left(Tc_a + \tau_{ma}^t\right) \sum_i fi_{mia}^t + \sum_{t,a} \delta^t Ik_a ik_a^t$$

such that :

$$\forall t,\ m,\ a, \quad \sum_p fp_{mpa}^t + \sum_i fi_{mia}^t - \left(Tk_a + \sum_{t' \le t-delay_i} ik_a^{t'}\right) \le 0 \quad (\tau_{ma}^t) \tag{6.8a}$$

$$\forall t,\ a, \quad ik_a^t - La_a \left(Tk_a + La_a \sum_{t' \le t-delay_i} ik_a^{t'}\right) \le 0 \quad (\iota a_a^t) \tag{6.8b}$$

$$\begin{aligned} \forall t,\ m,\ p,\ n, \quad & \sum_a M6_{an} fp_{mpa}^t (1 - loss_a) - \sum_a M5_{an} fp_{mpa}^t \\ & + \sum_f M1_{fn} q_{mpf}^t - \sum_d \sum_f M3_{dn} x_{mfpd}^t \\ & - \sum_i \sum_f M2_{in} zp_{mfpi}^t = 0 \quad (\alpha p_{mpn}^t) \end{aligned} \tag{6.8c}$$

$$\begin{aligned} \forall t,\ m,\ i,\ n, \quad & \sum_a M6_{an} fi_{mia}^t (1 - loss_a) - \sum_a M5_{an} fi_{mia}^t \\ & - \sum_d M3_{dn} y_{mid}^t + \sum_p M2_{in} zi_{mpi}^t \\ & - (-1)^m \sum_s M4_{sn} in_{is}^t = 0 \quad (\alpha i_{min}^t) \end{aligned} \tag{6.8d}$$

$$\forall t,\ m,\ a,\ p,\ i, \quad fp_{mpa}^t,\ fi_{mia}^t,\ ik_a^t \ge 0$$

The objective function contains both the transport/congestion and investment costs.
The congestion cost through arc a, τ_{ma}^t, is the dual variable associated with the constraint (9.21a). This constraint concerns the physical seasonal capacity of arc a, including the possible time-dependent investments.
The constraint (9.21b) bounds the capacity expansion of each arc a : each year, the investment decided to increase the transport capacity is less than $100 \times La_a$ percent the installed capacity at that year. In GaMMES, we used the value $La_a = 0.2$.
The other constraints are market-clearing conditions at each node, depending on whether this node is a production node, an independent trader location, a demand market or a storage node and depending on whether the transportation costs are supported by the producers or the independent traders.

We consider both pipeline and LNG routes for transport. The liquefaction and regasification costs are included in the transportation cost on the LNG arcs. We assume, in our representation that the physical losses occur at the end nodes of the arcs.

The storage operator optimization (cost minimization) program is given below. The corresponding decision variable is is_s^t. The storage operator minimizes the total operational and capacity investments costs.

$$\text{Min} \quad \sum_{t,s} \delta^t Is_s is_s^t + \sum_{t,i,s} \delta^t (Ic_s + Wc_s) in_{is}^t + \sum_{t,i,s} \delta^t Rc_s r_{is}^t$$

such that :

$$\forall t,\ s, \quad \sum_i r_{is}^t - Ks_s - \sum_{t' \le t-delay_s} is_s^{t'} \le 0 \quad (\beta s_s^t) \tag{6.9a}$$

$$\forall t,\ s, \quad is_s^t - Ls_s Ks_s - Ls_s \sum_{t' \le t-delay_s} is_s^{t'} \le 0 \quad (\iota s_s^t) \tag{6.9b}$$

$$\forall t,\ s, \quad is_s^t \ge 0$$

The storage operator minimizes the total operation cost that includes investment, storage, withdrawal and storage capacity reservation costs. His decision variable is is_s^t, which means that he only controls the different investments that dynamically increase the storage capacity of each storage node. The incentive this player has to invest is due to the constraint he must satisfy : the capacity available at each storage node must be sufficient to meet the volumes the independent traders have to store each year in the off-peak season. Capacity expansion is bounded and we used the value $Ls_s = 0.2$.

If we take a closer look at the optimization program of a producer, we will notice that his feasibility set depends on the decision variables of the independent traders. Also, the feasibility set of any independent trader's optimization program depends on the producers' decision variables. The situation is similar for the pipeline and storage operators. This particularity makes our formulation (the KKT conditions) a **Generalized Nash-Cournot problem**. Similarly, the Generalized Nash-Cournot problem can also be formulated as a Quasi-Variational Inequality problem (QVI). In order to solve our problem, we look for the particular solution that makes our problem a VI formulation (29). More details about the VI solution search are given in Section 10.

The concavity of all the players' objective functions is given in Appendix 1.
When the KKT conditions are written, we obtain the Mixed Complementarity Problem given in Appendix 2.

6.2.5 The (Quasi)-Variational Inequality and Generalized Nash-Cournot games

In this section, we recall Harker's result (29) in order to understand how to theoretically solve a Generalized Nash-Cournot problem. We refer to chapter 8 for more theoretical results and explanations.

A standard Nash-Cournot problem is a set of optimization programs where some of the players can influence other players' payoff via the objective functions. In a Generalized Nash-Cournot formulation, some players can also change the feasibility sets of other players, via their decision variables. In our particular model, if we consider an independent trader i, the constraint

$$\forall\ p,\ i,\ ui_{pi} = up_{pi}$$

contains the producers' decision variables up_{pi}. These decision variables influence trader i's feasibility set. The situation is symmetric for the producers. More generally, our double-layer economic structure makes the producers and independent traders influence each-other's feasibility sets. This is principally due to the formulation of the long-term contracts that are issued from a supply/demand equilibrium constraint. It is straightforward that a standard Nash-Cournot problem can be expressed as a VI formulation if the objective functions are differentiable (is suffices to write the necessary and sufficient conditions on the gradient of the objective functions that characterize the optimum, or the Euler's inequality).

A generalized Nash-Cournot problem can be expressed as a QVI formulation. Unlike VI problems, a QVI formulation often has an infinite set of equilibria. In some particular cases, a QVI problem can be slightly changed into a VI formulation. This is possible, in particular if the QVI is issued from a Generalized Nash-Cournot problem, which is our case. To do so, we make all the constraints that mix different players' decision variables common to all these players. From the KKT conditions point of view, Harker (29) demonstrated that the "VI solution" is obtained by giving the same dual variables to the common constraints.

If we apply the previous results to our model, this leads to the fact that the producers and independent traders, see the same dual variables η_{pi} and must consider the common constraint (9.17c) and (9.20e) in their optimization program. Economically speaking, this means that they have the same appreciation of the long-term contracts prices.

Using this technique, we make sure we end up with a VI solution (29).

§ 6.3 Conclusion

This chapter presents a Generalized Nash-Cournot model in order to describe the natural gas market's evolution. The demand representation is rich because it takes into account the possible energy substitution that can be made between oil, coal, and natural gas. This appears in the introduction of a competitive price, in the demand function. The exhaustibility of the resource is taken care of by the use of Golombek production cost functions.

The different actors' behavior is represented thanks to their optimization programs (they are assumed to be rational players). The strategic actors, who can exert market power, are the producers and the independent traders. Market power is exerted in the downstream and is modeled as a Nash-Cournot competition. Long-term contracts link the producers and the traders by a fixed gas volume that has to be exchanged each year (with a tolerated flexibility) at a contracted price. The producers constitute the only means of supply to the independent traders. This creates a double-layer economic structure that differs from the standard double-dividend problem : indeed, in GaMMES, we do not model a successive oligopoly because the traders do not show a demand function that can be exploited by the producers in their optimization program. On the contrary, the producers and independent traders see the same demand function in the downstream where they are in an imperfect competition.

The long-term contract prices and volumes are endogenously computed as dual variables to long-term contracts' constraints. This aspect makes our formulation a Generalized Nash-Cournot model, more generally a QVI formulation. In order to solve it, we derived the corresponding VI formulation.

GaMMES has been applied to describe the northwestern European gas trade till 2035. The results are provided in the next chapter.

- Chapitre 7 -

Application of the GaMMES model to the northwestern European gas trade.

§ 7.1 Introduction

The previous chapter presented the GaMMES model, a Generalized-Nash Cournot model to describe the natural gas market trade. The key features of the model are the following : energy substitution between coal, oil, and natural gas is taken into account and long-term contracts are endogenously described. Thus, GaMMES is particularly well suited to describe the European gas trade, which is still mainly dominated by long-term contracts in the upstream. This chapter is an application of our model to the northwestern European natural gas trade where the calibration process and the results are discussed. The results contain forecasts of the consumption, prices, production, and gas dependence in Europe. LTC prices and volumes are provided and analyzed. A comparison between our model, a more standard one where the demand does not take into consideration fuel substitution and the European Commission natural gas forecast is carried out in order to compare between the results. Finally, we take advantage of energy substitution allowed by the model to draw the evolution of the gas price as a function of the oil or coal prices.

In this chapter, we will use chapter 6's notation.

§ 7.2 The European natural gas markets model

This section puts the model at work and presents our numerical results.

7.2.1 The representation

The model we presented in Section 9.2.5 has been used in order to study the northwestern European natural gas trade. The following array summarizes the representation we have studied.

Producers		**Production nodes**		**Consuming markets**		**Independent traders**
Russia		Russia$_f$		France		France$_{tr}$
Algeria		Algeria$_f$		Germany		Germany$_{tr}$
Norway		Norway$_f$		The Netherlands		The Netherlands$_{tr}$
The Netherlands		NL$_f$		UK		UK$_{tr}$
UK		UK$_f$		Belgium		Belgium$_{tr}$

Storage nodes		**Seasons**		**Time**
France$_{st}$		off-peak		2000 – 2045
Germany$_{st}$		peak		
The Netherlands$_{st}$				
UK$_{st}$				
Belgium$_{st}$				

The model is run up through 2045 but only the results through 2035 are used to avoid end-of-horizon effects (depletion of all the production nodes, etc.).

We aggregate all the production nodes of each producer into one production node. We assume that each consuming market is associated with one independent local trader (indexed by tr). As an example, France$_{tr}$ would be GDF-SUEZ and Germany$_{tr}$ would be E-On Ruhrgas. All the storage nodes are also aggregated so that there is one storage node per consuming country. As for the transport, the different gas routes given in Figure 7.1 were considered.
The local production in the different consuming countries is also taken into consideration (the imports from non-represented producers, which are small, are also considered). We assume that these locally consumed volumes are exogenous to the model.
We consider Algeria as an LNG producer who can exert market power. The other LNG exchanges between producers "outside" the scope of the model (such as the UAE) and the represented consumers are considered exogenously in the model. Therefore, we assume that the LNG demand, except for Algeria, is inelastic to the gas price. This approach is an assumption that overestimates the market power allowed to standard (not LNG) natural gas producers. However, the missing LNG volumes are very small in (36) (less than 1%).

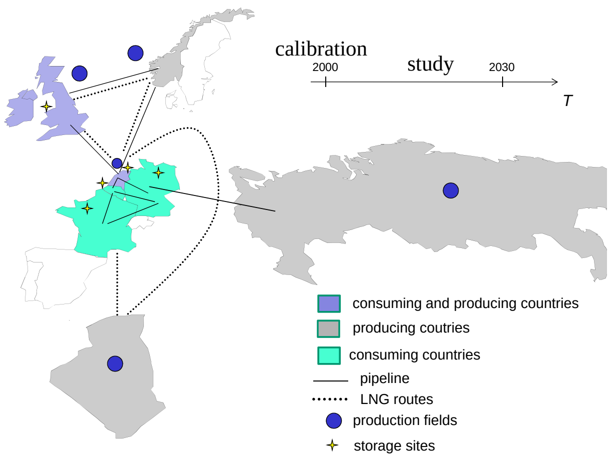

FIGURE 7.1 – *The northwestern European natural gas routes, production and storage nodes.*

7.2.2 The calibration

The calibration process has been carried out in order to best meet :
- the primary natural gas consumption,
- the industrial sector gas price and
- the volumes produced by each gas producer,

between 2000 and 2004 (the first time period).

The model has been solved using the solver PATH (20) from GAMS. In order to shorten the running time, we used a five-year time-step resolution. We chose five years because it is the typical length of time needed to construct investments in production, infrastructure or storage. Also, the demand function has been linearized.

The data for the market prices, consumed volumes, and imports is the publicly available set from IEA (36). We define a new variable $exch_{mpd}^{t}$ that represents the exported volume from producer p to market d. More precisely :

$$\forall t,\ m,\ p,\ d,\ exch_{mpd}^{t} = \sum_{i} B_{id}\ zp_{mpi}^{t} + x_{mpd}^{t}$$

The matrix B is such that $B_{id} = 1$ if the independent trader i is located in market d (e.g., GDF-SUEZ in France, E-On Ruhrgas in Germany) and $B_{id} = 0$ otherwise. Hence, one can notice that the exchanged volumes include both the spot and long-term contract trades.

The calibration elements we used are the inverse demand function parameters α_{md}^{t}, γ_{md}^{t}, pc_{md}^{t} and β_{md}^{t}. The idea is that the system dynamics (3) model is run in order to calculate all the inverse demand function parameters, for all the markets and at each year and season of our study. The calibration technique slightly adjusts these values to make the model correctly describe the historical data (between 2000 and 2004).

In order to calibrate the produced volumes properly, we introduced security of supply parameters that link each pair of producer/consuming countries (p, d). A security of supply measure forces each country not to import from any producer, more than a fixed percentage (denoted by SSP) of the overall imports. This property can be rewritten as follows :

$$\forall t,\ m,\ p,\ d,\ exch_{mpd}^{t} \leq SSP_{pd} \sum_{p} exch_{mpd}^{t}$$

The security of supply parameters are set exogenously. As mentioned before, the calibration concerned only the first time period.

The calibration tolerates a maximum error of 5% for the prices and consumed quantities and 10% for the imported/exported volumes. The tolerated error is higher for the exchanged volumes because they depend on the exports decided by the producers for all the targeted consumers, even those that are not in the scope of the model. As an example, the exported volumes from Russia to CIS (CEI) countries are exogenous to our model.

The discount factor δ is set to 0.95 (per year). This value is commonly used in the indusry. All the production, transport and investment costs are inflated by the CERA's UCCI index.

We did not provide all the data in this manuscript because of confidentiality issues.

7.2.3 Numerical results

In order to estimate the demand function parameters, our model requests exogenous inputs : the fossil primary energy demand and the evolution of the oil and coal prices. For that purpose, we used a scenario provided by the European Commission (18). The annual fossil primary consumption and prices growth per year that we used are given in the following array (starting from 2000) :

annual growth	*Total gross consumption (in %)*	*Oil price (in %)*	*Coal price (in %)*
France	0.46	3.71	2.61
Germany	0.06	3.71	2.61
United Kingdom	0.02	3.71	2.61
Belgium	0.06	3.71	2.61
The Netherlands	0.11	3.71	2.61

Figure 7.2 gives the evolution of the natural gas consumption between 2000 and 2030 provided by our model for the countries represented. The consumption is given in Bcm/year. The figure also shows the evolution of the natural gas prices ($/cm), in the industrial sector, for the represented countries. We recall that the industrial sector prices are taken as a proxy for natural gas prices. The figure also gives the evolution of the producing countries' sales between 2000 and 2030, in Bcm/year.

The average annual consumption growth between 2000 and 2030 is given in the following array :

Country	*Annual consumption growth (in %)*
France	0.61
Germany	0.23
UK	−1.35
Belgium	0.23
The Netherlands	−0.94

According to our simulation, France shows the highest annual consumption growth, averaging 0.61%, between 2005 and 2030. Both the UK and the Netherlands experience a significant decrease in their natural gas consumption, as their domestic supplies are replaced by more expensive foreign imports. This effect is magnified in our model by the fact that only existing reserves are taken into account, which are depleted relatively quickly due to high installed capacities.
The consumption of all the countries shown flattens out or decreases in 2030, compared to 2000, despite the increase of the fossil primary demand. This is mainly due to the fact that competition in the upstream market becomes less and less important with time. Indeed, in 2025, the continental Europe gas production (the UK and the Netherlands) is expected to be around 25 Bcm/year. This will increase the exercise of market power and the consumption growth will therefore be reduced.

The price average annual growth between 2000 and 2030 is given in the following chart :

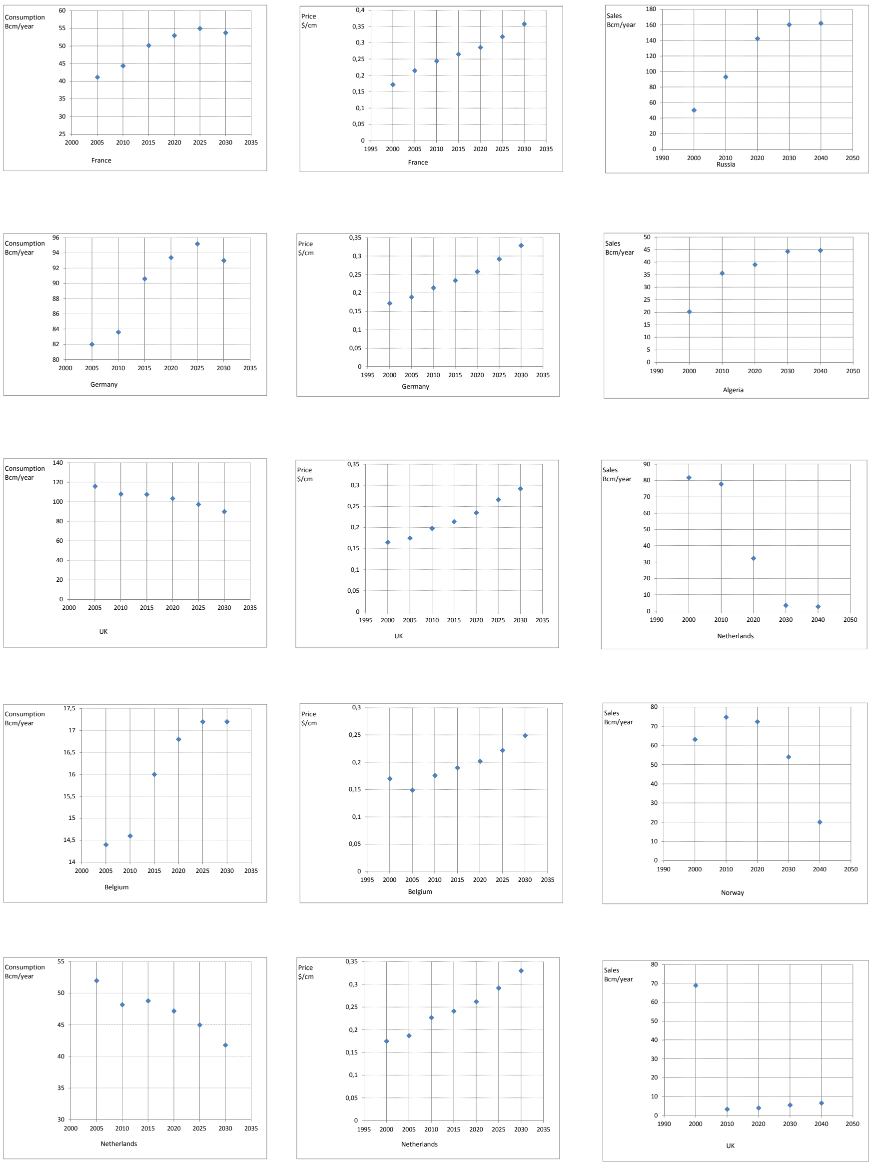

FIGURE 7.2 – *The natural gas consumption, prices, and sales between 2005 and 2030.*

Country	*annual price growth (in %)*
France	2.47
Germany	2.19
UK	1.28
Belgium	1.92
The Netherlands	2.14

As expected, the natural gas prices increase continuously in all the countries. The prices values are driven, as a resut of the Nash-Cournot interaction by the combination of two effects : the fossil primary energy demand and the competition between fuels (see equation 9.1). Since the fossil primary energy demand and the coal and oil prices increase with time, they force the gas price up. This combination explains why the natural gas price annual growth in all the countries is less important than the growth in both oil and coal. Indeed, this is due to the fact that the fossil primary energy consumption does not increase with time as quickly as the coal and oil prices.

The production in continental Europe is expected to greatly decrease in the forthcoming decades. The Norwegian production is expected to increase until 2012 before starting to decrease. The Dutch decrease is smooth (-4.5% per year between 2000 and 2020) whereas the UK one is very sharp. The model indicates that the United Kingdom will use up more than 75% of its natural gas reserves (starting from 2000) until 2015. This may seem surprising but can be understood by the fact that we take into account only the proven reserves in 2000 (10). Thus, we do not consider the reserves discoveries that may occur till 2045.

On the other hand, the Russian and Algerian shares in the European natural gas consumption is expected to grow in the coming decades : in 2020, the foreign imports will represent 47% of the northwestern European consumption.

In order to test the strength of the model, we compare its output versus historical values. For that purpose, we consider the consumption and prices in the European countries between 2005 and 2010 (second time-step) and compare them to what actually happened in that period. Let us recall that the second time-step has not been used in the calibration. Figure 7.3 gives the natural gas consumption between 2005 and 2010 in Bcm/year and prices in \$/cm in the countries represented. The left bars represent the model's output whereas the right bars represent the real historical data.

The average model estimation errors are 2.2% for the consumption and 3.5% for the prices. They are in the same range as the ones tolerated when calibrating the model (period 2000-2005).

Figure 7.4 gives the evolution of the northwestern European natural gas dependence on foreign imports (those considered in the model). The dependence is the ratio between the foreign exports to northwestern Europe and the domestic consumption [1].
The natural gas dependence is expected to reach 70% around 2030, which will bring about important security of supply concerns (2). However, these conclusions should be cautiously considered

1. The Norwegian sales are not taken into account in the foreign supplies for security of supply reasons.

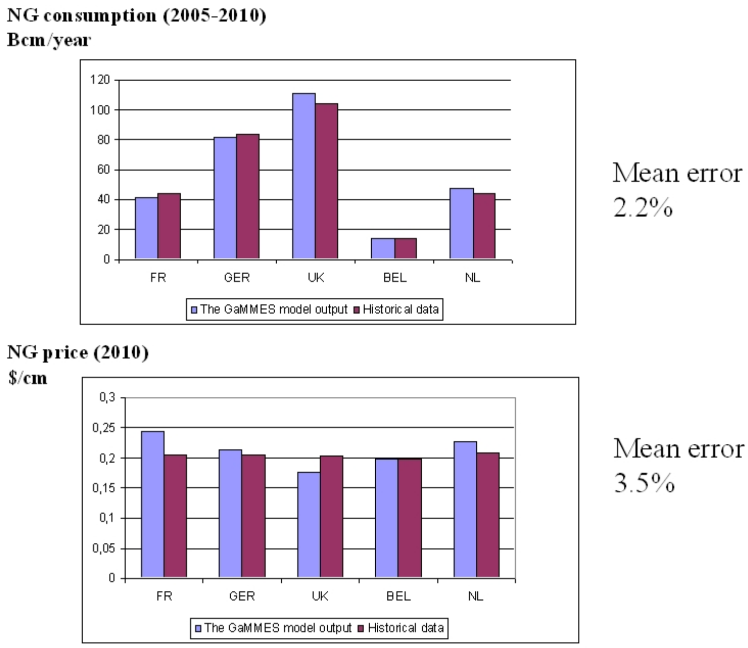

FIGURE 7.3 – *Comparison between the model's output and historical data.*

because they are based on strong assumptions. Indeed, in our study, we assume that no more natural gas reserves will be found in the future and no shale gas will be produced in Europe. [2]

$$dependence = \frac{foreign\ exports}{total\ consumption} \tag{7.1}$$

Now we present the results related to the long-term contracts (LTC) provided by GaMMES. The following tables give the LTC volumes and prices between the different producers and the independent traders :

Volume(Bcm/year)	$France_{tr}$	$Germany_{tr}$	UK_{tr}	$Belgium_{tr}$	$The\ Netherlands_{tr}$	Total
Russia	5.25	42.39	nc	1.25	nc	48.89
Algeria	7.18	nc	0.17	3.49	nc	10.85
The Netherlands	nc	nc	nc	1.66	6.18	7.84
Norway	0.36	nc	4.81	6.52	nc	11.69
UK	nc	nc	nc	nc	nc	0
Total	12.80	42.39	4.98	12.92	6.18	79.27

2. shale gas production is expected to be negligeable in Europe due to environmental concerns, for instance. As of now, few credible assumptions exist concerning the development of European domestic shale reserves (52).

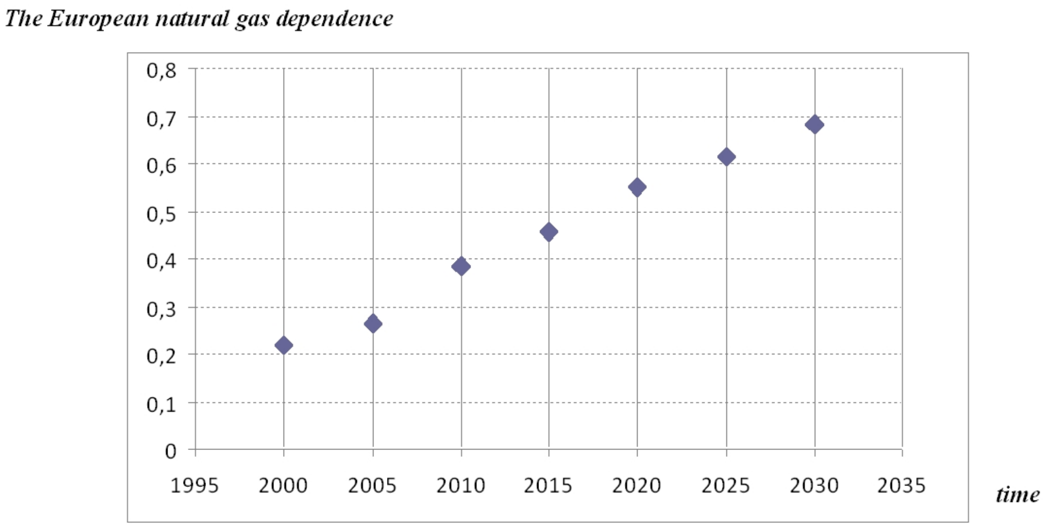

Figure 7.4 – *The northwestern European natural gas dependence over time.*

Price($/cm)	France_{tr}	Germany_{tr}	UK_{tr}	Belgium_{tr}	$\text{The Netherlands}_{tr}$
Russia	0.18	0.17	nc	0.20	nc
Algeria	0.18	nc	0.22	0.20	nc
The Netherlands	nc	nc	nc	0.20	0.20
Norway	0.18	nc	0.22	0.20	nc
UK	nc	nc	nc	nc	nc

One can notice that if a pair of producer-independent trader contract on the long-term, the corresponding LTC price is nonnegative, which is not straightforward since the correponding LTC price is a free dual variable. Also, the spot prices in the consuming countries reported in Figure 7.2 are in general higher than the LTC prices. The explanation is as follows : since long-term contracts are the only means for the independent traders to obtain natural gas, LTC prices can be considered as marginal supply costs. Similarly, the spot prices are directly related to the independent traders' revenue. Therefore, if an independent trader has an incentive to contract in the long-term, it implies that his revenues, over the time horizon, are greater than his costs. In a similar fashion, spot prices are greater than LTC prices.

The Belgian trader is the one that diversifies his gas supplies the most (four sources). This is due to its geographical location, which is close to three producing countries : Norway, The Netherlands and Algeria (recall that the Algerian production node is directly linked to Belgium via an LNG route). For a particular trader, the LTC price is the same with respect to all the possible supply sources (same price within the column). This suggests that the LTC prices are correlated to the spot prices : an independent trader may tolerate high supply marginal costs if his marginal revenue in his spot market is high enough. Besides, we assumed in our model that the producers do not exert market power when contracting in the long-term.

The UK does not contract in the long-term with the independent traders. This is due to its limited gas reserves that do not create an incentive to invest in production. Therefore, the producer does not have an investment-related risk hedging strategy and prefers directly targeting the spot markets without creating long-term contracts. This situation has been observed in recent years.

Regarding the LTC prices, the GaMMES results are close to reality (what can be found in private data). As for the LTC volumes, the results suggest that they represent, on average, 28% of the total (contract+spot) trade. This value is relatively low, compared to what is currently observed in Europe (70%) (40). This can be explained by the fact that in GaMMES, we only consider contracts endogenously determined after 2000 without taking into consideration the pre-existing ones signed before that time as part of the calibration process. Furthermore, from the point of view of the model, given installed production capacity as of 2000, the producers may not have a strong incentive to contract with the traders after this time because related investments have already been made.

The purpose of the next comparison is to show the effects of the fuel substitution-based demand function. To that end, we consider an alternative linear demand function of the following form :

$$q_{md}^t = a_{md}^t - b_d p_{md}^t \tag{7.2}$$

where the slope b_{md} should remain constant over time and the interceipt a_{md}^t changes as a function of the fossil primary energy demand. In our study, we made a_{md}^t evolve with the fossil primary energy demand annual growth. The slope b_{md} is a result of the calibration process. This description of the markets will be refered to as the *standard model* whereas the model we proposed in this chapter will be refered to as the *GaMMES model*. Note that the standard model is rather simplistic and does not correctly capture the demand behavior, because the inverse demand function's slope b_{md} is kept constant with time. However, the main purpose of the comparison is not to present a new model but rather to remove one feature of the GaMMES model (energy substitution) and see how this would alter the results.

Figure 7.5 provides the consumption and price levels for both models considered.

We notice that the standard model provides a lower consumption than the GaMMES results. The average difference in consumption is 13%. The standard model provides lower prices than the GaMMES results. The average difference between the two models is 23% which is quite large.

Now, let's compare between the results provided by the GaMMES model, the standard model and some official forecast. For that purpose, we choose the forecast of the European Commission (18).

Figure 7.6 shows the evolution of the global European energy consumption between 2000 and 2030 and the average European price, forecasted in three scenarios. Ths first one is issued from the European Commission report (baseline scenario) (18). The second one is our model forecast and the third one is the standard model forecast.

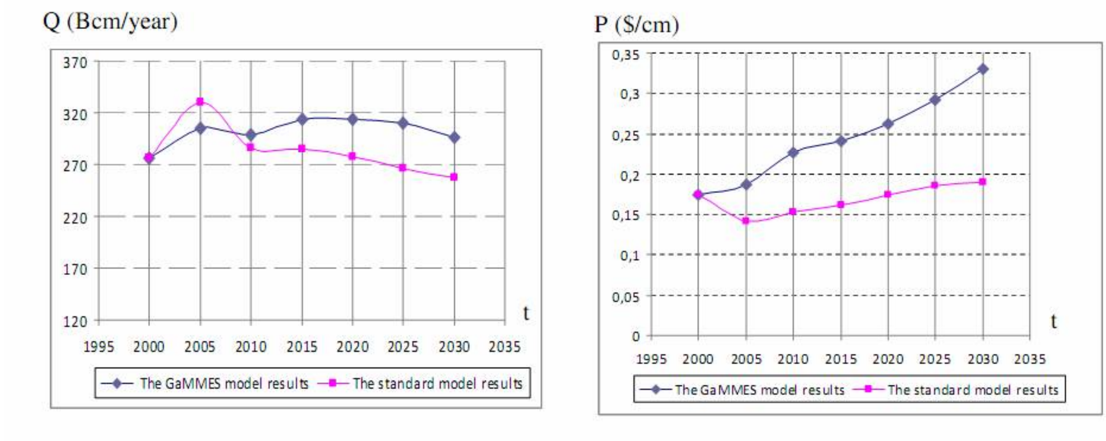

FIGURE 7.5 – *Comparison between the standard and the GaMMES model : consumption and prices.*

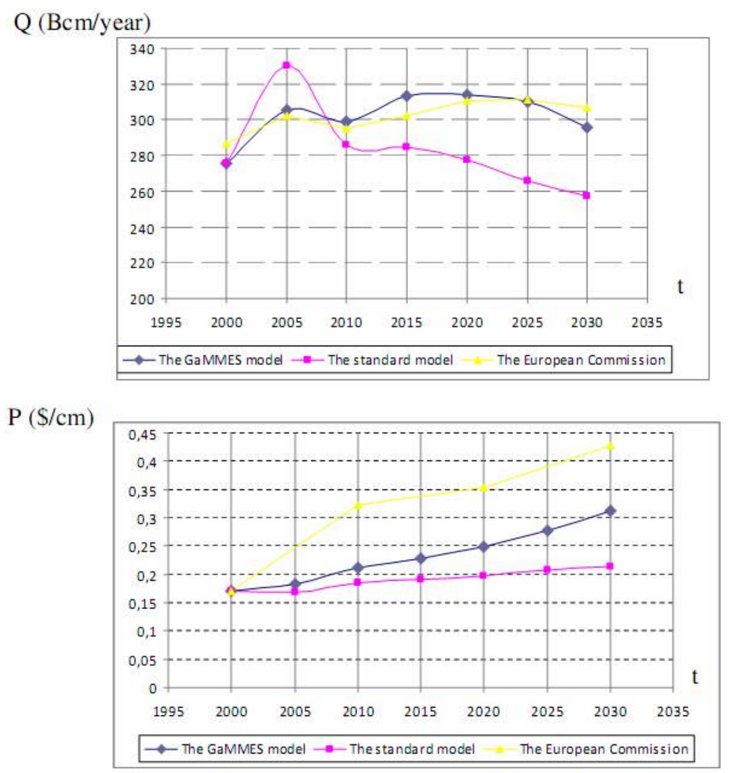

FIGURE 7.6 – *The European Commission, the GaMMES model and the standard model forecasts.*

Comparing the results of both the GaMMES model and the standard model with the 2007 European Commission forecasts (18) gives strong support to the need to take into account fuel substitution, especially in the long run. The standard model output shows a very fast decrease of natural gas consumption in the long-run. This seems at odds with the perspective of the market, since as fossil primary energy consumption is exogenous, the remaining energy consumption has to be met with oil and coal. This view clearly contradicts the global evolution of the different energy shares in the recent past as well as the strong support for cleaner fuels given by the European policy framework. On the contrary, the GaMMES model output gives a better outcome. The demand for gas slowly increases in the medium term, due to both higher fossil primary domestic consumption and a higher share for natural gas in the energy mix (39). The trend is compensated in the long run by the increased exercise of market power. The 2010 kink is mostly explained by the quick depletion of domestic reserves.

These previous results and those of figure 7.5 show that consumed quantities provided by the model are in line with the European Commission forecasts. This gives confidence in the GaMMES results, for the European Commission forecasts are subject to countries' review and acceptance. Regarding the prices, GaMMES is closer to the European Commission scenario than the standard model, even if both of these scenarios underestimate the prices.

In conclusion, compared to a standard description, the GaMMES model gives a better representation of the evolution of the natural gas prices and consumption. It is necessary to take into consideration the fuel substitution in the natural gas markets' modeling because is allows a better understanding of the consumers' behavior.

To test the effects of the systems dynamics approach, starting from time-step three (2010-2014), six sets of exogenous coal and oil price patterns over time were input varying only in time-step three. Then the different endogenous gas prices that resulted were analyzed. Hence, we are able to draw, in the third time-step, the dependence of the gas price on the oil and coal prices. Figure 7.7 gives the evolution of the (average) European natural gas price in the third time-step vs. the oil and coal prices. For the sake of clarity, we showed the evolution of the natural gas price over the competitive price p_c.

Obviously, this evolution is an increasing function of the substitution fuels' prices. The higher the oil and coal prices are, the greater the natural gas demand will be and, therefore, the higher the natural gas price will be. This property also concerns the long-term contract prices between the producers and the independent traders η_{pi}. Hence, our model allows us to capture part of the indexation (on coal and oil prices) effects via the substitution in the inverse demand function.

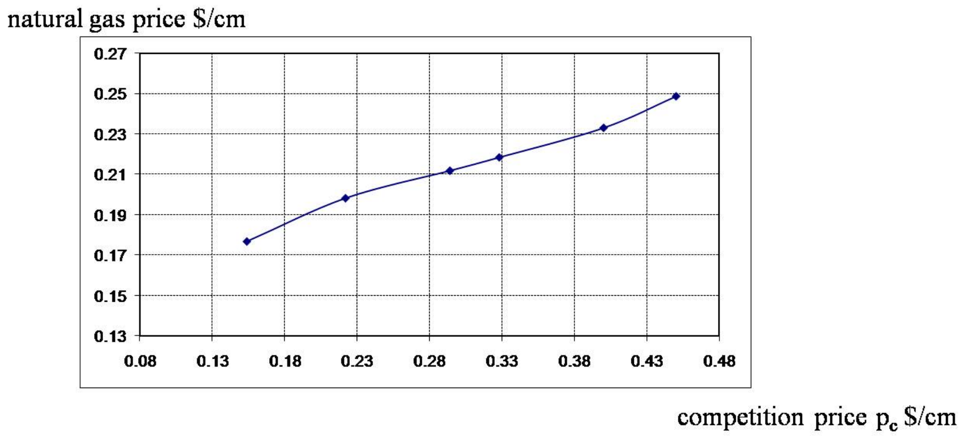

Figure 7.7 – *Evolution of the natural gas price over the competitive price in 2015.*

§ 7.3 Conclusion

This chapter presents an application of GaMMES to the nothwestern European gas trade between 2000 and 2035. The model has been solved using the PATH solver with GAMS. After the calibration process, the model was applied to the European natural gas trade between 2000 and 2035 to understand consumption, prices, production, and natural gas dependence. The consumption and price forecast are consistent with those found in the literature. A study of the evolution of the natural gas dependence on foreign supplies has been carried out. It shows that northwestern Europe will become more and more dependent on foreign supplies in the future. Long-term contract prices and volumes have been presented, analyzed, and compared with current data in order to understand the producers/traders' interaction.

Our results have been compared with other forecasts : one provided by the European Commission and another one issued from a standard model where the energy substitution is not present. The results show that it is important to capture, while studying the natural gas demand function, the possible energy substitution regarding other possible usable fuels market' prices.

In order to illustrate the possible use of fuel substitution, we studied the evolution of the natural gas price over the coal and oil prices. The coal-oil price indexation of the natural gas price in the spot markets or in the long-term contracts can be understood using these studies.

The GaMMES model coud be used in the future to address the impact of the Shale gas's penetration in Europe. This can be carried out by considering a more realistic representation of the European gas trade (more consuming and producing countries, a more complex infrastructure, etc.). It would allow us to understand and quantify the effect of the Shale gas on the long-term contracting behavior, regarding both prices and volumes. Also, stochasticity can be introduced when representing the impact of risk on the market or the seasonality of the demand. The demand can also be made random by modeling the fluctuations of the oil price. This would allow one to study, thanks to the energy substitution, the impacts of the oil price fluctuations on the natural gas price. The model can also be used to perform more policy focused analysis such as the impact of environmental policies on the gas trade evolution and the development of major infrastructures toward Europe, or to analyze more thoroughly the impact of long-term contracts on the competition in the downstream market.

Some of this will be presented in the following chapters.

- Chapitre 8 -

Generalized Nash-Cournot games.

§ 8.1 Introduction

The goal of this study is to present a very simple example of a Generalized Nash-Cournot game (GNCG) in order to understand the differences between the Quasi-Variational (QVI) and Variational (VI) Inequality formulations and to see in which cases a QVI solution is also the solution of a VI problem. The uniqueness of the solution will also be discussed in some cases.

We will first define the Standard (SNCG) and Generalized (GNCG) Nash-Cournot games and the Variational (VI) and Quasi-Variational (QVI) inequalities. Then, we prove the equivalence between SNCG and VI, and GNCG and QVI. In particular, we present necessary and sufficient conditions under which a GNCG equilibrium has a VI solution. The last part of the chapter is dedicated to a simple two-player game that illustrates the different theorems presented. We also develop an inexact penalization method that allows one to solve GNC games.

§ 8.2 Standard and Generalized Nash-Cournot games, Variational and Quasi-Variational Inequalities

A Standard Nash-Cournot game (SNCG) describes a situation where a certain number of players strive to optimize their respective utility, thanks to a particular set of strategies. A player's strategy choice can influence the other players' payoff, but not their strategy sets. On the contrary, in a Generalized Nash-Cournot game (GNCG), a player can change some of the other players' strategy sets. In an imperfect competition modeling context, a GNCG appears when the players have common constraints. As an example, if some firms produce natural gas from the same field, they may have to deal with a common resource constraint. If we denote by x_i the volume extracted by firm i and Q the reserve of natural gas in the field, the common reserve constraint will be $\sum_i x_i \leq Q$. Therefore, each firm i's feasibility set contains the following constraint :

$$\forall i,\ x_i \leq Q - \sum_{j \neq i} x_j$$

Hence, this common constraint makes the firms influence each other's feasibility (or strategy) sets, because of their decision variables.

Now, let us define theoretically a GNCG. We denote by $I = \{1, 2, ...N\}$ the set of players. Player i's decision variable is assumed to be a vector $x_i \in \mathbb{R}^{n_i}$. We will denote by x the vector $x = (x_1, x_2, ..., x_N)$ formed by all the decision variables. To distinguish between the different decision variables, we may write x as $x = (x_i, x_i^-)$, where x_i^- is the vector formed by all the decision variables, other than x_i. In order to represent the feasibility sets' inter-dependence, we denote by K_i a point-to-set mapping :

$$K_i :\ \mathbb{R}^{\sum_j n_j - n_i} \longrightarrow P\left(\mathbb{R}^{n_i}\right)$$

such as x_i must belong to $K_i(x_i^-)$. We will assume that $\forall x_i^-$, $K_i(x_i^-)$ is a convex set.

Each player i's payoff is $f_i(x) = f_i(x_i, x_i^-)$. f_i is assumed differentiable and strictly concave, with respect to the variable x_i. f_i depends on x_i but also on x_i^-, which is the standard non-cooperative game frame.

The general formulation of a GNC game is given by the following :

Definition 15. *A GNC game is a situation where each firm i deals with the following program :*

$$\begin{array}{ll} \mathit{Max} & f_i(x) \\ \mathit{s.t.} & x_i \in K_i(x_i^-) \end{array}$$

and the GNC equilibrium is defined by the following :

Definition 16. *x^* is a GNC equilibrium if :*

$$\forall i,\ x_i^* \in K_i(x_i^{*-}) \text{ and } f_i(x_i, x_i^{*-}) \text{ is optimal with respect to the variable } x_i \text{ when } x_i = x_i^*.$$

The condition $x_i^* \in K_i(x_i^{*-})$ reflects the fact that player i's feasibility set depends on the other players' decision variables x_i^{*-}. On the contrary, a Standard Nash-Cournot game is a situation where the strategy sets are not mixed. Thus, the feasibility set K_i no longer depends on x_i^{*-}.

The general formulation of an SNC game is given by the following :

Definition 17. *An SNC game is a situation where each firm i deals with the following program :*

$$\begin{array}{ll} \mathit{Max} & f_i(x) \\ \mathit{s.t.} & x_i \in K_i \end{array}$$

and the SNC equilibrium is defined by the following :

Definition 18. *x^* is an SNC equilibrium if :*

$$\forall i,\ x_i^* \in K_i \text{ and } f_i(x_i, x_i^{*-}) \text{ is optimal with respect to the variable } x_i \text{ when } x_i = x_i^*.$$

To simplify the notation, we will denote by K, in the SNCG situation, the set $K = K_1 \times K_2 ... \times K_N$.

We already know that, thanks to the Euler's inequality (see Part 1), the following problem :

$$\begin{array}{ll} \text{Max} & f_i(x) \\ \text{s.t.} & x_i \in K_i \end{array}$$

is equivalent to :

$$\text{find } x_i \in K_i \text{ such as } \forall y_i \in K_i,\ \nabla_{x_i} f_i(x)(y_i - x_i) \leq 0 \tag{8.1}$$

Theorem 9. *The SNCG is equivalent to :*

$$\text{find } x^* \in K \text{ such as } \forall y \in K,\ \sum_i \nabla_{x_i} f_i(x^*)(y_i - x_i^*) \leq 0 \tag{8.2}$$

Démonstration. Let us show that (8.1) $\Leftrightarrow$ (8.2). Obviously, (8.1) $\Rightarrow$ (8.2). Let x^* be a vector in K such as $\forall y \in K,\ \sum_i \nabla_{x_i} f_i(x^*)(y_i - x_i^*) \leq 0$. If $y_i \in K_i$, then $(y_i, x_i^{*-}) \in K$ and

$$\sum_j \nabla_{x_j} f_j(x^*)(y_j - x_j^*) = \nabla_{x_i} f_i(x^*)(y_i - x_i^*) \leq 0$$

Therefore, (8.2) $\Rightarrow$ (8.1). □

Similarly, a GNCG is equivalent to :

$$\text{find } x^* \text{ such as } \forall i,\ x_i^* \in K_i(x_i^{*-}) \text{ and } \forall y \text{ such as } \forall i,\ y_i \in K_i(x_i^{*-}),\ \sum_i \nabla_{x_i} f_i(x^*)(y_i - x_i^*) \leq 0$$

The previous problems constitute respectively Variational and Quasi-Variational inequalities. Given a point-to-point mapping $F : \mathbb{R}^{n_1+n_2...+n_N} \longrightarrow \mathbb{R}^{n_1+n_2...+n_N}$, these are defined by the following :

Definition 19. *The (F, K) QVI problem is defined by :*

$$\text{find } x^* \text{ such as } \forall i,\ x_i^* \in K_i(x_i^{*-}) \text{ and } \forall y \text{ such as } \forall i,\ y_i \in K_i(x_i^{*-}),\ F(x^*)(y^* - x^*) \leq 0$$

Definition 20. *The (F, K) VI problem is defined by :*

$$\text{find } x^* \in K \text{ such that } \forall y \in K,\ F(x^*)(y^* - x^*) \leq 0$$

We have demonstrated that the GNCG is equivalent to QVI$((\nabla_{x_1} f_1, \nabla_{x_2} f_2, ...\nabla_{x_n} f_n), K)$ and the SNCG is equivalent to VI$((\nabla_{x_1} f_1, \nabla_{x_2} f_2, ...\nabla_{x_n} f_n), K)$. In definition 19 , K is defined by $K = K_1(x_1^{*-}) \times K_2(x_2^{*-}) \times ...K_N(x_N^{*-})$.

§ 8.3 QVI and VI solutions

As previously said, we assume that the payoff functions are strictly concave. We also assume that the GNCG is issued from a common constraints situation. Hence, we will assume that there exists two functions g and h such as the decision variables x_i must satisfy : $g(x) = 0$ and $h(x) \leq 0$, where $x = (x_1, ...x_N)$. The non common constraints are represented by two functions g_i and h_i : $g_i(x_i) = 0$ and $h_i(x_i) \leq 0$. h and h_i are convex and g and g_i affine. Player i's optimization program is as follows :

$$\begin{array}{ll} \text{Max} & f_i(x) \\ \text{s.t.} & g_i(x_i) = 0 \\ & h_i(x_i) \leq 0 \\ & g(x) = 0 \\ & h(x) \leq 0 \end{array}$$

One can notice that constraint qualifications hold. In this case, the mappings K_i are such as :

$$K_i(x_i^-) = \left\{x_i \in \mathbb{R}^{n_i} / g_i(x_i) = 0,\ h_i(x_i) \leq 0,\ g(x_i, x_i^-) = 0,\ h(x_i, x_i^-) \leq 0\right\}$$

The corresponding QVI is :

$$g_i(x_i^*) = 0,\ h_i(x_i^*) \leq 0,\ g(x^*) = 0,\ h(x^*) \leq 0,\ \forall y \text{ such that } \forall i,\ y_i \in K_i(x_i^{*-}),\ \sum_i \nabla_{x_i} f_i(x^*)(y_i - x_i^*) \leq 0$$

and the corresponding KKT conditions, which are necessary and sufficient for optimality, are :

$$\begin{array}{l} \text{find } x_i^*,\ \alpha_i,\ \beta_i,\ \lambda_i,\ \mu_i, \text{ such that} \\ g_i(x_i^*) = 0,\ h_i(x_i^*) \leq 0,\ g(x^*) = 0,\ h(x^*) \leq 0 \\ \nabla_{x_i} f_i(x^*) + \alpha_i \nabla_{x_i} g_i(x_i^*) + \beta_i \nabla_{x_i} h_i(x_i^*) + \lambda_i \nabla_{x_i} g(x^*) + \mu_i \nabla_{x_i} h(x^*) = 0 \\ 0 \geq \beta_i \perp h_i \\ 0 \geq \mu_i \perp h \end{array} \tag{8.3}$$

This problem will be refered to as Problem 1.

If we define by $K = \{x \ / \ g_i(x_i) = 0,\ h_i(x_i) \leq 0,\ g(x) = 0,\ h(x) \leq 0\}$, we call Problem 2 the VI version of problem 1. More precisely, Problem 2 is the following VI :

$$g_i(x_i^*) = 0,\ h_i(x_i^*) \leq 0,\ g(x^*) = 0,\ h(x^*) \leq 0,\ \forall y \in K,\ \sum_i \nabla_{x_i} f_i(x^*)(y_i - x_i^*) \leq 0$$

and the corresponding KKT conditions are :

$$\begin{array}{l} \text{find } x_i^*,\ \alpha_i,\ \beta_i,\ \lambda,\ \mu, \text{ such that} \\ g_i(x_i^*) = 0,\ h_i(x_i^*) \leq 0,\ g(x^*) = 0,\ h(x^*) \leq 0 \\ \nabla_{x_i} f_i(x^*) + \alpha_i \nabla_{x_i} g_i(x_i^*) + \beta_i \nabla_{x_i} h_i(x_i^*) + \lambda \nabla_{x_i} g(x^*) + \mu \nabla_{x_i} h(x^*) = 0 \\ 0 \geq \beta_i \perp h_i \\ 0 \geq \mu \perp h \end{array} \tag{8.4}$$

Theorem 10. *A solution of Problem 2 (VI) is also a solution of Problem 1 (QVI).*

Démonstration. Let us consider x^* a VI solution and demonstrate that it is also a QVI solution. We already know that $g_i(x_i^*) = 0,\ h_i(x_i^*) \leq 0,\ g(x^*) = 0,\ h(x^*) \leq 0$, which means that $x^* \in K$. If y is such as $y_i \in K_i(x_i^{*-})$, then :
$\forall i,\ (y_i, x_i^{*-}) \in K$ and the VI formulation allows us to state that $\nabla_{x_i} f_i(x^*)(y_i - x_i^*) \leq 0$. Hence $\sum_i \nabla_{x_i} f_i(x^*)(y_i - x_i^*) \leq 0$. Since $x^* \in K$, the demonstration is complete. □

The first conclusion to draw from the previous theorem is that a QVI formulation is more general than a VI one, because the VI solutions are also solutions for the QVI. Now it may be interesting to characterize the VI solutions of a QVI. This can be done thanks to this theorem :

Theorem 11. *Let us consider $(x^*, \alpha_i, \beta_i, \lambda_i, \mu_i)$ a solution of Problem 1 (QVI), issued from its KKT conditions. $(x^*, \alpha_i, \beta_i, \lambda_i, \mu_i)$ is solution to Problem 2 (VI) if and only if :*

$$\lambda_1 = \lambda_2 = ... = \lambda_N \text{ and } \mu_1 = \mu_2 = ... = \mu_N$$

.

Démonstration. The demonstration is straightforward, considering equations (8.3) and (8.4) □

Theorem 11 give a simple condition that ensures a QVI solution to be a VI. This condition makes all the dual variables associated with the common constraints of the concerned players equal. We will see in the next section that a GNCG usually has an infinite continuous solution set. On the contrary, an SNCG has usually a discrete (often finite) solution set because it is less degenerate. Therefore, in a GNCG context, we will often pick the VI solution, if possible, when looking for an equilibrium. In that case, Theorem 11 gives a simple selection criterion, based on the common constraints dual variables.

§ 8.4 A simple Generalized Nash-Cournot game

8.4.1 Introduction

This section illustrates the different definitions and theorems presented previously in this chapter.

For that purpose, we consider the situation of two utility maximizing players 1 and 2. The players' strategies sets are supposed to be continuous and are denoted by X_1 and X_2. The strategies x_1 and x_2 are such that $x_1 \in X_1(x_2)$ and $x_2 \in X_2(x_1)$. To take into account the fact that these sets are nondisjoint, we make X_1 depend on x_2 and X_2 on x_1. Player i's utility is denoted by $\Pi_i(x_1, x_2)$. The GNC game can be written as follows :

$$\begin{array}{ll} \text{Max} & \Pi_1(x_1, x_2) \\ \text{s.t.} & x_1 \in X_1(x_2) \end{array}$$

$$\begin{array}{ll} \text{Max} & \Pi_2(x_1, x_2) \\ \text{s.t.} & x_2 \in X_2(x_1) \end{array}$$

It is easy to demonstrate that the previous problem can be rewritten as follows :

find x_1 and x_2 such as $x_1 \in X_1(x_2)$ and $x_2 \in X_2(x_1)$

$$\forall y_1 \in X_1(x_2),\ \forall y_2 \in X_2(x_1),\ -\left(\frac{\partial \Pi_1}{\partial x_1}(x_1, x_2)(y_1 - x_1) + \frac{\partial \Pi_2}{\partial x_2}(x_1, x_2)(y_2 - x_2)\right) \geq 0 \qquad (8.5)$$

Equation 8.5 is the QVI formulation of our GNC game. The functions Π_i are supposed strictly concave with respect to both variables $x_{1,2} \in X_{1,2}$.

8.4.2 The MNCP and QVI formulations

Let us simplify our problem by assuming that $\Pi_i(x_1, x_2) = f(x_1 + x_2)x_i - c_i x_i$ (where f is a decreasing differentiable one variable function and c_i is a nonnegative parameter). This expression of the utilities can represent, for instance, a firm's profit, f being the inverse demand function. We study two problems. The first situation is the following :

Problem 1

$$
\begin{array}{ll}
\text{Max} & f(x_1 + x_2)x_1 - c_1 x_1 \\
\text{s.t.} & x_1 \geq 0
\end{array}
$$

$$
\begin{array}{ll}
\text{Max} & f(x_1 + x_2)x_2 - c_2 x_2 \\
\text{s.t.} & x_2 \geq 0, \ x_2 = x_1
\end{array}
$$

The second game rule is slightly different. The condition $\{x_1 = x_2\}$ is taken out from the second player's optimization program and becomes a matter for both players (economically speaking, this can represent a market-clearing condition).

Problem 2

$$
\begin{array}{ll}
\text{Max} & f(x_1 + x_2)x_1 - c_1 x_1 \\
\text{s.t.} & x_1 \geq 0, \ x_1 = x_2
\end{array}
$$

$$
\begin{array}{ll}
\text{Max} & f(x_1 + x_2)x_2 - c_2 x_2 \\
\text{s.t.} & x_2 \geq 0, \ x_2 = x_1
\end{array}
$$

The main difference between problems 1 and 2 is as follows : in the first situation, the condition $x_2 = x_1$ is important only for player 2 (player 1 realizes his optimization program with no consideration to this condition) whereas in the second situation, the condition $x_1 = x_2$ is important for both players. This difference will appear with more clarity while writing the MCP corresponding to each problem and introducing the dual variables.

In the following calculations, we will assume that $f(0) > max(c_1, c_2)$ and $c_1 \neq c_2$.

If the usual concavity assumptions are satisfied, the KKT conditions allow us to characterize the Nash Cournot equilibrium (they are both necessary and sufficient conditions for the solutions set). More particularly, for problem 1 the equilibrium is reached if and only if there exists $x_1 \in \mathbb{R}_+$, $x_2 \in \mathbb{R}_+$ and $\lambda \in \mathbb{R}$ such as

$$
\begin{array}{llll}
\multicolumn{4}{l}{\text{Problem 1(KKT)}} \\
0 \leq & x_1 \perp & (f'(x_1 + x_2)x_1 + f(x_1 + x_2) - c_1) & \leq 0 \\
0 \leq & x_2 \perp & (f'(x_1 + x_2)x_2 + f(x_1 + x_2) - c_2 - \lambda) & \leq 0 \\
 & & x_1 = x_2 &
\end{array}
\tag{8.6}
$$

For problem 2, the equilibrium is reached if and only if there exists $x_1 \in \mathbb{R}_+$, $x_2 \in \mathbb{R}_+$, $\lambda_1 \in \mathbb{R}$ and $\lambda_2 \in \mathbb{R}$ such as

$$\begin{array}{llll} \text{Problem 2(KKT)} & & & \\ 0 \leq & x_1 \perp & (f'(x_1+x_2)x_1 + f(x_1+x_2) - c_1 + \lambda_1) & \leq 0 \\ 0 \leq & x_2 \perp & (f'(x_1+x_2)x_2 + f(x_1+x_2) - c_2 - \lambda_2) & \leq 0 \\ & & x_1 = x_2 & \end{array} \tag{8.7}$$

Both Problems 1 and 2 are MCP problems. It is interesting to write the corresponding QVI formulations of our two games :

$$\begin{array}{l} \text{Problem 1(QVI)} \\ X_1(x_2) = \mathbb{R}_+,\ X_2(x_1) = \{x_2 \geq 0, / x_2 = x_1\} \\ \text{find } x_{1,2} \text{ such as} \\ x_1 \in X_1,\ x_2 \in X_2(x_1) \\ \forall y_1 \in X_1,\ y_2 \in X_2(x_1) \\ -\left(f'(x_1+x_2)x_1 + f(x_1+x_2) - c_1\right)(y_1 - x_1) - \left(f'(x_1+x_2)x_2 + f(x_1+x_2) - c_2\right)(y_2 - x_2) \geq 0 \end{array} \tag{8.8}$$

and

$$\begin{array}{l} \text{Problem 2(QVI)} \\ X_1(x_2) = \{x_1 \geq 0, / x_1 = x_2\},\ X_2(x_1) = \{x_2 \geq 0, / x_2 = x_1\} \\ \text{find } x_{1,2} \text{ such as} \\ x_1 \in X_1(x_2),\ x_2 \in X_2(x_1) \\ \forall y_1 \in X_1(x_2),\ y_2 \in X_2(x_1) \\ -\left(f'(x_1+x_2)x_1 + f(x_1+x_2) - c_1\right)(y_1 - x_1) - \left(f'(x_1+x_2)x_2 + f(x_1+x_2) - c_2\right)(y_2 - x_2) \geq 0 \end{array} \tag{8.9}$$

The main difference between the two problems appears in the expressions of X_1 and X_2. The first game allows the first player not to consider the condition $x_2 = x_1$ which makes the set X_1 "free" whereas the second game binds it to the variable x_2.
Problem 1(QVI) can be rewritten more simply :

find $x_1 \geq 0$ and $x_2 = x_1$ such as

$$\forall y_1 \geq 0,\ -\big(f'(2x_1)x_1 + f(2x_1) - c_1\big)(y_1 - x_1) \geq 0$$

It is then easy to notice that Problem 1(QVI) cannot be expressed as a VI problem.

Problem 2 is obviously more general than Problem 1. By "more general", we mean that Problem 1's solution set is included in Problem 2's one. Indeed, considering equations (8.6) and (8.7), Problem 2's particular solution where $\lambda_1 = 0$ and $\lambda_2 = c_1 - c_2$ satisfies all of Problem 1's equations. Hence, adding the equation $x_1 = x_2$ in player 1's feasibility set does not reduce our solution set.

We can easily express Problem 1's solutions. We will assume that the function f is such that $\forall x \in \mathbb{R}_+,\ f(x) > 0$. We will also assume that the equation

$$xf'(2x) + f(2x) - c_1 = \mu$$

has a unique solution $x^*(\mu)$ in $\mathbb{R}$, $\forall \mu \in \mathbb{R}$. This is the case, for instance, if f is linear (i.e. $f(x) = ax + b$ with $a < 0$). Hence, Problem 1's solution is :

$x_1 = x_2 = x^*(\mu = 0)$

In a similar way, we can express Problem 2's solution as follows :

$x_1 = x_2 = x^*(\mu)$
$\mu \in \{\mu \in \mathbb{R} / x^*(\mu) \geq 0\}$

One can notice that the QVI problem has a continuous infinite solution set. This particularity is general for QVI problems.

The standard Nash game can be expressed as a VI problem because all the players do not have power over each other's strategy sets. It is then natural to look for particular VI solutions.

Let us consider the following VI problem :

$$\begin{array}{l}
\text{Problem 2(VI)} \\
X = \{(x_1, x_2) \geq 0, / x_1 = x_2\} \\
\text{find } x_1 \text{ and } x_2 \text{ such as} \\
(x_1, x_2) \in X \\
\forall (y_1, y_2) \in X \\
-\left(f'(x_1 + x_2)x_1 + f(x_1 + x_2) - c_1\right)(y_1 - x_1) - \left(f'(x_1 + x_2)x_2 + f(x_1 + x_2) - c_2\right)(y_2 - x_2) \geq 0
\end{array} \tag{8.10}$$

Problem 2(VI) is equivalent to :

$$\begin{array}{l}
\text{Problem 2(VI)} \\
\text{find } x_1 \in \mathbb{R}_+ \text{ such as} \\
\forall y_1 \in \mathbb{R}_+ \\
-\left(f'(2x_1)x_1 + f(2x_1) - \frac{c_1+c_2}{2}\right)(y_1 - x_1) \geq 0 \\
x_2 = x_1
\end{array} \tag{8.11}$$

It is now easy to notice that Problem 2(VI)'s solutions are also solutions of Problem 2(KKT) (and hence Problem 2(QVI)). They correspond to the particular situation where $\lambda_1 = \lambda_2$. Indeed, if $\lambda_1 = \lambda_2$, equations (8.7)' solution is a solution for equations (8.11), and *vice versa.*

This VI solution is not solution for Problem 1(KKT) : otherwise, since $f(0) > max(c_1, c_2)$ we would have (thanks to equations 8.6) $f'(2x_1)x_1 + f(2x_1) = c_1 = \frac{c_1+c_2}{2}$, which is not possible because $c_1 \neq c_2$.

The first conclusions to draw from this study are the following : If the condition $x_1 = x_2$ constrains only player 2's feasibility set, we have a GNC game which may not have VI solutions. This situation can lead to an infinite continuous set of solutions (which is not the case in our particular example, but it can happen in more complex situations). Adding the condition $x_1 = x_2$ in player 1's maximization program leads to a more general QVI that accepts a particular VI solution. This solution is found if we impose the same dual variable to all the players, for the common equation.

To be more explicit, let us consider the particular example of a linear functional form for f, $\forall x \geq 0, f(x) = ax + b$, where the parameters a and b are such that $a < 0$ and $\text{Max}(c_1, c_2) < b$. If we denote by S the solution set of our problems, we find :

$$S_{Problem1} = \left\{ (-\frac{b - c_1}{3a}, -\frac{b - c_1}{3a}) \right\}$$

and

$$S_{Problem2} = \left\{ (x_1, x_2) \in \mathbb{R}^2,\ x_1 = x_2 = -\frac{\lambda + b - c_1}{3a} / \lambda \geq c_1 - b \right\}$$

$S_{Problem2}$ is an infinite continuous set, which is a particularity of a QVI problem solution set. We notice that in $S_{Problem2}$ the VI solution is obtained when the dual variables λ_1 and λ_2 are such as $\lambda_1 + \lambda_2 = c_1 - c_2$ and $\lambda_1 = \lambda_2$.

To conclude, we can state that if we want to take out some equations from the feasibility set of a particular player, which includes strategies of other players to avoid thorny GNC problems (which have usually an infinite continuous set of equilibria) and make it common to all the players, we need to impose the same dual variables for all the players, associated with that equation, in order to have a VI formulation.

8.4.3 The penalization method

There exists a simpler way to transform a GNC game into a standard NC game and avoid having QVI formulations (and hence avoid taking the risk of having an infinite set of equilibria). It consists on removing the equations that make the problem a GNC game and add them to the concerned players objective functions, as a penalty. For example, Problem 1 becomes :

Problem 3

$$\begin{array}{ll} \text{Max} & f(x_1 + x_2)x_1 - c_1 x_1 \\ \text{s.t.} & x_1 \geq 0 \end{array}$$

$$\begin{array}{ll} \text{Max} & f(x_1 + x_2)x_2 - c_2 x_2 - \lambda(x_1 - x_2)^2 \\ \text{s.t.} & x_2 \geq 0 \end{array}$$

where the penalty factor λ is such that $\lambda > 0$. The idea is that if we make λ big enough, we hope that Problem 3's solution will be a good approximation of Problem 1's solution. Mathematically speaking, we want to demonstrate that Solution$_{penalty} \longrightarrow$ Solution$_{GNC}$, when the penalty factor $\lambda \longrightarrow +\infty$. The main advantage of the penalty method is that it transforms a GNC situation into a standard NC game. Indeed, Problem 3 makes the players influence each other's choices only via the objective function.

When the KKT conditions are written, Problem 3 can be reformulated as :

find x_1 and x_2 such that :

$$\begin{array}{lll} \text{Problem 3} & & \\ 0 \leq x_1 \perp & (f'(x_1+x_2)x_1 + f(x_1+x_2) - c_1) & \leq 0 \\ 0 \leq x_2 \perp & (f'(x_1+x_2)x_2 + f(x_1+x_2) - c_2 + 2\lambda(x_1 - x_2)) & \leq 0 \end{array} \tag{8.12}$$

Obviously, the condition $x_1 = x_2$ has been dropped and is taken care of by the penalty factor.

Let us denote Problem 3's solutions by $x_{1,2}(\lambda)$. We assume that $\exists \lambda_0 > 0 \;/\; \forall \lambda > \lambda_0,\; x_1(\lambda) > 0$ and $x_2(\lambda) > 0$ (this assumption will allow us to avoid studying, when the KKT conditions are written, the corner solutions where $x_1 = 0$ or $x_2 = 0$). We also assume that $x_{1,2}(\lambda)$ converges toward a finite limit $x^*_{1,2}$ (which is the case if f is linear), the second equation of (8.12) allows us to write

$$x_2(\lambda) - x_1(\lambda) = \frac{1}{2\lambda}\left(f'(x_1(\lambda)+x_2(\lambda))x_2(\lambda) + f(x_1(\lambda)+x_2(\lambda)) - c_2\right)$$

Since f and f' are continuous, if we make $\lambda \longrightarrow +\infty$ we can state that

$$\left(f'(x_1(\lambda)+x_2(\lambda))x_2(\lambda) + f(x_1(\lambda)+x_2(\lambda))\right) \longrightarrow \left(f'(x_1^*+x_2^*)x_2^* + f(x_1^*+x_2^*)\right)$$

and

$$x_1^* = x_2^* = x^*$$

where x^* can be calculated thanks to the first equation of (8.12). Indeed x^* is solution of :

$$f'(2x^*)x^* + f(2x^*) - c_1 = 0$$

Hence, we have demonstrated that under some particular assumptions, the penalization technique leads to a good approximation of the GNC solution.

Let us study the linear functional form. $f(x) = ax + b$. $b > \text{Max}(c_1, c_2)$. In that case, we can easily calculate Problem 3's solution. If λ is big enough, we can discard the solutions $x_1 = 0$ or $x_2 = 0$ (for example, when $\lambda > a\left(1 - \frac{c_2-b}{2(c_1-b)}\right)$).

$$(x_1^*(\lambda), x_2^*(\lambda)) = \left(\frac{1}{3a(a-2\lambda)}(2(a-\lambda)(c_1-b) - a(c_2-b)), \frac{c_1-b}{a} - \frac{2}{3a(a-2\lambda)}(2(a-\lambda)(c_1-b) - a(c_2-b))\right)$$

Hence,

$$(x_1(\lambda), x_2(\lambda)) \longrightarrow (-\frac{b-c_1}{3a}, -\frac{b-c_1}{3a})$$

which is the property we are looking for. The penalized solution converges like $\frac{1}{\lambda}$, when $\lambda \longrightarrow +\infty$ toward the GNC solution. The penalty method provides a good way to change the formulation of a GNC game (QVI) into an NC game (VI). However, if we want to end up with the VI solution, it is necessary to impose the same penalization factor to all the players.

The penalization technique has provided very good results when applied to the GaMMES model.

§ 8.5 Conclusion

This chapter focuses on Generalized Nash-Cournot games, in order to study different ways to solve such problems. For that purpouse, we highlighted the difference between Standard (SNCG) and Generalized Nash-Cournot (GNCG) games in order to show the usual degeneracy of the Generalized Nash-Cournot equilibria solutions set. Thanks to the variational formulation, a GNCG can be formulated as a Quasi-Variational Inequality (QVI) whereas an SNCG is equivalent to a Variational Inequality (VI). We have shown how an SNCG problem is less degenerate than a GNCG. The QVI's solution set contains the solution set of an SNCG whose VI formulation is more restrictive than the initial QVI. In other words the QVI's solution set of a GNCG always contains VI solutions.

In our natural gas markets modeling, the GNC form appears when we have to deal with mixed constraints that belong to different players' feasibility sets. We have demonstrated that the difference between VI and QVI solutions can be explained thanks to the dual variables associated with the mixed constraints. The VI solution is obtained when all the corresponding dual variables are equal. These conditions remove the degeneracy inherent to the QVI formulation.

We have presented a simple GNCG, based on the interaction between two players, in order to illustrate the differences beween GNCGs and SNCGs. Once the QVI and VI formulations were written, we showed that the SNCG is less degenerate than the GNCG and found simple conditions to characterize the VI solutions. These conditions are consistent with the theorems presented in the first section.

We have tested another theoretical way to solve a GNC problem, by changing it into an SNC one. This technique uses a penalization factor that penalizes a particular player, in his objective function, if the corresponding mixed constraints do not hold. Therefore, it is possible to drop the mixed constraints in the feasibility sets and to deal with SNC problems. We have demonstrated that, if the calculated equilibria converge when the penalization factor is high enough and if we use the same penalization factor for all the players, the equilibria converge toward the VI solution.

Bibliographie

[1] I. Abada & O. Massol, 2011. *Security of supply and retail competition in the European gas market. Some model-based insights.* Energy Policy 39 (2011), 4077-4088.

[2] I. Abada, V. Briat & O. Massol, 2011. *Construction of a fuel demand function portraying interfuel substitution, a system dynamics approach.* Economix Working Paper, available at http ://economix.fr/fr/dt/2011.php.

[3] F. R. Aune, K. E. Rosendahl, E. L. Sagen, 2009. *Globalisation of natural gas markets - effects on prices and trade patterns.* Special Issue of The Energy Journal 30, 39-54.

[4] M. G. Boots & F. A. M. Rijkers & B. F. Hobbs, 2004. *Trading in the Downstream European Gas Market : A Successive Oligopoly Approach.* The Energy Journal, 25(3), 73-102.

[5] BP Statistical Review of World Energy, 2009. www.bp.com.

[6] R. Egging, S. A. Gabriel, 2006. *Examining market power in the European natural gas market.* Energy Policy 34 (17), 2762-2778.

[7] R. Egging, S. A. Gabriel, F. Holtz, J. Zhuang, 2008a. *A complementarity model for the European natural gas market.* Energy Policy 36 (7), 2385-2414.

[8] R. Egging, F. Holz, S. A. Gabriel 2010. *The World Gas Model : A Multi-Period Mixed Complementarity Model for the Global Natural Gas Market.* Energy. 35 (10), pp. 4016-4029.

[9] European Commission, 2007. DG Competition Report on Energy Sector Inquiry 2007, Luxembourg : Office for Official Publ. of the Europ. Communities. *Available at http ://ec.europa.eu/dgs/energy.*

[10] European Commission, 2008. European energy and transport : trends to 2030, update 2007, Luxembourg : Office for Official Publ. of the Europ. Communities. *Available at http ://ec.europa.eu/dgs/energy_transport/figures/trends_2030_update_2007/.*

[11] F. Facchinei & J.-S. Pang, 2003. *Finite-Dimensional Variational Inequalities and Complementarity Problems.* Springer, New York.

[12] M. C. Ferris & T. S. Munson, 1987. *The PATH solver*. Elsevier Science Publishers B.V. (North-Holland).

[13] S. A. Gabriel, S. Kiet, J. Zhuang, 2005a. *A Mixed Complementarity-Based Equilibrium Model of Natural Gas Markets*. Operations Research, 53(5), 799-818.

[14] S. A. Gabriel, J. Zhuang, S. Kiet, 2005b *A Large-scale Complementarity Model of the North American Gas Market*. Energy Economics, 27, 639-665.

[15] S. A. Gabriel, K. E. Rosendahl, R. Egging, H. Avetisyan, S. Siddiqui, 2010. *Cartelization in Gas Markets : Studying the Potential for a 'Gas OPEC'*. In review.

[16] GDF-SUEZ Reference Document, 2009. *http ://www.gdfsuez.com/*.

[17] R. Golombek, E. Gjelsvik, K. E. Rosendahl, 1995. *Effects of liberalizing the natural gas markets in Western Europe*. Energy Journal 16, 85-111.

[18] R. Golombek, E. Gjelsvik, K. E. Rosendahl, 1998. *Increased competition on the supply side of the Western European natural gas market*. Energy Journal 19 (3), 1-18.

[19] P. T. Harker, 1991. *Generalized Nash games and Quasi-Variational inequalities*. European Journal of Operational Research, Vol.54 81-94.

[20] P. T. Harker & J. Pang, 1998. *Finite-dimensional variational inequality and nonlinear complementarity problems : A survey of theory, algorithms and applications*. Mathematical Programming 48 (1990) 161-220 (North-Holland).

[21] F. Holz, C. von Hirschhausen, C. Kemfert, 2008. *A Strategic Model of European Gas Supply (GASMOD)*. Energy Economics 30, 766-788.

[22] F. Holz, 2009. *Modeling the European Natural Gas Market - Static and Dynamic Perspectives of an Oligopolistic Market*. Ph.D. Dissertation, Technische Universitat-Berlin.

[23] R. G. Hubbard & R. J. Weiner, 1986. *Regulation and Long-Term Contracting in U.S. Natural Gas Markets*. Journal of Industrial Economics, Vol. 35, No. 1, pp. 71-79

[24] IEA, 2004. *Natural Gas Information* .

[25] IEA, 2009. *Natural Gas Information* .

[26] International Energy Agency, 2007. *World Energy Outlook 2007*, OECD/IEA.

[27] International Energy Agency, 2008. *World Energy Outlook 2008*, OECD/IEA.

[28] International Energy Agency, 2009. *World Energy Outlook 2009*, OECD/IEA.

[29] International Gas Union, Gastech Conference Amsterdam March 2011, *New Trends in Gas Price Formation*.

[30] W. Lise & B. F. Hobbs, 2008. *Future evolution of the liberalised European gas market. Simulation results with the dynamic GASTALE model*, Energy Policy 36 (6), 1890-1906.

[31] W. Lise & B. F. Hobbs, 2009. *A Dynamic Simulation of Market Power in the Liberalised European Natural Gas Market.* The Energy Journal 30, 119-136.

[32] L. Mathiesen, K. Roland, K. Thonstad, 1987. *The European Natural Gas Market : Degrees of Market Power on the Selling Side.* In R. Golombek, M. Hoel, and J. Vislie (eds.) : Natural Gas Markets and Contracts, North-Holland.

[33] E. Moxnes, 1985. *Price of Oil, Gas, Coal and Electricity in four European Countries 1960-1983.* Christian Michelsen Institue, No. 852260-1, Bergen, Norway.

[34] E. Moxnes & E. Nesset, 1985. *Substitution between Oil, Gas and Coal in UK Industrial Steam Raising.* Christian Michelsen Institue, No. 842240-4, Bergen, Norway.

[35] E. Moxnes, 1987. *The dynamics of Interfuel Substitution in the OECD-Europe Industrial Sector.* Elsevier Science Publishers B.V. (North-Holland).

[36] Perner & Seeliger, 2004. *Prospects of gas supplies to the European market until 2030. Results from the simulation model EUGAS.* Utilities Policy 12 (4), 291-302.

[37] World Trade Gas Model, Rice, 2004.
www.rice.edu/energy/publications/docs/GSP_ WorldGasTradeModel_ Part1_ 05_ 26_ 04.pdf.

[38] Rice, 2005. *http ://www.rice.edu/energy/publications/docs/GAS_ BIWGTM_ March2005.pdf.*

[39] Y. Smeers, 2008. *Gas Models and Three Difficult Objectives*, ECORE discussion paper available at http ://www.ecore.be/.

[40] P. Stevens, 2011. 'Shale Gas Revolution' : Hype and Reality. A Chatham House Report.

§ 8.6 Appendix 1

This appendix demonstrates the concavity of all the players' objective functions.
We will demonstrate that the production cost function is convex with respect to the quantity produced. The storage/withdrawal/investments costs are convex functions because they are linear.

Let's consider a producer p. First we demonstrate the convexity of the Golombek production cost function. We consider a production node f. To simplify the notation, let us denote by q the produced volume (a variable) and by Rf_f the reserve (a constant). We recall that the cost function Pc_f is as follows :

$$\frac{d\ Pc_f}{d\ q} : \quad \begin{array}{l} [0, Rf_f) \longrightarrow R^+ \\ q \longrightarrow a_f + b_f q + \ c_f \ \ln\left(\frac{Rf_f - q}{Rf_f}\right) \end{array}$$

where $c_f \leq 0$ and $b_f \geq 0$.
Pc_f is a $C^2([0, Rf_f))$ function (twice continuously differentiable) and we have :

$$\forall q \in [0, Rf_f) \ \frac{d^2 Pc_f}{d^2 q} = b_f - \frac{c_f}{Rf_f - q} \geq 0$$

Thus, Pc_f is convex.

Producer p's objective function is :
$+\sum_{t,m,f,i} \delta^t \eta_{pi}(zp^t_{mfpi})$
$+\sum_{t,m,f,d} \delta^t \left(p^t_{md}(x^t_{mfpd} + \overline{x^t_{mfpd}})\right) x^t_{mfpd}$
$-\sum_{t,f} \delta^t \left(Pc_f\left(\sum_{t' \leq t} \sum_m q^{t'}_{mfp}, Rf_f\right) - Pc_f\left(\sum_{t' < t} \sum_m q^{t'}_{mfp}, Rf_f\right)\right)$
$-\sum_{t,f} \delta^t Ip_f ip^t_{fp}$
$-\sum_{t,m,p,a} \delta^t ((Tc_a + \tau^t_{ma}) fp^t_{mpa})$

As mentioned before, the inverse demand function has been linearized. Let's write the natural gas price in market d as follows :

$$p^t_{md} = a^t_{md} - b^t_{md}(x^t_{mfpd} + \overline{x^t_{mfpd}})$$

where $b^t_{md} > 0$. The function $\sum_{t,m,f,d} \delta^t \left(p^t_{md}(x^t_{mfpd} + \overline{x^t_{mfpd}})\right) x^t_{mfpd}$ is therefore a concave function of the variables x^t_{mfpd}. Indeed the Hessian matrix H^t_{md} associated with the spot market profit is diagonal and such that the diagonal terms are $H^t_{md} = -2b^t_{md} < 0$. Hence, the Hessian matrix is negative definite.

Let us consider the global cost function GP :
$q^t_{mfp} \longrightarrow GP(q^t_{mfp}) = \sum_{t,f} \delta^t \left(Pc_f\left(\sum_{t' \leq t} \sum_m q^{t'}_{mfp}, Rf_f\right) + Pc_f\left(\sum_{t' < t} \sum_m q^{t'}_{mfp}, Rf_f\right)\right)$. And let's demonstrate that GP is convex. Let's consider two variable vectors $q1^t_{md}$ and $q2^t_{md}$ and

$\lambda \in [0,1]$.

$$
\begin{aligned}
&GP(\lambda q1^t_{md} + (1-\lambda)q2^t_{md})\\
&=\\
&\sum_{t,f} \delta^t \left(Pc_f\left(\sum_{t'\le t}\sum_m(\lambda q1^{t'}_{md} + (1-\lambda)q2^{t'}_{md}), Rf_f\right)\right)\\
&-\sum_{t,f} \delta^t \left(Pc_f\left(\sum_{t'<t}\sum_m(\lambda q1^{t'}_{md} + (1-\lambda)q2^{t'}_{md}), Rf_f\right)\right)\\
&=\\
&\sum_f\sum_{t=0}^{Num} \delta^t \left(Pc_f\left(\sum_{t'\le t}\sum_m(\lambda q1^{t'}_{md} + (1-\lambda)q2^{t'}_{md}), Rf_f\right)\right)\\
&-\sum_f\sum_{t=0}^{Num-1} \delta^{t+1} \left(Pc_f\left(\sum_{t'\le t}\sum_m(\lambda q1^{t'}_{md} + (1-\lambda)q2^{t'}_{md}), Rf_f\right)\right)\\
&=\\
&\sum_f\sum_{t=0}^{Num-1} (\delta^t - \delta^{t+1}) \left(Pc_f\left(\sum_{t'\le t}\sum_m(\lambda q1^{t'}_{md} + (1-\lambda)q2^{t'}_{md}), Rf_f\right)\right)\\
&+\sum_f \delta^{Num} \left(Pc_f\left(\sum_{t'\le Num}\sum_m(\lambda q1^{t'}_{md} + (1-\lambda)q2^{t'}_{md}), Rf_f\right)\right)\\
&=\\
&\sum_f\sum_{t=0}^{Num-1} \delta^t(1-\delta) \left(Pc_f\left(\sum_{t'\le t}\sum_m(\lambda q1^{t'}_{md} + (1-\lambda)q2^{t'}_{md}), Rf_f\right)\right)\\
&+\sum_f \delta^{Num} \left(Pc_f\left(\sum_{t'\le Num}\sum_m(\lambda q1^{t'}_{md} + (1-\lambda)q2^{t'}_{md}), Rf_f\right)\right)
\end{aligned}
$$

Since $0 \le \delta \le 1$ and Pc_f is convex, we can write :

$$
\begin{aligned}
&\sum_f\sum_{t=0}^{Num-1} \delta^t(1-\delta) \left(Pc_f\left(\sum_{t'\le t}\sum_m(\lambda q1^{t'}_{md} + (1-\lambda)q2^{t'}_{md}), Rf_f\right)\right)\\
&+\sum_f \delta^{Num} \left(Pc_f\left(\sum_{t'\le Num}\sum_m(\lambda q1^{t'}_{md} + (1-\lambda)q2^{t'}_{md}), Rf_f\right)\right)\\
&\le\\
&\lambda\sum_f\sum_{t=0}^{Num-1} \delta^t(1-\delta) \left(Pc_f\left(\sum_{t'\le t}\sum_m q1^{t'}_{md}, Rf_f\right)\right)\\
&+(1-\lambda)\sum_f\sum_{t=0}^{Num-1} \delta^t(1-\delta) \left(Pc_f\left(\sum_{t'\le t}\sum_m q2^{t'}_{md}, Rf_f\right)\right)\\
&+\lambda\sum_f \delta^{Num} \left(Pc_f\left(\sum_{t'\le Num}\sum_m q1^{t'}_{md}, Rf_f\right)\right)\\
&+(1-\lambda)\sum_f \delta^{Num} \left(Pc_f\left(\sum_{t'\le Num}\sum_m q2^{t'}_{md}, Rf_f\right)\right)\\
&=\\
&\lambda GP(q1^t_{md}) + (1-\lambda)GP(q2^t_{md})
\end{aligned}
$$

Hence, the cost function is convex. The rest of the profit is made of linear functions of the decision variables. The concavity of the producers objective function is thus demonstrated.

The independent traders' objective function's concavity can be demonstrated in a similar way. Like for the producers, the spot market benefit is also concave.

The pipeline and storage operators objective functions are convex (to minimize) because they are linear. The feasibility sets are all convex due to linearity of the constraint functions.

§ 8.7 Appendix 2

This appendix presents the KKT conditions derived from our model. Once the KKT conditions are written, we get the Mixed Complementarity Problem (MCP) given below.

The producers KKT conditions

$$\forall t,\ m,\ f,\ p,\ i, \quad 0 \le zp^t_{mfpi} \quad \perp \quad \delta^t \eta_{pi} - \gamma^t_{mfp} - \epsilon 2^t_{mfpi} - \eta p^t_{pi} - \sum_n M2_{in} \alpha p^t_{mpn} \quad \le 0 \tag{8.13a}$$

$$\forall t,\ m,\ f,\ p,\ d, \quad 0 \le x^t_{mfpd} \quad \perp \quad \delta^t p^t_{md}(x^t_{mfpd} + \overline{x^t_{mfpd}}) + \delta^t \frac{\partial p^t_{md}}{\partial x^t_{mfpd}}(x^t_{mfpd} + \overline{x^t_{mfpd}}) x^t_{mfpd} - \gamma^t_{mfp} - \epsilon 1^t_{mfpd} - \sum_n M3_{dn} \alpha p^t_{mpn} \quad \le 0 \tag{8.13b}$$

$$\forall t,\ m,\ f,\ p, \quad 0 \le q^t_{mfp} \quad \perp \quad -\sum_{t' \ge t} \delta^{t'} \frac{\partial Pc_f}{\partial q}(\sum_{t'' \le t'} \sum_m q^{t''}_{mfp}, Rf_f) + \sum_{t' > t} \delta^{t'} \frac{\partial Pc_f}{\partial q}(\sum_{t'' < t'} \sum_m q^{t''}_{mfp}, Rf_f) - \sum_{t' \ge t} \phi^{t'}_f - \chi^t_{mf} + \gamma^t_{mfp} - (-1)^m (\vartheta 1^t_f - \vartheta 2^t_f) - \epsilon 3^t_{mfp} + \sum_n M1_{fn} \alpha p^t_{mpn} \quad \le 0 \tag{8.13c}$$

$$\forall t,\ f,\ p, \quad 0 \le ip^t_{fp} \quad \perp \quad -\delta^t Ip_f - \epsilon 4^t_{fp} + \sum_m \sum_{t' \ge t + delay_p} \chi^{t'}_{mf} (1 - dep_f)^{t'-t} - \iota p^t_f + Lf_f \sum_{t' \ge t + delay_p} \iota p^{t'}_f (1 - dep_f)^{t'-t} \quad \le 0 \tag{8.13d}$$

$$\forall t,\ p,\ i, \quad 0 \le up_{pi} \quad \perp \quad \sum_t \eta p^t_{pi} - \eta_{pi} \quad \le 0 \tag{8.13e}$$

$$\forall t,\ f, \quad 0 \le \phi^t_f \quad \perp \quad \sum_p \sum_{t' \le t} \sum_m q^{t'}_{mfp} - Rf_f \quad \le 0 \tag{8.13f}$$

$$\forall t,\ m,\ f, \qquad 0 \le \chi^t_{mf} \quad \perp \quad \sum_p q^t_{mfp} - Kf_f(1-dep_f)^t - \sum_p \sum_{t' \le t-delay_p} ip^{t'}_{fp}(1-dep_f)^{t-t'} \le 0 \tag{8.14a}$$

$$\forall t,\ m,\ f,\ p, \qquad 0 \le \gamma^t_{mfp} \quad \perp \quad -q^t_{mfp} + \sum_i zp^t_{mfpi} + \sum_d x^t_{mfpd} \le 0 \tag{8.14b}$$

$$\forall t,\ f, \qquad 0 \le \vartheta 1^t_f \quad \perp \quad \sum_m \sum_p (-1)^m q^t_{mfp} - fl_f \le 0 \tag{8.14c}$$

$$\forall t,\ f, \qquad 0 \le \vartheta 2^t_f \quad \perp \quad -\sum_m \sum_p (-1)^m q^t_{mfp} - fl_f \le 0 \tag{8.14d}$$

$$\forall t,\ f, \qquad 0 \le \iota p^t_f \quad \perp \quad \sum_p ip^t_{fp} - Lf_f Kf_f(1-dep_f)^t - Lf_f \sum_p \sum_{t' \le t-delay_p} ip^{t'}_{fp}(1-dep_f)^{t-t'} \le 0 \tag{8.14e}$$

$$\forall t,\ f, m,\ p,\ d, \qquad 0 \le \epsilon 1^t_{mfpd} \quad \perp \quad x^t_{mfpd} - O_{fp}H \le 0 \tag{8.14f}$$

$$\forall t,\ m, f,\ p,\ i, \qquad 0 \le \epsilon 2^t_{mfpi} \quad \perp \quad zp^t_{mfpi} - O_{fp}H \le 0 \tag{8.14g}$$

$$\forall t,\ m, f,\ p, \qquad 0 \le \epsilon 3^t_{mfp} \quad \perp \quad q^t_{mfp} - O_{fp}H \le 0 \tag{8.14h}$$

$$\forall t,\ f,\ p, \qquad 0 \le \epsilon 4^t_{fp} \quad \perp \quad ip^t_{fp} - O_{fp}H \le 0 \tag{8.14i}$$

$$\forall t,\ m,\ p,\ n, \qquad \text{free} \quad \alpha p^t_{mpn} \qquad \begin{aligned}[t] &\sum_a M_6(a,n) f p^t_{mpa}(1 - loss_a) \\ &- \sum_a M5_{an} f p^t_{mpa} + \sum_f M1_{fn} q^t_{mpf} \\ &- \sum_d \sum_f M3_{dn} x^t_{mfpd} \\ &- \sum_i \sum_f M2_{in} z p^t_{mfpi} \end{aligned} \qquad = 0 \tag{8.15a}$$

$$\forall t,\ p,\ i, \qquad \text{free} \quad \eta p^t_{pi} \qquad up_{pi} - \sum_{f,m} z p^t_{mfpi} \qquad = 0 \tag{8.15b}$$

$$\forall\ p,\ i, \qquad \text{free} \quad \eta_{pi} \qquad ui_{pi} - up_{pi} \qquad = 0 \tag{8.15c}$$

The independent traders' KKT conditions

$$\forall t,\ m,\ p,\ i, \qquad 0 \le zi^t_{mpi} \quad \perp \quad \begin{aligned}[t] &- \delta^t \eta_{pi} - \eta i^t_{pi} \\ &+ \psi^t_{mi} \\ &+ \sum_n M2_{in} \alpha i^t_{min} \\ &+ (1 - min_{pi}) \upsilon^t_{mpi} \end{aligned} \qquad \le 0 \tag{8.16a}$$

$$\forall t,\ m,\ i,\ d, \qquad 0 \le y^t_{mid} \quad \perp \quad \begin{aligned}[t] &\delta^t p^t_{md}(y^t_{mfpd} + \overline{y^t_{mfpd}}) \\ &\delta^t \frac{\partial p^t_{md}}{\partial y^t_{mid}} (y^t_{mfpd} + \overline{y^t_{mfpd}}) y^t_{mid} \\ &- \psi^t_{mi} - \sum_n M3_{dn} \alpha i^t_{min} \end{aligned} \qquad \le 0 \tag{8.16b}$$

$$\forall t,\ i,\ s, \qquad 0 \le r^t_{is} \quad \perp \quad - \delta^t Rc_s + \mu^t_{is} - \beta s^t_s \qquad \le 0 \tag{8.16c}$$

$$\forall t,\ i,\ s, \qquad 0 \le in^t_{is} \quad \perp \quad \begin{aligned}[t] &- \delta^t (Ic_s + Wc_s) \\ &- \mu^t_{is} - \sum_m (-1)^m \psi^t_{mi} \\ &- \sum_n M4_{sn} \alpha i^t_{min} (-1)^m \end{aligned} \qquad \le 0 \tag{8.16d}$$

$$\forall t,\ p,\ i, \qquad 0 \le ui_{pi} \quad \perp \quad \sum_t \eta i^t_{pi} + \eta_{pi} \qquad \le 0 \tag{8.16e}$$

$$
\begin{array}{llllr}
\forall t,\ m,\ i, & \text{free} \quad \psi^t_{mi} & & \sum_p zi^t_{mpi} - \sum_d y^t_{mid} + (-1)^m \sum_s in^t_{is} & = 0 \quad (8.17\text{a}) \\
\forall t,\ i,\ s, & 0 \leq \mu^t_{is} & \perp & in^t_{is} - r^t_{is} & \leq 0 \quad (8.17\text{b}) \\
\forall t,\ m,\ i,\ n, & \text{free} \quad \alpha i^t_{min} & & \sum_a M6_{an} fi^t_{mia}(1 - loss_a) & = 0 \quad (8.17\text{c}) \\
 & & & - \sum_a M5_{an} fi^t_{mia} - \sum_d M3_{dn} y^t_{mid} & \\
 & & & + \sum_p M2_{in} zi^t_{mpi} & \\
 & & & - (-1)^m \sum_s M4_{sn} in^t_{is} & \\
\forall t,\ p,\ i, & \text{free} \quad \eta i^t_{pi} & & ui_{pi} - \sum_m zi^t_{mpi} & = 0 \quad (8.17\text{d}) \\
\forall\ p,\ i, & \text{free} \quad \eta_{pi} & & ui_{pi} - up_{pi} & = 0 \quad (8.17\text{e}) \\
\forall t,\ m,\ p,\ i, & 0 \leq \upsilon^t_{mpi} & & - zi^t_{mpi} + min_{pi} \sum_m zi^t_{mpi} & \leq 0 \quad (8.17\text{f}) \\
\forall t,\ s, & 0 \leq \beta s^t_s & \perp & \sum_i r^t_{is} - Ks_s - \sum_{t' \leq t - delay_s} is^{t'}_s & \leq 0 \quad (8.17\text{g})
\end{array}
$$

The pipeline operator KKT conditions

$$\forall t,\ m,\ p,\ a, \qquad 0 \le fp^t_{mpa} \quad \perp \quad -\delta^t(Tc_a + \tau^t_{ma}) - \tau^t_{ma} + \sum_n M6_{an}\alpha p^t_{mpn}(1 - loss_a) - \sum_n M5_{an}\alpha p^t_{mpn} \le 0 \tag{8.18a}$$

$$\forall t,\ m,\ i,\ a, \qquad 0 \le fi^t_{mia} \quad \perp \quad -\delta^t(Tc_a + \tau^t_{ma}) - \tau^t_{ma} + \sum_n M6_{an}\alpha i^t_{min}(1 - loss_a) - \sum_n M5_{an}\alpha i^t_{min} \le 0 \tag{8.18b}$$

$$\forall t,\ a, \qquad 0 \le ik^t_a \quad \perp \quad -\delta^t Ik_a + \sum_{t' \ge t + delay_i} \tau^{t'}_{ma} - \iota a^t_a + La_a \sum_{t' \ge t + delay_i} \iota a^{t'}_a \le 0 \tag{8.18c}$$

$$\forall t,\ m,\ a, \qquad 0 \le \tau^t_{ma} \quad \perp \quad \sum_p fp^t_{mpa} + \sum_i fi^t_{mia} - Tk_a - \sum_{t' \le t - delay_i} ik^t_a \le 0 \tag{8.18d}$$

$$\forall t,\ a, \qquad 0 \le \iota a^t_a \quad \perp \quad ik^t_a - Tk_a - \sum_{t' \le t - delay_i} ik^t_a \le 0 \tag{8.18e}$$

$$\forall t,\ m,\ p,\ n, \qquad \text{free} \quad \alpha p^t_{mpn} \qquad \sum_a M_6(a,n) fp^t_{mpa}(1 - loss_a) - \sum_a M5_{an} fp^t_{mpa} + \sum_f M1_{fn} q^t_{mpf} - \sum_d \sum_f M3_{dn} x^t_{mfpd} - \sum_i \sum_f M2_{in} zp^t_{mfpi} = 0 \tag{8.18f}$$

$$\forall t,\ m,\ i,\ n, \qquad \text{free} \quad \alpha i^t_{min} \qquad \begin{aligned} &\sum_a M6_{an} fi^t_{mia}(1 - loss_a) \\ &- \sum_a M5_{an} fi^t_{mia} - \sum_d M3_{dn} y^t_{mid} \\ &+ \sum_p M2_{in} zi^t_{mpi} \\ &- (-1)^m \sum_s M4_{sn} in^t_{is} \end{aligned} \qquad = 0 \tag{8.19a}$$

The storage operator KKT conditions

$$\forall t,\ s, \qquad 0 \le is^t_s \quad \perp \quad \begin{aligned} &-\delta^t Is_s + \sum_{t' \ge t + delay_s} \beta s^{t'}_s \\ &- \iota s^t_s + Ls_s \sum_{t' \ge t + delay_s} \iota s^{t'}_s \end{aligned} \qquad \le 0 \tag{8.20a}$$

$$\forall t,\ s, \qquad 0 \le \beta s^t_s \quad \perp \quad \sum_i r^t_{is} - Ks_s - \sum_{t' \le t - delay_s} is^{t'}_s \qquad \le 0 \tag{8.20b}$$

$$\forall t,\ s, \qquad 0 \le \iota s^t_s \quad \perp \quad is^t_s - Ls_s Ks_s - Ls_s \sum_{t' \le t - delay_s} is^{t'}_s \qquad \le 0 \tag{8.20c}$$

Sixième partie

Introducing stochasticity in GaMMES : the S-GaMMES model

- Chapitre 9 -

Introducing stochasticity in GaMMES : the S-GaMMES model.

§ 9.1 Introduction

When trying to represent the natural gas industry, the modeler may have to deal with uncertainty, at practically all the different gas chain levels. If we consider production, for instance, the exploration activities contain a lot of uncertainty since a producing firm does not know, *a priori*, the amount of gas trapped under the ground before drilling. Regarding the infrastructure, technical hazards may constitute an important uncertainty source in the gas transport. As for the demand, its fluctuations among the months of the year (or the seasonality) is mainly driven by the temperature variation, which is fundamentally a random phenomenon from the point of view of energy economics. Adding to that, uncertainty may be the consequence of political or technical issues that are sometimes hard to take care of in detailed mathematical models. As an example, the Russia/Ukraine dispute over the Russian gas dedicated to Europe that led to an important shortage of supplies, was mainly motivated by political reasons. The unpredictedness of the shortage, which happened twice between 2006 and 2010, may make us consider such situations as random.

Taking into account randomness in the decisions of a gas industry actor may radically change its planning, as compared with a deterministic foresight's outcome. Indeed, a trader for example who has to choose its gas supplies may want to diversify its sources if he has to deal with security of supply issues. A random demand will deeply influence a producer or a storage operator's investment decisions. Therefore, to be more realistic, it is important to capture randomness of the gas markets when trying to mathematically model them. Nevertheless, though this leads to more realism, considering stochasticity in models is not costless. Indeed, stochastic models are often huge in terms of number of variables and hold computational problems when solving them, which forces the modeler to use decomposition techniques, such as the Benders' decomposition (6), (25) or scenario reduction methods (12). Therefore, one must carefully select the type of randomness (production, demand, etc.) to consider.

Security of supply and randomness in European gas supplies have been studied in Part 2 of this manuscript. Among all the other types of random gas market's characteristics, we decided to model the uncertainty associated with the demand because on the one hand we believe that its impact on the markets' outcome (especially prices and consumption) is the most important and, on the other hand, the demand function specification is the most serious drawback of current gas markets models (51) because it presents an arbitrary aspect in the calibration. The economic literature provides an important panel of numerical models whose objective is to describe the natural gas trade structure while taking into account stochasticity. As an example, we can cite the "Stochastic World Gas Model" (University of Maryland) ((16)), which presents a stochastic extension of the "World Gas Model" ((14)), where the demand is made random. Other interesting works include (24) and (54). Most of these models consider only randomness of the demand.

A casual look at the oil and gas prices in the spot markets suggests that they are strongly correlated (39). This is mainly due to two reasons : The first is the LTC prices' oil price indexation and the second is related to energy substitution.

Long-term contract prices between producers and traders have always been indexed by the oil products' price [1], to allow natural gas to be a competitive fuel. [2] Since LTC prices constitute a supply marginal cost for the traders, they are correlated to the gas spot prices and, therefore, spot gas and oil prices become correlated too.

Energy substitution also plays an important role in linking the fuels' prices. Indeed, if the consumers are allowed to choose their energy consumption's source, they will go for the cheapest fuel to satisfy their demand (notwithstanding capacity consumption and investment inertia). Therefore, such a consumption feature will ensure all the fuels remain competitive in the market and will link their prices.

Taking into account long-term contracts' oil price indexation in gas markets modeling requires exogenous data, such as the indexation formula between each pair of producer/trader. Because of a lack of data and the fact that we wanted the LTCs to be endogenous in the model, we decided to focus mainly on energy substitution to capture the gas and oil prices correlation.

The model we present, named S-GaMMES, Stochastic Gas Market Modeling with Energy Substitution, is based on an oligopolistic approach to the natural gas markets. The interaction between all the players is a Generalized Nash-Cournot competition and we explicitly take into consideration, in an endogenous way, the long-term contractual aspects (prices and volumes) of the markets. The representation of the demand is new and rich because it includes the possible substitution, within the overall energy consumption, between different types of fuels. Hence, in our work, we mitigate the market power exerted by the strategic players : they cannot force the natural gas price up freely because some consumers would switch to other fuels to satisfy their demand. This chapter presents the stochastic extension of GaMMES by making the demand random.

The economic structure we modeled is the one used in the deterministic GaMMES. In particular, we divide the markets into two stages : the upstream part that represents production and the downstream one, constituted by the different spot markets (end-use consumption markets). Both stages are linked by a set of independent traders. The traders buy gas from the producers on a long-term contract basis and bring it to the spot markets where market power is exerted. Both producers and traders have market power and compete via a Nash-Cournot competition. Long-term contracts, production, transportation, and storage investments are endogenous to the model and this property makes our formulation a Generalized Nash-Cournot game.

The specification of the demand function is the one derived from the system dynamics approach presented in Part 3. Besides, in order to capture the oil price's fluctuation and the oil/gas price correlation, we decided to model the oil price as a random variable. This property makes the demand function stochastic.

1. This was also the case for Netback pricing.
2. Currently, some coal prices indexation formulas are being introduced in the contracts.

The remaining of this part is as follows : this chapter is a general description of the chosen economic structure representation. All the players are presented and are divided into two categories : the strategic and the non-strategic ones. The strategic interaction is also detailed. The notation is presented and we explain how we introduced stochasticity in the demand representation and construct the scenario tree. The third part is dedicated to the mathematical representation of the markets : the optimization programs associated with all the strategic and non-strategic players are presented and discussed. We also explain in this part how we make the long-term contract prices and volumes endogenous to the model. A set of theorems and theoretical results inherent to S-GaMMES is provided and discussed. They principally concern long-term contract prices and volumes characteristics. Chapter 10 gives an application of our model to the European natural gas trade where the calibration process and the results are discussed. The results provide scenarios of the evolution of consumption, prices, and production in northwestern Europe. LTC aspects are also provided and discussed in depth. In addition, we define, calculate, and discuss the value (loss and gain) of the stochastic solution adapted to our model.

The same notation will be used in chapters 9 and 10.

§ 9.2 The model

9.2.1 Economic description

The economic structure is similar to the one described in GaMMES. We refer to chapter 6 for more details.

The main advantage of the S-GaMMES model is that it takes into account, in an endogenous way, long-term contracts between the independent traders and the producers. Obviously, this representation is quite realistic for the European natural gas trade since the latter is still dominated by long-term selling/purchase prices and volumes. Another advantage inherent to this description is that the inverse demand function explicitly takes into consideration the possible substitution between consumption of natural gas and the competing fuels.

Market power is exerted by the producers and the independent traders in the spot markets, where the competition is modeled thanks to a Nash-Cournot equilibrium.

Considering the energy substitutions in the natural gas demand mitigates the market power that can be exerted by all the strategic players in the end-use markets. Indeed, this is due to the fact that the consumers have the ability to reduce the natural gas share in their energy mixes if the gas market price is much higher than the substitution fuel's (such as oil and coal) price. Therefore, the producers may not have any considerable incentive to reduce their natural gas production in order to force the price up. This model property allows us to take into account the oil/natural gas prices indexation : the Nash-Cournot interaction will link the natural gas price to the coal and oil prices because of the demand function dependence on these parameters.

Standard stochastic natural gas market models, like (24), (54) and (31) usually consider randomness in the demand. If the demand function is considered linear, which is the case in most of these models, $consumption = a - b \times price$, then the parameter a is usually made stochastic using a discrete probability law. This leads to the construction of a scenario tree that captures the dynamics of the model. Unfortunately most of these models give arbitrary probability laws to the demand levels and do not carry out a realistic calibration process. As an example, the parameter a may follow a Gaussian distribution with an arbitrary mean value and variance. In the Stochastic GaMMES model, randomness is also taken care of by the demand level. Indeed, in order to capture the demand fluctuations and make the model more realistic, we introduced stochasticity in the demand via the fluctuations of the oil price. For that purpose, an econometric study of the oil price is carried out in order to deduce and calibrate the probability law of the oil price's dynamic evolution.

The model also takes advantage of a scenario tree representation where each node represents the intersection of randomness and time. The oil price, at each time-step, is hence a random variable that influences the demand function parameters at each scenario node.

The transport and storage infrastructure is modeled using competitive pipeline and storage operators whose objective is to minimize the operation costs. Regarding the transport, the cost includes transportation, congestion, and investment fees. Regarding the storage, the cost includes capacity reservation, storage, withdrawal, and investment fees.

9.2.2 Notation

The units chosen for the model are the following : quantities in toe (i.e., Ton Oil Equivalent) or Bcm (i.e., 10^9 cubic meters) and unit prices in \$/toe or \$/cm.

The following table summarizes the notation chosen for the exogenous parameters and the endogenous variables.

Exogenous factors

P	set of producers-dedicated traders
I	set of independent traders
D	set of gas consuming countries in the downstream market (no distinction between the sectors) $D \subset N$
T	time $T = \{0, 1, 2, ..., Num\}$
M	set of seasons. Off-peak (low-consumption) and peak (high-consumption) regimes
F	set of all the gas production fields. $F \subset N$
N	set of the nodes
S	set of the storage sites $S \subset N$
A	set of the arcs (topology)
Ω	set of scenario nodes
Ω_l	set of the tree leaves $\Omega_l \subset \Omega$
Rf_f	field f's total gas resources (endowment)
Kf_f	field f's initial capacity of production, year 0
Lf_f	production node f's maximum increase of the production capacity (in %)
Ic_s	injection marginal cost at storage site s (constant)
Wc_s	withdrawal marginal cost at storage site s (constant)
Rc_s	reservation marginal cost at storage site s (constant)
Ls_s	storage node s's maximum increase of the storage capacity (in %)
Pc_f	production cost function, field f
Tc_a	transport marginal cost through arc a (constant)
Tk_a	pipeline initial capacity through arc a, year 0
Ks_s	initial storage capacity at site s, year 0
Is_s	investment marginal costs in storage (constant)
Ip_f	investment marginal costs in production (constant)
Ik_a	investment marginal costs in pipeline capacity through arc a (constant)
La_a	arc a's maximum increase of the transport capacity (in %)
O	incidence matrix $\in \mathrm{M}_{F\times P}$. $O_{fp} = 1$ if and only if producer p owns field f
B	incidence matrix $\in \mathrm{M}_{I\times D}$. $B_{id} = 1$ if and only if trader i is located at the consumption node d
$M1$	incidence matrix $\in \mathrm{M}_{F\times N}$. $M1_{fn} = 1$ if and only if node n has field f
$M2$	incidence matrix $\in \mathrm{M}_{I\times N}$. $M2_{in} = 1$ if and only if trader i is located at node n
$M3$	incidence matrix $\in \mathrm{M}_{D\times N}$. $M3_{dn} = 1$ if and only if node n has market d
$M4$	incidence matrix $\in \mathrm{M}_{S\times N}$. $M4_{sn} = 1$ if and only if node n has storage site s
$M5$	incidence matrix $\in \mathrm{M}_{A\times N}$. $M5_{an} = 1$ if and only if arc a starts at node n
$M6$	incidence matrix $\in \mathrm{M}_{A\times N}$. $M6_{an} = 1$ if and only if arc a ends at node n
$\pi(\omega)$	probability of occurence of scenario node ω
$t(\omega)$	time associated with scenario node ω
H	maximum value for the quantities produced and consumed

δ^{ω}_{md}	an inverse demand function parameter
β^{ω}_{md}	an inverse demand function parameter
γ^{ω}_{md}	an inverse demand function parameter
pc^{ω}_{md}	an inverse demand function parameter
fl_f	field f's flexibility : the maximum spread between production during off-peak and peak seasons
min_{pi}	percentage of the minimum quantity that has to be exchanged on the long-term contract trade between i and p
δ	discount factor
$delay_{s,i,p}$	period of time necessary to undertake technical investments
$loss_a$	loss factor through arc a
dep_f	depreciation factor of the production capacity at field f

Endogenous variables

x^{ω}_{mfpd}	quantity of gas produced by p from field f for the end-use market d, scenario node ω, season m in Bcm
zp^{ω}_{mfpi}	quantity of gas produced by p from field f dedicated to the long-term contract with trader i, scenario node ω, season m in Bcm
zi^{ω}_{mpi}	quantity of gas bought by trader i from producer p with a long-term contract scenario node ω, season m in Bcm
up_{pi}	quantity of gas sold by producer p to trader i with a long-term contract, each year in Bcm
ui_{pi}	quantity of gas bought by trader i from producer p on the long-term contract, each year in Bcm
y^{ω}_{mid}	quantity of gas sold by i to the market d, scenario node ω, season m in Bcm
ip^{ω}_{fp}	producer p's increase of field f's production capacity, due to investments in production, scenario node ω in Bcm/time unit
q^{ω}_{mfp}	production of producer p from field f, scenario node ω, season m in Bcm
p^{ω}_{md}	market d's gas price, result of the Cournot competition between all the traders, scenario node ω, season m in \$/cm
η_{pi}	long-term contract price contracted between producer p and trader i in \$/cm
r^{ω}_{is}	amount of storage capacity reserved by trader i at site s, scenario node ω in Bcm
in^{ω}_{is}	volume injected by trader i at site s, scenario node ω in Bcm
is^{ω}_{s}	increase of storage capacity at site s, scenario node ω due to the storage operator investments in Bcm/time unit

ik_a^ω	increase of the pipeline capacity through arc a, scenario node ω, due to the TSO investments in Bcm/time unit
$fp_{m,p,a}^\omega$	gas quantity that flows through arc a from producer p scenario node ω, season m in Bcm
$fi_{m,i,a}^\omega$	gas quantity that flows through arc a from trader i scenario node ω, season m in Bcm
$\tau_{m,a}^\omega$	the dual variable associated with arc a capacity constraint scenario node ω, season m in Bcm/season. It represents the congestion transportation cost over arc a

The previous table is divided into two parts. The upper half represents the exogenous parameters or functions whereas the lower half represents the different decision variables and the inherent retail prices.

The indices p, d, i, f, n, s, a, m, ω and t are such that $p \in P$, $d \in D$, $i \in I$ $f \in F$, $n \in N$, $s \in S$, $a \in A$, $m \in M$, $\omega \in \Omega$ and $t \in T$. In the remainder of the chapter and according to the context, a node can either represent a geographical location (of a production field, a consumption market or a storage site) or a location in the scenario tree.

The long-term contract between producer p and trader i fixes both a unit selling price and an amount to be purchased by the independent trader i each year from producer p. Both price and quantity will be specified endogenously by the model.
Matrix O is such that $O_{fp} = 1$ if producer p owns field f and $O_{fp} = 0$ otherwise.

Figure 9.1 represents a schematic overview of S-GaMMES.

9.2.3 The inverse demand function

We need to specify a functional form for the inverse demand function which links the price p_d at market d to the quantity brought to the market. Most of the natural gas models (49), (48), (41), (14) do not take into account fuel substitution. Let h_{md}^ω be the specific inverse demand function in market d, season m of scenario node ω. We assume that the long-term contract quantities do not directly influence the market competition price, which is to say that $p_{md}^\omega = h_{md}^\omega(\sum_i y_{mid}^\omega + \sum_f \sum_p x_{mfpd}^\omega)$. (Actually, this assumption is necessary to guarantee the concavity of the objective functions of each strategic player's maximization problem, regardless of the quantities decided by the other competitors. Otherwise, this assumption can be dropped if linear functions are used).
As mentioned in the introduction, we want to capture the inter-fuel substitution in the global energy consumption. To be able to do so, we used a system dynamics approach that models the behavior of the consumers who have to decide whether they invest in new burners that use either oil, coal or natural gas. The model is fully developed in part 3 and (3). If we denote by Q_{md}^ω

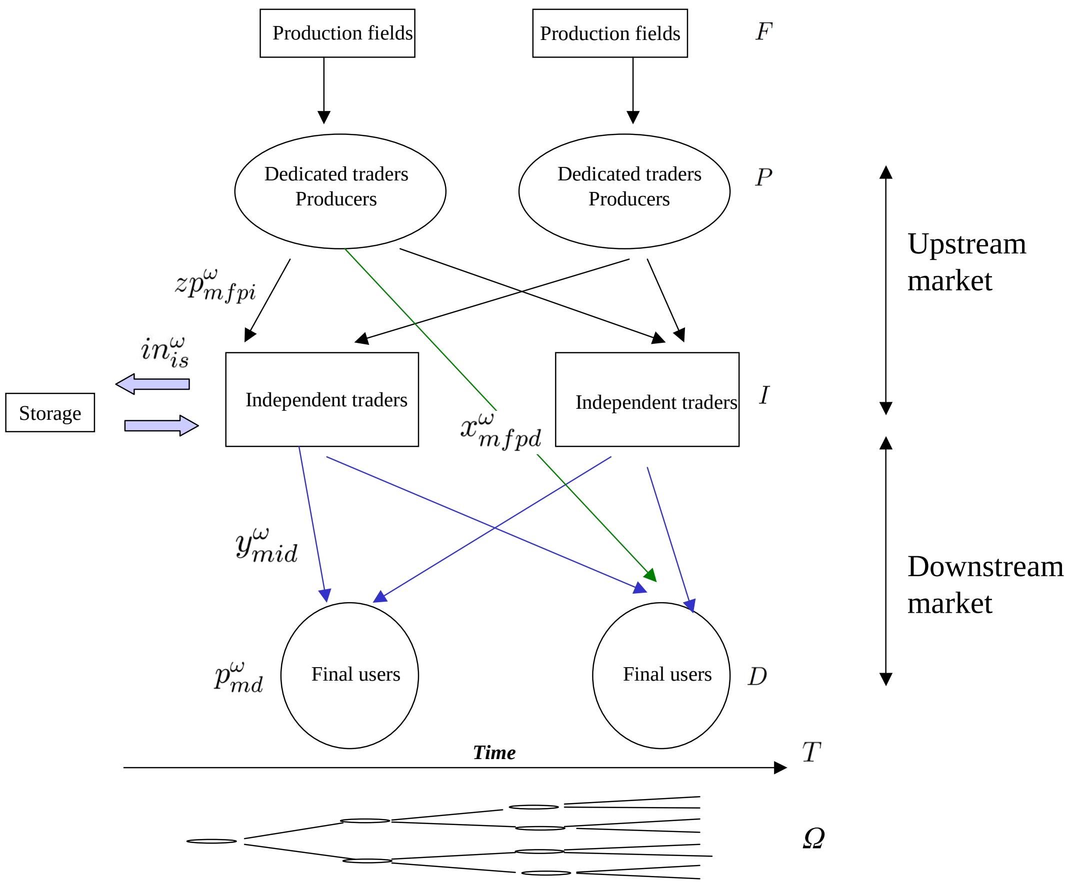

Figure 9.1 – *The market representation in S-GaMMES.*

the quantity $\sum_i y^{\omega}_{mid} + \sum_f \sum_p x^{\omega}_{mfpd}$, the gas demand study (3) provides the following inverse demand function :

$$p^{\omega}_{md} = \begin{array}{ll} pc^{\omega}_{md} + \frac{1}{\gamma^{\omega}_{md}}\text{atanh}\left(\frac{\delta^{\omega}_{md}+\beta^{\omega}_{md}-Q^{\omega}_{md}}{\delta^{\omega}_{md}}\right) & \text{if } Q^{\omega}_{md} \geq \beta^{\omega}_{md} + \frac{\delta^{\omega}_{md}\beta^{\omega}_{md}}{\delta^{\omega}_{md}+\beta^{\omega}_{md}} \\ p'c^{\omega}_{md} + \frac{1}{\gamma'^{\omega}_{md}}\text{atanh}\left(\frac{\delta'^{\omega}_{md}+\beta'^{\omega}_{md}-Q^{\omega}_{md}}{\delta'^{\omega}_{md}}\right) & \text{if } Q^{\omega}_{md} \leq \beta^{\omega}_{md} + \frac{\delta^{\omega}_{md}\beta^{\omega}_{md}}{\delta^{\omega}_{md}+\beta^{\omega}_{md}} \end{array} \tag{9.1}$$

where the parameters δ, β, γ and pc, which are time- and season-dependent must be calibrated.

The distinction between the domains $Q^{\omega}_{md} \geq \beta^{t}_{md} + \frac{\delta^{\omega}_{md}\beta^{\omega}_{md}}{\delta^{\omega}_{md}+\beta^{\omega}_{md}}$ and $Q^{\omega}_{md} \leq \beta^{\omega}_{md} + \frac{\delta^{\omega}_{md}\beta^{\omega}_{md}}{\delta^{\omega}_{md}+\beta^{\omega}_{md}}$ is needed to take into account the anticipated scrapping of burners and to avoid absurd situations where the price rises toward $+\infty$ and also to guarantee the concavity of the objective functions. The parameters δ', β', γ' and $p'c$ are calculated to guarantee the continuity of h and its derivative h'. To make the price converge toward 0 when the quantity goes to $+\infty$, we need to force $\beta' = 0$.

The function atanh is such that :

$$\forall x \in (-1, 1) \ \text{atanh}(x) = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$$

To calibrate the demand function for the future, we need to specify a scenario for the global fossil energy demand and the oil and coal market prices. Our system dynamics approach (3) will allow us to understand how the global demand is going to be shared between the consumption of the three fuels and explicitly find the natural gas demand function.

In order to have an algorithm convergence in a reasonable time, the inverse demand function has been linearized in S-GaMMES.

9.2.4 The scenario tree

This section specifies how the scenario tree is constructed in the model. The demand is made random in order to capture the strong fluctuations of the oil price in Europe. The oil price's dynamic evolution will influence the inverse demand function parameters δ, β, γ and pc. Indeed, if the oil price is high in a certain year, consumers will invest more in natural gas burners (the substitute) and therefore, the future demand for natural gas will rise [3]. On the contrary, a low oil price will reduce the future demand for natural gas. The study of the coal price's evolution over time indicates that its fluctuation is negligible compared to the oil one (10). Therefore, the coal price is not taken as random. To simplify the model, the total gross fossil energy demand is also deterministic.

3. This argument holds for a constant evolution of the coal price.

Let us denote by p_b the chain of the Brent price, with a six-month time-step[4] and ζ_b the corresponding logarithmic percentage price change :

$$\zeta_b = \frac{\ln(p_{b+1}) - \ln(p_b)}{\ln(p_b)} \tag{9.2}$$

The data base we use for the Brent price is given in (10).
More precisely, p_b is the mean value, over six months, of the Brent price and ζ_b the six-month logarithmic percentage change.

Figure 9.2 gives the evolution of the price p_b and ζ_b, $b \in \{1, 2, ...64\}$. $b = 1$ corresponds to the period July 1977 to December 1977 and $b = 64$ to the period January 2009 to June 2009.

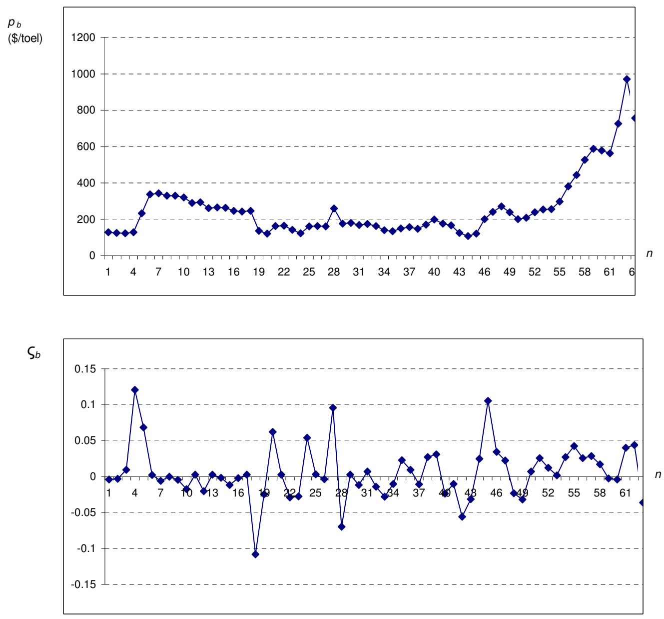

FIGURE 9.2 – *The evolution of p_b in \$/toe and ζ_b over time.*

Figure 9.3 is a histogram of the variable ζ_b. A visual inspection of the correlogram shows no sign of linear auto-correlation between the variables ζ_b, $b \in \{1, 2, ...64\}$. In addition, the variables'

4. This time-step is the one that gives the best correlation in our study.

independence has been checked using the BDS test (11) (the BDS statistics with two dimensions 0.008 with probability 0.52). The ζ_b variables can therefore be considered as independent and identically distributed random variables. The Kolmogorov-Smirnov (43) test allows us to state that they have a normal distribution. Indeed, the test did not reject the 0-hypothesis of normality (Adj. value 1.04 with probability 0.22). The Gaussian fit is provided in Figure 9.3. This fit is obtained by minimizing the normalized error between a Gaussian distribution and the histogram points of ζ_b. The normalized error is given by the following : if (x_i, y_i), $i \in \{1, 2...n\}$ are the histogram points and $\mathcal{N}_{x0,\sigma}$ a Gaussian distribution, the error $e_{x0,\sigma}$ is :

$$e_{x0,\sigma} = \frac{1}{n}\sum_{i=1}^{n} |\frac{y_i - \mathcal{N}_{x0,\sigma}(x_i)}{y_i}| \tag{9.3}$$

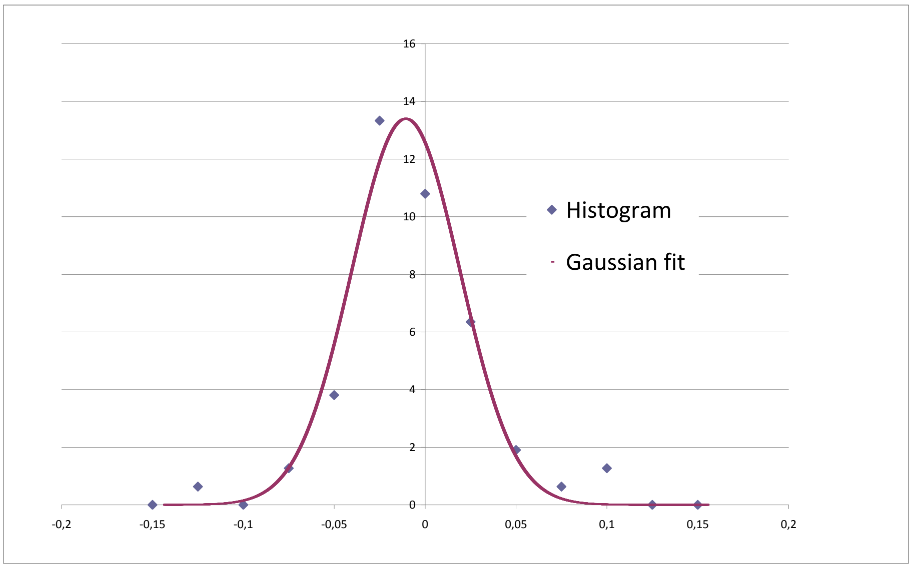

FIGURE 9.3 – *The histogram of ζ and the Gaussian fit.*

The statistical study we carried out provided a normalized error of 0.2, for the Gaussian fit shown in Figure 9.3. The other numerical results (mean value, variance) will be provided later.

In the representation of the European natural gas trade, we may need to use a time-step longer than six months [5]. Hence, it is worthwhile to explain how we can deduce the new log percentage

5. Like in the deterministic version, we typically use a five-year time-step in the stochastic version.

change's probability density that can be used directly by the model. Let us assume that the model's study time-step is $\kappa\times$ 6 months where $\kappa \in \mathbb{N}$, and call λ the new log percentage change :

$$\lambda_b = \frac{\ln(p_{b+\kappa}) - \ln(p_b)}{\ln(p_b)} \tag{9.4}$$

λ_b takes into account the $\kappa\times 6$ months offset. In our case, κ=10 (relation between five years and six months time-steps). The relationship between λ_b and ζ_b is given using the following lemmas and theorems :

Lemma 2. $\forall b \in \mathbb{N},\ p_{b+\kappa} = p_b^{\Pi_{i=0}^{\kappa-1}(1+\zeta_{b+i})}$

Démonstration. Lemma 2's proof is straightforward : using equation (9.2), we can deduce that :

$$\forall b,\ p_{b+1} = p_b^{1+\zeta_b} \tag{9.5}$$

Hence

$$\begin{aligned}
p_{b+\kappa} &= p_{b+\kappa-1}^{1+\zeta_{b+\kappa-1}} \\
&= p_{b+\kappa-2}^{(1+\zeta_{b+\kappa-2})(1+\zeta_{b+\kappa-1})} \\
&= p_{b+\kappa-3}^{(1+\zeta_{b+\kappa-3})(1+\zeta_{b+\kappa-2})(1+\zeta_{b+\kappa-1})} \\
&= \ldots \\
&= p_b^{\Pi_{i=0}^{\kappa-1}(1+\zeta_{b+i})}
\end{aligned}$$

□

The previous equation can be rewritten as follows :

$$\ln(p_{b+\kappa}) = \Pi_{i=0}^{\kappa-1}(1+\zeta_{b+i})\ \ln(p_b) \tag{9.6}$$

Figure 9.3 shows that the random variable ζ is such that $|\zeta| \leq 0.05$ with a more than 90% probability. Hence, we can write that, in first approximation, $\forall b \in \mathbb{N}, \zeta_b << 1$ and

$$\Pi_{i=0}^{\kappa-1}(1+\zeta_{b+i}) \simeq 1 + \sum_{i=0}^{\kappa-1} \zeta_{b+i} \tag{9.7}$$

The approximation error can be bounded via the following theorem :

Theorem 12. *If we denote by* $\epsilon = \Pi_{i=0}^{\kappa-1}(1+\zeta_{b+i}) - \left(1 + \sum_{i=0}^{\kappa-1}\zeta_{b+i}\right)$ *the error and* ζ_{max} *the maximum absolute value of* ζ_k *:* $\zeta_{max} = Max\{|\zeta_k|,\ k \in \{b, ..., b+\kappa-1\}\}$*, then :*

$$|\epsilon| \leq (1+\zeta_{max})^\kappa - 1 - \kappa\zeta_{max} \tag{9.8}$$

The proof is as follows :

Démonstration. If we develop $\Pi_{i=0}^{\kappa-1}(1+\zeta_{b+i})$, we find :

$$\Pi_{i=0}^{\kappa-1}(1+\zeta_{b+i}) = \sum_{j=0}^{\kappa} \sum_{\substack{(k_1,k_2,...,k_j)\in\{b,...,b+\kappa-1\}\\ k_1<k_2<...<k_j}} \zeta_{k_1}\zeta_{k_2}...\zeta_{k_j} \tag{9.9}$$

In the sum, the term that corresponds to $j=0$ is 1 and to $j=1$ is $\left(\sum_{i=0}^{\kappa-1}\zeta_{b+i}\right)$. Therefore, we can write :

$$\sum_{i=0}^{\kappa-1}(1+\zeta_{b+i}) = \left(1+\sum_{i=0}^{\kappa-1}\zeta_{b+i}\right) + \sum_{j=2}^{\kappa} \sum_{\substack{(k_1,k_2,...,k_j)\in\{b,...,b+\kappa-1\}\\ k_1<k_2<...<k_j}} \zeta_{k_1}\zeta_{k_2}...\zeta_{k_j} \tag{9.10}$$

Therefore, we have :

$$|\epsilon| = \left| \sum_{j=2}^{\kappa} \sum_{\substack{(k_1,k_2,...,k_j)\in\{b,...,b+\kappa-1\}\\ k_1<k_2<...<k_j}} \zeta_{k_1}\zeta_{k_2}...\zeta_{k_j} \right| \tag{9.11}$$

and we can write :

$$\begin{aligned}
|\epsilon| &\leq \sum_{j=2}^{\kappa} \sum_{\substack{(k_1,k_2,...,k_j)\in\{b,...,b+\kappa-1\}\\ k_1<k_2<...<k_j}} |\zeta_{k_1}||\zeta_{k_2}|...|\zeta_{k_j}| \\
&\leq \sum_{j=2}^{\kappa} \sum_{\substack{(k_1,k_2,...,k_j)\in\{b,...,b+\kappa-1\}\\ k_1<k_2<...<k_j}} \zeta_{max}^{j} \\
&= \sum_{j=2}^{\kappa} \binom{\kappa}{j} \zeta_{max}^{j} \\
&= (1+\zeta_{max})^{\kappa} - 1 - \kappa\zeta_{max}
\end{aligned}$$

The last equality is obtained exploiting the Newton binomial theorem.

□

As stated before, figure 9.3 shows that the random variable ζ_{max} is such that $\zeta_{max} \leq 0.05$ with a more than 90% probability. Therefore, $|\epsilon| \leq 0.1$ with more than 90% probability.

Using equations (9.6) and (9.7), we can deduce that :

$$\lambda_b = \sum_{i=0}^{\kappa-1}\zeta_{b+i} \tag{9.12}$$

Since we assumed that ζ_b are independent and identically distributed random variables and since we know that they follow the Gaussian distribution $\mathcal{N}_{x_0,\sigma}$, then we can derive that λ_b are also independent and identically distributed and follow a Gaussian probability distribution $\mathcal{N}_{\kappa x_0,\sqrt{\kappa}\sigma}$.

In order to solve the model in a reasonable time, we decided to use only two scenarios for the oil price at each time-step. Therefore, we have to approximate the logarithmic yield λ's Gaussian probability density $\mathcal{N}_{\kappa x_0,\sqrt{\kappa}\sigma}$ by a two-value probability law. Let us call λ_1 and λ_2 the two possible values of the random variable λ that will be used by the model, p and $1-p$ the associated probabilities. The goal now is to find λ_1, λ_2 and p.

The mean value and the standard deviation of λ are respectively κx_0 and $\sqrt{\kappa}\sigma$. Therefore, we can write :

Lemma 3. *λ_1, λ_2 and p verify*

$$p\lambda_1 + (1-p)\lambda_2 = x_0 \tag{9.13a}$$
$$p\lambda^2 + (1-p)\lambda_2^2 - x_0^2 = \kappa\sigma^2 \tag{9.13b}$$

Démonstration. Equation (9.13a) equates the average of the two-value probability law $(\lambda_1, \lambda_2, p)$ and the Gaussian distribution. Equation (9.13b) does the same with the variance. □

Equations (9.13a) and (9.13b) allow us to state that (assuming that $p \notin \{0,1\}$) :

$$\lambda_1 = x_0 + \frac{\sigma}{\sqrt{p}}\sqrt{1-p} \tag{9.14a}$$
$$\lambda_2 = x_0 - \frac{\sigma}{\sqrt{p}}\left(\frac{1}{\sqrt{1-p}} - \sqrt{1-p}\right) \tag{9.14b}$$

Since we are looking for three variables $(\lambda_1, \lambda_2, p)$, we need to impose a third equation. In our case, we added the following relation :

$$\lambda_1 = -\lambda_2$$

in order to capture the increasing and decreasing fluctuations of the oil price. A nonnegative value for λ implies an increase of the oil price, whereas a negative value means a decrease of the oil price. In our case, $\lambda_1 \geq 0$ and $\lambda_2 \leq 0$ correspond respectively to an increase and decrease of the Brent price.

The study of the Brent price between 1977 and 2009 gives the follwing values for λ_1, λ_2 and p :

λ_1	0.1
λ_2	-0.1
p	0.15

These values have been calculated for a five-year evolution of the Brent price.
To summarize, the oil price evolves via the following formula :

$$p_{b+\kappa} = p_b^{1+\lambda_1} \quad \text{with probability } p \tag{9.15a}$$
$$p_{b+\kappa} = p_b^{1+\lambda_2} \quad \text{with probability } 1-p \tag{9.15b}$$

Relations (9.15a) and (9.15b) suggest that the oil price is modeled as a Markov chain. This assumption has been verified and used in some statistical studies of the oil price (53).

To calibrate the demand function for the future, we need to specify a deterministic scenario for the global fossil energy demand and the coal markets' prices. The oil price evolution will create the scenario tree as follows : at each time-step the oil price can follow respectively equation (9.15a) or (9.15b) with probability p and $1-p$. In the stochastic version, the model's time scope is 2000-2035, with a time resolution of five years. In order to keep the model solvable in a reasonable time, we considered randomness only for the first five time-steps, until 2025. Starting from 2025, the oil price follows the trend forecast by the European Commission (18) : an increase by 3.7% per year. The corresponding log-change percentage in that case is called μ.

Figure 9.4 gives a schematic description of the scenario tree for the oil price and therefore for the demand function parameters. There are 31 nodes and seven time-steps (35 years). Node 0, which is the top of the scenario tree corresponds to the 2000-2004 time period. Note that randomness occurs starting from 2010 (node 1).

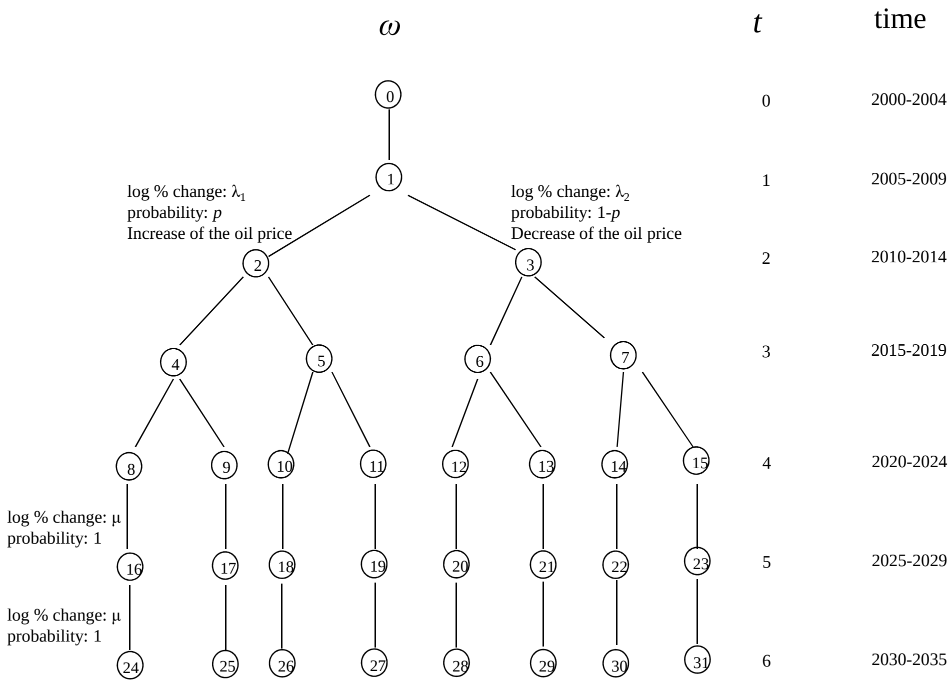

FIGURE 9.4 – *The scenario tree.*

Figure 9.5 gives the values of the different scenario nodes weights $\pi(\omega)$ of the tree.

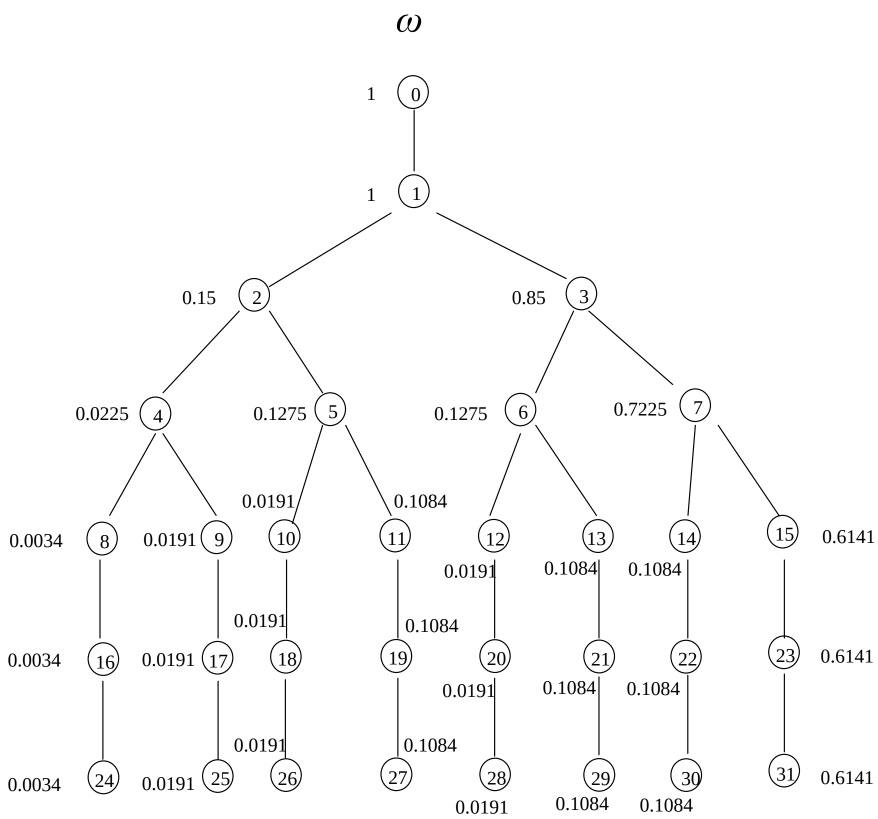

FIGURE 9.5 – *The scenario tree weights.*

9.2.5 The mathematical description

This section details the mathematical description of the model. It presents the optimization problems of all the supply chain players. [6]

Each node of the scenario tree represents the intersection of randomness with time. The first-stage variables are all the ones decided by all the players at node 0 and 1, which are deterministic. Once these variables have been chosen, they cannot be changed later, in the rest of the time periods (or nodes). Similarly, the decisions made at nodes 2 and 3 will influence the market outcome at all the forthcoming nodes $\omega \in \{4, 5, ...31\}$ especially the production, transport, and storage investments. More generally, an investment or a contractual decision made at node ω will remain unchanged and will influence the market structure at all the nodes ω' that follow ω. In the rest of the chapter, when two scenario nodes ω and ω' are related, we will write :

$$\omega \leq \omega'$$

if ω' is a successor of ω (or ω is a predecessor of ω'). For example, in the scenario tree, *node* $4 \leq$ *node* 4 and *node* 25.

In order to take into account the different investment delays, we need to consider the strict successors of a particular node. When two scenario nodes ω and ω' are related, we will write :

$$\omega < \omega' \Leftrightarrow \omega \leq \omega' \text{ and } \omega \neq \omega'$$

if ω' is a strict successor of ω (or ω is a strict predecessor of ω'). For example, in the scenario tree, *node* $4 <$ *node* 8 and *node* 25.

Using this scenario tree approach, we do not need to take into account non-anticipativity conditions, because we define a relation between the nodes (successors and predecessors). From the programming perspective, these relations have been included by using incidence matrices $M7$ and $M8$: $M7(\omega, \omega') = 1$ if and only if $\omega \leq \omega'$, otherwise, $M7(\omega, \omega') = 0$ and $M8(\omega, \omega') = 1$ if and only if $\omega < \omega'$, otherwise, $M8(\omega, \omega') = 0$.

All the players are assumed to be risk-neutral. They optimize their respective expected utilities over the time horizon and all the scenarios are computed simultaneously.

Producer p's maximization program is given below. The corresponding decision variables are zp^{ω}_{mfpi}, x^{ω}_{mfpd}, ip^{ω}_{fp}, q^{ω}_{mfp} and up_{pi}.

6. Note that the dual variables are written in parentheses next to their associated constraints.

Max

$$\sum_{\omega,m,f,i} \pi(\omega)\delta^{t(\omega)}(\eta_{pi})(zp^{\omega}_{mfpi})$$
$$+ \sum_{\omega,m,f,d} \pi(\omega)\delta^{t(\omega)} \left(p^{\omega}_{md}(x^{\omega}_{mfpd} + \overline{x^{\omega}_{mfpd}})\right) x^{\omega}_{mfpd}$$
$$- \sum_{\omega,f} \pi(\omega)\delta^{t(\omega)} Pc_f \left(\sum_{\omega' \leq \omega} \sum_{m} q^{\omega'}_{mfp}, Rf_f\right)$$
$$+ \sum_{\omega,f} \pi(\omega)\delta^{t(\omega)} Pc_f \left(\sum_{\omega' < \omega} \sum_{m} q^{\omega'}_{mfp}, Rf_f\right)$$
$$- \sum_{\omega,f} \pi(\omega)\delta^{t(\omega)} Ip_f(ip^{\omega}_{fp})$$
$$- \sum_{\omega,m,a} \pi(\omega)\delta^{t(\omega)}((Tc_a + \tau^{\omega}_{m,a})fp^{\omega}_{m,p,a})$$

such that :

$$\forall \omega,\ f, \qquad \sum_{p} \sum_{\omega' \leq \omega} \sum_{m} q^{\omega}_{mfp} - Rf_f \leq 0 \qquad (\phi^{\omega}_{f}) \tag{9.16a}$$

$$\forall \omega,\ f,\ m, \qquad \sum_{p} q^{\omega}_{mfp} - Kf_f(1 - dep_f)^{t(\omega)} - \sum_{p} \sum_{\omega' < \omega} ip^{\omega'}_{fp}(1 - dep_f)^{t(\omega) - t(\omega')} \leq 0 \qquad (\chi^{\omega}_{mf}) \tag{9.16b}$$

$$\forall \omega,\ m,\ f, \qquad -q^{\omega}_{mfp} + \left(\sum_{i} zp^{\omega}_{mfpi} + \sum_{d} x^{\omega}_{mfpd}\right) \leq 0 \qquad (\gamma^{\omega}_{mfp}) \tag{9.16c}$$

$$\forall \omega,\ f,\ p, \qquad \sum_{m}((-1)^m q^{\omega}_{mfp}) - fl_f \leq 0 \qquad (\vartheta 1^{\omega}_{fp}) \tag{9.16d}$$

$$\forall \omega,\ f,\ p, \qquad -\sum_{m}((-1)^m q^{\omega}_{mfp}) - fl_f \leq 0 \qquad (\vartheta 2^{\omega}_{fp}) \tag{9.16e}$$

$$\forall \omega,\ f,\ d,\ m, \qquad x^{\omega}_{mfpd} - O_{fp}H \leq 0 \qquad (\epsilon 1^{\omega}_{mfpd}) \tag{9.16f}$$

$$\forall \omega,\ f,\ i,\ m, \qquad zp^{\omega}_{mfpi} - O_{fp}H \leq 0 \qquad (\epsilon 2^{\omega}_{mfpi}) \tag{9.16g}$$

$$\forall \omega,\ f,\ m, \qquad q^{\omega}_{mfp} - O_{fp}H \leq 0 \qquad (\epsilon 3^{\omega}_{mfp}) \tag{9.16h}$$

$$\forall \omega,\ f, \qquad ip^{\omega}_{fp} - O_{fp}H \leq 0 \qquad (\epsilon 4^{\omega}_{fp}) \tag{9.16i}$$

$$\forall \omega,\ f, \qquad \sum_{p} ip^{\omega}_{fp} - Lf_f Kf_f(1 - dep_f)^{t(\omega)} - Lf_f \sum_{p} \sum_{\omega' < \omega} ip^{\omega'}_{fp}(1 - dep_f)^{t(\omega) - t(\omega')} \leq 0 \qquad (\iota p^{\omega}_{f}) \tag{9.16j}$$

$$
\begin{array}{llll}
\forall \omega,\ m,\ n, & \sum_a M6_{an} f p^{\omega}_{m,p,a}(1-loss_a) - \sum_a M5_{an} f p^{\omega}_{m,p,a} & & \\
& + \sum_f q^{\omega}_{mpf} M1_{fn} - \sum_d \sum_f x^{\omega}_{mfpd} M3_{dn} & & \\
& - \sum_i \sum_f zp^{\omega}_{mfpi} M2_{in} & = 0 & (\alpha p^{\omega}_{m,p,n}) \\
& & & (9.17a) \\
\forall \omega,\ i, & up_{pi} - \sum_{f,m} zp^{\omega}_{mfpi} & = 0 & (\eta p^{\omega}_{pi}) \quad (9.17b) \\
\forall\ p,\ i, & ui_{pi} - up_{pi} & = 0 & (\eta_{pi}) \quad (9.17c) \\
\forall \omega,\ m,\ d,\ i,\ f, & zp^{\omega}_{mfpi},\ x^{\omega}_{mfpd},\ ip^{\omega}_{fp},\ q^{\omega}_{mfp},\ up_{pi} & \geq 0 &
\end{array}
$$

We denote by $\overline{x^{\omega}_{mfpd}}$ the total amount of gas brought at node ω, season m to the market d by all the players different from producer p.

The term

$$\sum_{\omega,m,f,i} \pi(\omega)\delta^{t(\omega)}(\eta_{pi})(zp^{\omega}_{mfpi}) + \sum_{\omega,m,f,d} \pi(\omega)\delta^{t(\omega)}\left(p^{\omega}_{md}(x^{\omega}_{mfpd} + \overline{x^{\omega}_{mfpd}})\right) x^{\omega}_{mfpd}$$

is the revenue, which is obtained from the sales from the long-term contracts' sales to the independent traders or directly from the retail markets.
The term

$$\sum_{\omega,f} \pi(\omega)\delta^{t(\omega)} Pc_f\left(\sum_{\omega'\leq\omega}\sum_m q^{\omega'}_{mfp}, Rf_f\right) - \sum_{\omega,f} \pi(\omega)\delta^{t(\omega)} Pc_f\left(\sum_{\omega'<\omega}\sum_m q^{\omega'}_{mfp}, Rf_f\right)$$

is the actualized production cost. This term's explanation is as follows :
The production cost (at field f) Pc_f depends on two variables, the total quantity produced, which will be denoted q and the natural gas resources Rf_f. The Golombek production cost function we used is as follows :

$$\forall q \in [0, Rf_f),\ Pc_f(q, Rf_f) = a_f q + b_f \frac{q^2}{2} - \ Rf_f c_f \left(\frac{Rf_f - q}{Rf_f} \ln\left(\frac{Rf_f - q}{Rf_f}\right) + \frac{q}{Rf_f}\right) \quad (9.18)$$

or if written for the marginal production cost

$$\forall q \in [0, Rf_f),\ \frac{\mathrm{d}Pc_f}{\mathrm{d}q} = a_f + b_f q \ + \ c_f \ln\left(\frac{Rf_f - q}{Rf_f}\right) \quad (9.19)$$

In our model, the production cost function is dynamic. The gas volume available to be extracted is dynamically reduced at each period, taking into account the exhaustivity of the resource. If at time-step 1, the production is $q1$ and at time-step 2 $q2$, the total cost is hence :

$$cost = Pc_f(q1, RES_f) + \delta(Pc_f(q1 + q2, RES_f) - Pc_f(q1, RES_f))$$

Thus, to estimate the cost at scenario node ω, we need to calculate the production cost of the sum over all the extracted volumes until node ω and subtract the cost we have cummulated at all the strict predecessor nodes to ω.

The term

$$\sum_{\omega,f} \pi(\omega)\delta^{t(\omega)} Ip_f(ip^{\omega}_{fp})$$

is the investment cost in production at the different production fields.

The term

$$\sum_{\omega,m,a} \pi(\omega)\delta^{t(\omega)}((Tc_a + \tau^{\omega}_{m,a})fp^{\omega}_{m,p,a})$$

is the transport and congestion costs charged by the pipeline operator to producer p. The dual variable $\tau^{\omega}_{m,ar}$ is associated with the pipeline capacity constraint through the arc a. It represents the congestion price on the corresponding pipeline (see the transport operator optimization problem for a more detailed explanation).

The explanation of the constraints is straightforward :
The constraint (9.16a) bounds each field's production by its reserves.
The constraint (9.16b) bounds the seasonal quantities produced by each field's production capacity, taking explicitly into account the different dynamic investments, that decrease with time because of the production depreciation factor. To take into consideration the investment delays, we account only for the invested capacities at the strict predecessor nodes. This corresponds to a five-year investment delay (the time-step of the model).
The constraint (9.16c) states that the total production must be greater than the sales (to the long-term and spot markets). The constraints (9.16d) and (9.16e) can be rewritten as follows :

$$\forall \omega,\ f,\ p,\ |\sum_m ((-1)^m q^{\omega}_{mfp})| \leq fl_f$$

This fixes a maximum spread between the off-peak/peak production at each field. $(-1)^m$ is equal to 1 in the off-peak season and -1 in the peak season.
The constraint (9.17a) is a market-clearing condition at each node, regarding the flows from producer p depending on whether this node is a production field, an independent trader location or a demand market.
The constraint (9.16j) bounds the capacity expansion of each production node f : each year, the investment decided to increase the production capacity is less than $100 \times Lf_f$ percent the installed capacity at that year. A historical study of the capacity expansion of some production nodes

allowed us to calibrate the value of Lf_f : $Lf_f = 0.20$.
The constraint (9.17b) equates the sales of producer p for the long-term contracts to the contracted volume up_{pi}, each scenario node.
The constraint (9.17c) describes the following : For each pair of producer/independent trader (p, i), the gas quantity sold by p in the long-term contract market must be equal to the gas quantity purchased by i. Therefore, this is a supply/demand equation in the long-term contracts market. The associated dual variable η_{pi} is the corresponding contract unit selling/purchase price, because we do not assume the existence of market power in the long-term contract trade. Using this technique, it is possible to make the long-term contract prices and volumes endogenous to the description so that they become an output of the model.
The constraint (and the similar other ones) (9.16f) allows producer p to use only the fields he owns (for production, investments, sales, etc.). We recall that the incidence matrix O is such as $O_{fp} = 1$ if and only if producer p owns field f, otherwise, $O_{fp} = 0$.

Independent trader i's maximization program is given below. The corresponding decision variables are zi^{ω}_{mpi}, y^{ω}_{mid}, r^{ω}_{is}, in^{ω}_{is} and ui_{pi}.

$$
\begin{aligned}
&\text{Max} \\
&\quad \sum_{\omega,m,d} \pi(\omega)\delta^{t(\omega)} \left(p^{\omega}_{md}(y^{\omega}_{mid} + \overline{y^{\omega}_{mid}})y^{\omega}_{mid}\right) \\
&\quad - \sum_{\omega,p,m} \pi(\omega)\delta^{t(\omega)} \left(\eta_{pi} zi^{\omega}_{mpi}\right) \\
&\quad - \sum_{\omega,s} \pi(\omega)\delta^{t(\omega)} \left(Rc_s(r^{\omega}_{is})\right) \\
&\quad - \sum_{\omega,s} \pi(\omega)\delta^{t(\omega)} \left((Ic_s + Wc_s) in^{\omega}_{is}\right) \\
&\quad - \sum_{\omega,m,a} \pi(\omega)\delta^{t(\omega)} \left(Tc_a + \tau^{\omega}_{m,a}\right) fi^{\omega}_{m,i,a}
\end{aligned}
$$

such that :

$$
\begin{array}{llllr}
\forall \omega,\ m, & \displaystyle\sum_p zi^{\omega}_{mfpi} - \left(\sum_d y^{\omega}_{mid} + (-1)^m \sum_s in^{\omega}_{is}\right) & = 0 & (\psi^{\omega}_{mi}) & (9.20a) \\
\forall \omega,\ s, & in^{\omega}_{is} - r^{\omega}_{is} & \leq 0 & (\mu^{\omega}_{is}) & (9.20b) \\
\forall \omega,\ m,\ n, & \displaystyle\sum_a M6_{an} fi^{\omega}_{m,i,a}(1 - loss_a) - \sum_a M5_{an} fi^{\omega}_{m,i,a} & & & \\
 & \displaystyle - \sum_d y^{\omega}_{mfid} M3_{dn} + \sum_p zi^{\omega}_{mpi} M2_{in} & & & \\
 & \displaystyle - (-1)^m \sum_s M4_{sn} \left(\sum_i ij^{\omega}_{is}\right) & = 0 & (\alpha i^{\omega}_{m,p,n}) & (9.20c) \\
\forall \omega,\ p, & \displaystyle ui_{pi} - \sum_m zi^{\omega}_{mpi} & = 0 & (\eta i^{\omega}_{pi}) & (9.20d) \\
\forall\ p,\ i, & ui_{pi} - up_{pi} & = 0 & (\eta_{pi}) & (9.20e) \\
\forall \omega,\ m,\ p,\ i, & \displaystyle - zi^{\omega}_{mpi} + min_{pi} \sum_m zi^{\omega}_{mpi} & \leq 0 & (\upsilon^{\omega}_{mpi}) & (9.20f) \\
\forall \omega,\ m,\ s,\ d, & zi^{\omega}_{mpi},\ y^{\omega}_{mid},\ r^{\omega}_{is},\ in^{\omega}_{is},\ ui_{pi} & \geq 0 & &
\end{array}
$$

The term

$$\sum_{\omega,m,d} \pi(\omega)\delta^{t(\omega)} \left(p^{\omega}_{md}(y^{\omega}_{mid} + \overline{y^{\omega}_{mid}})y^{\omega}_{mid}\right) - \sum_{\omega,p,m} \pi(\omega)\delta^{t(\omega)} \left(\eta_{pi} zi^{\omega}_{mpi}\right)$$

is the net profit.
The term

$$\sum_{\omega,s} \pi(\omega)\delta^{t(\omega)} \left(Rc_s(r^{\omega}_{is})\right)$$

is the storage capacity reservation cost.
The term

$$\sum_{\omega,s} \pi(\omega)\delta^{t(\omega)} \left((Ic_s + Wc_s)in_{is}^{\omega}\right)$$

are the storage/withdrawal costs. [7]
The term

$$\sum_{\omega,m,a} \pi(\omega)\delta^{t(\omega)} \left(Tc_a + \tau_{m,a}^{\omega}\right) fi_{m,i,a}^{\omega}$$

is the transport and congestion costs charged by the pipeline operator to the independent trader i.

As for the feasibility set, it is also easy to specify :

The constraint (9.20a) is a gas quantity balance for each trader. The term $(-1)^m$ is equal to 1 in the off-peak season and -1 otherwise. An implicit assumption we use in the description is that all the storage sites must be "empty" (regardless of the working gas quantitities) at the end of each year.
The equation (9.20b) implies that each independent trader has to pay for a storage reservation quantity, each year and at each storage site s, to be able to store his gas.
The constraint (9.20d) forces each trader to purchase the same quantity, in long-term contracts, from each producer and scenario node.
The constraint (9.20e) is similar to the constraint (9.17c) of the producers' optimization program. For each pair of producer/independent trader (p, i), the gas quantity sold by p in the long-term contract market must be equal to the gas quantity purchased by i. Therefore, this is a supply/demand equation in the long-term contracts market. The associated dual variable η_{pi} is the corresponding contract unit selling/purchase price, because we do not assume the existence of market power in the long-term contract trade. Using this technique, it is possible to make the long-term contract prices and volumes endogenous to the description so that they become an output of the model.
The constraint (9.20f) fixes a minimum percentage of the contracted volume, per time unit, min_{pi} that has to be exchanged between p and i each season of each scenario node. Obviously, this constraint is expected to be more saturated in the summer when there is little need for the traders to have an important amount of gas supply.

On the transportation side of our model, we will assume that the producers pay the transport costs to bring natural gas from the production fields to the independent traders' locations and the end-use markets. The traders support the transport costs to store/withdraw gas or bring it to the end-users for their sales. All the distribution costs are implicitly included in the transportation costs we use.

7. There are no storage losses in the model. They can easily be taken into account by increasing the tansportation losses of the arcs that start at the storage nodes.

The pipeline operator optimization (cost minimization) program is given below. The corresponding decision variables are $fp^{\omega}_{m,p,a}$, $fi^{\omega}_{m,i,a}$ and ik^{ω}_{a}.

$$\begin{aligned}
&\text{Min} \\
&\quad \sum_{\omega,m,a} \pi(\omega)\delta^{t(\omega)} \left(Tc_a + \tau^{\omega}_{m,a}\right) \sum_{p} fp^{\omega}_{m,p,a} \\
&\quad + \sum_{\omega,m,a} \pi(\omega)\delta^{t(\omega)} \left(Tc_a + \tau^{\omega}_{m,a}\right) \sum_{i} fi^{\omega}_{m,i,a} \\
&\quad + \sum_{\omega,a} \pi(\omega)\delta^{t(\omega)} Ik_a(ik^{\omega}_a)
\end{aligned}$$

such that :

$$\forall \omega,\ m,\ a, \quad \sum_{p} fp^{\omega}_{m,p,a} + \sum_{i} fi^{\omega}_{m,i,a} - \left(Tk_a + \sum_{\omega'<\omega} ik^{\omega'}_a\right) \le 0 \qquad (\tau^{\omega}_{m,a}) \tag{9.21a}$$

$$\forall \omega,\ a, \quad ik^{\omega}_a - La_a\left(Tk_a + La_a \sum_{\omega'<\omega} ik^{\omega'}_a\right) \le 0 \qquad (\iota a^{\omega}_a) \tag{9.21b}$$

$$\begin{aligned}
\forall \omega,\ m,\ p,\ n, \quad & \sum_{a} M6_{an} fp^{\omega}_{m,p,a}(1 - loss_a) - \sum_{a} M5_{an} fp^{\omega}_{m,p,a} \\
& + \sum_{f} q^{\omega}_{mpf} M1_{fn} - \sum_{d}\sum_{f} x^{\omega}_{mfpd} M3_{dn} \\
& - \sum_{i}\sum_{f} zp^{\omega}_{mfpi} M2_{in} = 0 \qquad (\alpha p^{\omega}_{m,p,n})
\end{aligned} \tag{9.21c}$$

$$\begin{aligned}
\forall \omega,\ m,\ i,\ n, \quad & \sum_{a} M6_{an} fi^{\omega}_{m,i,a}(1 - loss_a) - \sum_{a} M5_{an} fi^{\omega}_{m,i,a} \\
& - \sum_{d} y^{\omega}_{mfid} M3_{dn} + \sum_{p} zi^{\omega}_{mpi} M2_{in} \\
& - (-1)^m \sum_{s} M4_{sn}\left(\sum_{i} ij^{\omega}_{is}\right) = 0 \qquad (\alpha i^{\omega}_{m,p,n})
\end{aligned} \tag{9.21d}$$

$$\forall \omega,\ m,\ a,\ p,\ i, \quad fp^{\omega}_{m,p,a},\ fi^{\omega}_{m,i,a},\ ik^{\omega}_a \ge 0$$

The objective function contains both the transport/congestion and invesment costs. The congestion cost through arc a, $\tau^{\omega}_{m,a}$, is the dual variable associated with the constraint (9.21a). This constraint concerns the physical seasonal capacity of arc a, including the possible node-dependent investments.
The constraint (9.21b) bounds the capacity expansion of each arc a : each year, the investment decided to increase the transport capacity is less than $100 \times La_a$ percent the installed capacity at that year. In S-GaMMES, we used the value $La_a = 0.2$.

The other constraints are market-clearing conditions at each node, depending on whether this node is a production field, an independent trader location, a demand market or a storage site, and depending on whether the transportation costs are supported by the producers or the independent traders.
We consider both pipeline and LNG routes for transport. The liquefaction and regasification costs are included in the transportation costs on the LNG arcs. We assume, in the representation that the physical losses occur at the end nodes of the arcs.

The storage operator optimization (cost minimization) program is given below. The corresponding decision variable is is_s^ω.

$$\text{Min} \quad \sum_{\omega,s} \pi(\omega)\delta^{t(\omega)} Is_s(is_s^\omega) + \sum_{i,\omega,s} \pi(\omega)\delta^{t(\omega)} \left((Ic_s + Wc_s) in_{is}^\omega + Rc_s r_{is}^\omega\right)$$

such that :

$$\forall \omega,\ s, \quad \sum_i r_{is}^\omega - Ks_s - \sum_{\omega' < \omega} is_s^{\omega'} \leq 0 \quad (\beta s_s^\omega) \tag{9.22a}$$

$$\forall \omega,\ s, \quad is_s^\omega - Ls_s Ks_s - Ls_s \sum_{\omega' < \omega} is_s^{\omega'} \leq 0 \quad (\iota s_s^\omega) \tag{9.22b}$$

$$\forall \omega,\ s, \quad is_s^\omega \geq 0$$

The storage operator controls the different investments that dynamically increase the storage capacity of each storage node. The incentive this player has to invest is due to the constraint he must satisfy : the capacity available at each storage site must be sufficient to meet the volumes the independent traders have to store each year in the off-peak season. Capacity expansion is bounded and we used the value $Ls_s = 0.2$.

If we take a closer look at the optimization program of a producer, we will notice that his feasibility set depends on the decision variables of the independent traders. Also, the feasibility set of any independent trader's optimization program depends on the producers' decision variables. The situation is similar for the pipeline and storage operators. This particularity makes our formulation (the KKT conditions) a **Generalized Nash-Cournot problem**. Similarly, the Generalized Nash-Cournot problem can also be formulated as a Quasi-Variational Inequality problem (QVI). In order to solve the problem, we look for the particular solution that makes the problem a VI formulation (29).

When the KKT conditions are written, we obtain the Mixed Complementarity Problem given in Appendix 2.

9.2.6 Theoretical results

We refer to Appendix 2 for the MCP formulation of S-GaMMES. This section uses this appendix's equations numbers.
One of the S-GaMMES model's key features is that it cpatures the markets' long-term aspects in an endogenous way, for both long-term contract prices and volumes. In the deterministic version of GaMMES, it can be proved that long-term contract prices, or LTC prices, are smaller than the spot market prices.[8] Indeed, since long-term contracts are the only means for the independent traders to obtain gas, LTC prices are to be considered as supply costs for them. Besides, they make a profit by selling natural gas directly to the consumers, in the spot markets. Therefore, if the traders have any incentive to sell gas to the consumers, their revenue must be greater than their costs and consequently, spot prices should be, on average, greater than LTC prices.

These conclusions still hold for the stochastic version of GaMMES. They are explained in the following theorems.

First we prove that our representation of the long-term contracts leads to nonnegative LTC prices. This property is not straightforward because these prices are computed as free dual variables associated with equality constraints.

Theorem 13. *If producer p and trader i contract on the long-term, then the long-term contract price η_{pi} is such as $\eta_{pi} \geq 0$*

Démonstration. We assume that producer p contracts on the long-term with trader i. This means that the LTC volume up_{pi} is such that $up_{pi} > 0$. Let us denote by d the market where i is located, i.e., d is the only market such as $B_{id} = 1$. Hence, we can write that

$$\forall n \in N,\ M2_{in} = M3_{dn}$$

.

We already know from equation (10.8b) that :

$$\forall \omega,\ up_{pi} - \sum_{f,m} zp^{\omega}_{mfpi} = 0$$

Hence we can deduce that :

$$\forall \omega,\ \exists f(\omega) \in F \text{ and } m(\omega) \in M \text{ such as } zp^{\omega}_{m(\omega)f(\omega)pi} > 0$$

where we denote by $f(\omega)$ the particular field that producer p may use, at scenario node ω to respect the LTC volume he has to sell to i in season $m(\omega)$.

8. Though this situation is less realistic nowadays, given the current high, long-term contract gas prices that are indexed on the oil price.

Because of relation (10.10e), we can deduce that

$$\forall \omega,\ \exists f(\omega) \in F \text{ such as } \forall m \in M,\ zp^{\omega}_{mf(\omega)pi} > 0$$

Using the complementarity condition of equation (10.6a), we can deduce that : $\forall \omega,\ \exists f(\omega) \in F$ such as $\forall m \in M$,

$$\pi(\omega)\delta^{t(\omega)}\eta_{pi} - \gamma^{\omega}_{mf(\omega)p} - \epsilon 2^{\omega}_{mf(\omega)pi} - \eta p^{\omega}_{pi} - \sum_{n} M2_{in}\alpha p^{\omega}_{m,p,n} = 0 \tag{9.23}$$

Since $zp^{\omega}_{mf(\omega)pi} > 0$, producer p owns the particular field $f(\omega)$ and constraint (9.16g) is not saturated. Therefore, $\epsilon 2^{\omega}_{mf(\omega)pi} = 0$ and

$$\pi(\omega)\delta^{t(\omega)}\eta_{pi} - \gamma^{\omega}_{mf(\omega)p} - \eta p^{\omega}_{pi} - \sum_{n} M2_{in}\alpha p^{\omega}_{m,p,n} = 0 \tag{9.24}$$

To simplify the notation we will denote the term

$$\left(p^{\omega}_{md}(x^{\omega}_{mf(\omega)pd} + \overline{x^{\omega}_{mf(\omega)pd}}) + \frac{\partial p^{\omega}_{md}}{\partial x^{\omega}_{mfpd}}(x^{\omega}_{mf(\omega)pd} + \overline{x^{\omega}_{mf(\omega)pd}}) x^{\omega}_{mf(\omega)pd} \right)$$

by

$$\left(p^{\omega}_{md} + \frac{\partial p^{\omega}_{md}}{\partial x^{\omega}_{mfpd}} x^{\omega}_{mf(\omega)pd} \right)$$

Using relation (10.6b) we have :

$$\pi(\omega)\delta^{t(\omega)} \left(p^{\omega}_{md} + \frac{\partial p^{\omega}_{md}}{\partial x^{\omega}_{mfpd}} x^{\omega}_{mf(\omega)pd} \right) - \gamma^{\omega}_{mf(\omega)p} - \epsilon 1^{\omega}_{mf(\omega)pd} - \sum_{n} M3_{dn}\alpha p^{\omega}_{m,p,n} \leq 0 \tag{9.25}$$

Since producer p owns the particular field $f(\omega)$, constraint ($9.16f$) is not saturated. Therefore, $\epsilon 1^{\omega}_{mf(\omega)pd} = 0$ and

$$\pi(\omega)\delta^{t(\omega)} \left(p^{\omega}_{md} + \frac{\partial p^{\omega}_{md}}{\partial x^{\omega}_{mfpd}} x^{\omega}_{mf(\omega)pd} \right) - \gamma^{\omega}_{mf(\omega)p} - \sum_{n} M3_{dn}\alpha p^{\omega}_{m,p,n} \leq 0 \tag{9.26}$$

Combining equations (9.24) and (9.26) and using the fact that $\forall n \in N,\ M2_{in} = M3_{dn}$ [9], we obtain :

$$\pi(\omega)\delta^{t(\omega)} \left(p^{\omega}_{md} + \frac{\partial p^{\omega}_{md}}{\partial x^{\omega}_{mfpd}} x^{\omega}_{mf(\omega)pd} - \eta_{pi} \right) + \eta p^{\omega}_{pi} \leq 0 \tag{9.27}$$

9. We recall that trader i is located at market d.

We know by equation (10.6e) and the fact that $up_{pi} > 0$, that :

$$\eta_{pi} = \sum_{\omega} \eta p^{\omega}_{pi} \tag{9.28}$$

Therefore, since equation (9.27) is satisfied for all $\omega \in \Omega$, summing it over ω and using relation (9.28) gives :

$$\eta_{pi}\left(\sum_{\omega}\pi(\omega)\delta^{t(\omega)} - 1\right) + \sum_{\omega}\pi(\omega)\delta^{t(\omega)}\left(p^{\omega}_{md} + \frac{\partial p^{\omega}_{md}}{\partial x^{\omega}_{mfpd}} x^{\omega}_{mf(\omega)pd}\right) \leq 0 \tag{9.29}$$

Since $\left(\sum_{\omega}\pi(\omega)\delta^{t(\omega)} - 1\right) < 0$, we have :

$$\eta_{pi} \geq \frac{-\sum_{\omega}\pi(\omega)\delta^{t(\omega)}\left(p^{\omega}_{md} + \frac{\partial p^{\omega}_{md}}{\partial x^{\omega}_{mfpd}} x^{\omega}_{mf(\omega)pd}\right)}{\left(\sum_{\omega}\pi(\omega)\delta^{t(\omega)} - 1\right)} \geq 0 \tag{9.30}$$

□

The next theorem allows us to compare LTC and spot prices. Before, let us define the LTC constraints cost supported by an independent trader i. From the point of view of an independent trader, long-term contracts constrains him to purchase gas from the producers (he contracts with) each year, with a minimum proportional amount each season. In S-GaMMES, this is taken care of by constraints (9.20d) and (9.20f). Using the KKT conditions and the Lagrangian formulation, it is possible to define a cost inherent to the respect of these LTC constraints. Obviously, this cost that depends on the scenario node ω, the season m and the producer p involved in the contract, is function of the dual variables associated with constraints (9.20d) and (9.20f) : ηi^{ω}_{pi} and υ^{ω}_{mpi}.

Definition 21. *The LTC cost between trader i and producer p is defined at each scenario node ω and each season m by :*

$$LTCcost^{\omega}_{mpi} = \eta i^{\omega}_{pi} - (1 - min_{pi})\upsilon^{\omega}_{mpi}$$

In the LTC cost definition, the term ηi^{ω}_{pi} takes care of the annual LTC constraint (i.e., the trader must purchase the same volume from the producer, at each scenario node) and the term $-(1 - min_{pi})\upsilon^{\omega}_{mpi}$ captures the seasonal LTC constraint (i.e., the trader must buy at least $100 \times min_{pi}$ percent of the annual LTC volume, at each season). Since the variable ηi^{ω}_{pi} is free (it is associated with an equality constraint), the LTC cost can be positive or negative.

In the following theorem and proof, we consider a particular pair of producer p and independent trader i who contract on the long-term. We will denote by d the consumption market where i is located.

Theorem 14. *If producer p and trader i contract on the long-term and $LTCcost^{\omega}_{mpi}$ is nonnegative then the spot price at market d is greater than the LTC price as long as trader i sells gas to market d :*

$$\forall \omega,\ m,\ y^{\omega}_{mid} > 0 \Longrightarrow p^{\omega}_{md} \geq \eta_{pi}$$

Démonstration. Producer p and trader i are assumed to contract on the long-term. Hence, $ui_{pi} > 0$. We already know, using equation (10.10c), that

$$\forall\omega, ui_{pi} = \sum_m zi^\omega_{mpi}$$

Thus, $\forall\omega$, $\exists m(\omega) \in M$ such as $zi^\omega_{mpi} > 0$. Because of equation (10.10e), the previous inequation holds for all the seasons :

$$\forall\omega, \forall m,\ zi^\omega_{mpi} > 0$$

Using equation (10.9a), it is possible to write : $\forall\omega, m$

$$-\pi(\omega)\delta^{t(\omega)}\eta_{pi} - \eta i^\omega_{pi} + \psi^\omega_{mi} + \sum_n M2_{in}\alpha i^\omega_{min} + (1 - min_{pi})\upsilon^\omega_{mpi} = 0 \tag{9.31}$$

If we assume that trader i sells gas to market d, then using equation (10.9b) and by denoting (for the sake of simplicity)

$$p^\omega_{md} + \frac{\partial p^\omega_{md}}{\partial y^\omega_{mid}} y^\omega_{mid}$$

the term

$$p^\omega_{md}(y^\omega_{mid} + \overline{y^\omega_{mid}}) + \frac{\partial p^\omega_{md}}{\partial y^\omega_{mid}}(y^\omega_{mid} + \overline{y^\omega_{mid}})y^\omega_{mid}$$

we find that :

$$\pi(\omega)\delta^{t(\omega)}\left(p^\omega_{md} + \frac{\partial p^\omega_{md}}{\partial y^\omega_{mid}} y^\omega_{mid}\right) - \psi^\omega_{mi} - \sum_n M3_{dn}\alpha i^\omega_{m,i,n} = 0 \tag{9.32}$$

Since trader i is located at market d, we can write :

$$\forall n \in N,\ M3_{dn} = M2_{in}$$

Combining equations (9.31) and (9.32), we find that :

$$\pi(\omega)\delta^{t(\omega)}\left(p^\omega_{md} + \frac{\partial p^\omega_{md}}{\partial y^\omega_{mid}} y^\omega_{mid} - \eta_{pi}\right) - \eta i^\omega_{pi} + (1 - min_{pi})\upsilon^\omega_{mpi} = 0 \tag{9.33}$$

or

$$\pi(\omega)\delta^{t(\omega)}\left(p^\omega_{md} + \frac{\partial p^\omega_{md}}{\partial y^\omega_{mid}} y^\omega_{mid} - \eta_{pi}\right) = \eta i^\omega_{pi} - (1 - min_{pi})\upsilon^\omega_{mpi} = LTCcost^\omega_{mpi} \tag{9.34}$$

In particular, if the LTC cost is nonnegative, we find that :

$$\pi(\omega)\delta^{t(\omega)}\left(p^\omega_{md} + \frac{\partial p^\omega_{md}}{\partial y^\omega_{mid}} y^\omega_{mid} - \eta_{pi}\right) \geq 0 \tag{9.35}$$

or

$$\pi(\omega)\delta^{t(\omega)}\left(p^{\omega}_{md}-\eta_{pi}\right)\geq-\pi(\omega)\delta^{t(\omega)}\left(\frac{\partial p^{\omega}_{md}}{\partial y^{\omega}_{mid}}y^{\omega}_{mid}\right)\tag{9.36}$$

Since the inverse demand function is decreasing, we can deduce that :

$$\frac{\partial p^{\omega}_{md}}{\partial y^{\omega}_{mid}}\leq 0\tag{9.37}$$

Hence,

$$\pi(\omega)\delta^{t(\omega)}\left(p^{\omega}_{md}-\eta_{pi}\right)\geq-\pi(\omega)\delta^{t(\omega)}\left(\frac{\partial p^{\omega}_{md}}{\partial y^{\omega}_{mid}}y^{\omega}_{mid}\right)\geq 0\tag{9.38}$$

and

$$\eta_{pi}\leq p^{\omega}_{md}\tag{9.39}$$

□

From the point of view of an independent trader i, it may be interesting to study the variation of the LTC price among the different producers. Intuitively, since LTC prices are modeled as supply marginal costs for the trader i and since we assume that no market power is exerted by the producers on the LTC trade, we can deduce that all the producers will contract at the same price with i. The LTC price will therefore be correlated to the spot price because the latter is related to the profit earned by i, whereas the former is related to his supply cost. The following theorem details the relation between the different LTC prices.

Theorem 15. *If trader i contracts with producers p and p' on the long-term, then the LTC prices are equal :*

$$\eta_{pi}=\eta_{p'i}$$

Démonstration. We assume that trader i has LTCs with producers p and p', which means that $ui_{pi}>0$ and $ui_{p'i}>0$. To simplify the proof, we will assume that constraint (9.20f) is not binding. Therefore, the corresponding dual variables are such as :

$$\forall\omega,m,\ \upsilon^{\omega}_{mpi}=0$$

and

$$\forall\omega,m,\ \upsilon^{\omega}_{mp'i}=0$$

Let us demonstrate that $\eta_{pi}=\eta_{p'i}$.
Since $ui_{pi}>0$ and $ui_{p'i}>0$, we can use equation (10.9e) to deduce that

$$\sum_{\omega}\eta i^{\omega}_{pi}+\eta_{pi}=0\tag{9.40}$$

and

$$\sum_{\omega} \eta i^{\omega}_{p'i} + \eta_{p'i} = 0 \tag{9.41}$$

Since i contracted on the long-term with p and p', we can deduce, like in the previous proofs that :

$$\forall \omega,\ m,\ zi^{\omega}_{mpi} > 0 \text{ and } zi^{\omega}_{mp'i} > 0$$

Hence, using equation (10.9a), it is possible to write :

$$\forall \omega,\ m,\ -\pi(\omega)\delta^{t(\omega)}\eta_{pi} - \eta i^{\omega}_{pi} + \psi^{\omega}_{mi} + \sum_{n} M2_{in}\alpha i^{\omega}_{min} = 0 \tag{9.42}$$

and

$$\forall \omega,\ m,\ -\pi(\omega)\delta^{t(\omega)}\eta_{p'i} - \eta i^{\omega}_{p'i} + \psi^{\omega}_{mi} + \sum_{n} M2_{in}\alpha i^{\omega}_{min} = 0 \tag{9.43}$$

Summing equations (9.42) and (9.43) over ω and m and using relations (9.40) and (9.41), we can deduce that :

$$\left((\sum_{\omega} \pi(\omega)\delta^{t(\omega)}) - 1 \right) \eta_{pi} = \left((\sum_{\omega} \pi(\omega)\delta^{t(\omega)}) - 1 \right) \eta_{p'i} \tag{9.44}$$

or

$$\eta_{pi} = \eta_{p'i}$$

□

The following theorem concerns the stored volumes decided by the independent traders and the related reservation capacity.

Theorem 16. *The stored and reserved capacities for storage are such as :*

$$\forall \omega,\ \forall i,\ s,\ r^{\omega}_{is} > 0 \Rightarrow r^{\omega}_{is} = in^{\omega}_{is}$$

The previous theorem allows us to assert that at each scenario node, each storage site, the capacity reserved by an independent trader is always equal to the volume he actually decides to store. This result is very intuitive because the independent traders do not take care of storage investments. Hence, they are not affected, in their storage decision variables by the randomness of the demand. Theorem 16's proof is straightforward :

Démonstration. Let us assume that a trader i decides to make a storage reservation at storage site s : $r^{\omega}_{is} > 0$.

If he does not use completely the reserved capacity $r_{is}^{\omega} > in_{is}^{\omega}$, then using equation (10.10b), we deduce that :

$$\mu_{is}^{\omega} = 0 \tag{9.45}$$

If we consider relation (10.9c), we find that :

$$-\pi(\omega)\delta^{t(\omega)}Rc_s = \beta s_s^{\omega} \tag{9.46}$$

Since $\beta s_s^{\omega} \geq 0$ (using equation (10.13b)), we would have :

$$-\pi(\omega)\delta^{t(\omega)}Rc_s \geq 0 \tag{9.47}$$

which is absurd because $Rc_s > 0$.

□

§ 9.3 Conclusion

This chapter presents a Stochastic Generalized Nash-Cournot model in order to describe the natural gas markets' evolution, taking into account the fluctuations of the oil price. The demand representation takes into consideration the possible energy substitution that can be made between oil, coal, and natural gas. The exhaustibility of the resource is taken care of by the use of specific production cost functions (Golombek production cost functions).

The long-term contract prices and volumes are endogenously taken into account with the use of dual variables. This aspect makes our formulation a Generalized Nash-Cournot model, or similarly a QVI formulation. In order to solve it, we derived the VI formulation that usually presents a unique solution.

The demand is made random by considering the oil price's fluctuation with time. The model uses a scenario tree representation to capture the oil price fluctuation. The oil price's dynamic evolution is modeled as a Markov chain. The transition probabilities have been calibrated using an econometric study of the Brent price's historical evolution. The scenario tree representation allows us to not take care of non-anticipativity conditions. The consequence is that the model's formulation is very similar to the deterministic version GaMMES, with a bigger number of variables.

We have presented, proved and discussed a set of results and theorems related to our formulation. Most of these concern a comparison between the long-term contracts and spot markets gas prices. They allow one to understand the economic correlation between LTC and spot prices. Besides, when considering an independent trader, a comparison between all the LTC prices among all his possible supply sources is provided in order to understand the competition between the producers, in the upstream market.

S-GaMMES has been applied to model the northwestern European gas trade. The results are given in the following chapter.

- Chapitre 10 -

Application of the S-GaMMES model to the northwestern European gas trade.

§ 10.1 Introduction

The previous chapter presented the S-GaMMES model, a Stochastic Generalized-Nash Cournot model to describe the natural gas market trade. The key features of the model are the following : energy substitution between coal, oil and natural gas is taken into account, long-term contracts are endogenously described and the oil price's fluctuation is captured by modeling the oil price as a random variable. Thus, S-GaMMES is particularly well suited to describe the European gas trade, which is still mainly dominated by long-term contracts in the upstream and where the LTCs are oil price-indexed. This chapter is an application of our model to the northwestern European natural gas trade where the calibration process and the results are discussed. The results contain scenario forecasts of the consumption, prices, production, and gas dependence in Europe. LTC prices and volumes are provided and analyzed. We defined and calculated the value, the loss, and the gain of the stochastic solution in order to quantify the usefulness of taking into account stochasticity in the model.

In this chapter, we will use chapter 9's notation.

§ 10.2 The European natural gas markets model

This section puts the model at work and presents our numerical results.

10.2.1 The representation

This section presents an application of the model to the northwestern European natural gas market. The representation is very similar to the one presented in the deterministic version of GaMMES (1). The following array summarizes the main actors, production, and storage sites and the seasons studied :

Producers		**Fields**		**Consuming markets**		**Independent traders**		**Storage sites**
Russia		Russia$_f$		France		France$_{tr}$		France$_{st}$
Algeria		Algeria$_f$		Germany		Germany$_{tr}$		Germany$_{st}$
Norway		Norway$_f$		The Netherlands		The Netherlands$_{tr}$		The Netherlands$_{st}$
The Nethherlands$_f$		NL$_f$		UK		UK$_{tr}$		UK$_{st}$
UK		UK$_f$		Belgium		Belgium$_{tr}$		Belgium$_{st}$

Seasons		**Time**		**Time step**		**# Scenario nodes**		**# Branches at each time step**
off-peak		$2000 - 2035$		$5\ years$		31		2 or 1
peak								

We aggregate all the production fields of each producer into one production node. We assume that each consuming market is associated with one independent local trader (indexed by tr). As an example, France$_{tr}$ would be GDF-SUEZ and Germany$_{tr}$ would be E-On Ruhrgas. All the storage sites are also aggregated so that there is one storage node per consuming country. As for the transport, the different gas routes given in Figure 10.1 were considered.

The local production in the different consuming countries and the imports from non-represented producers, which are small, are exogenously taken into consideration.

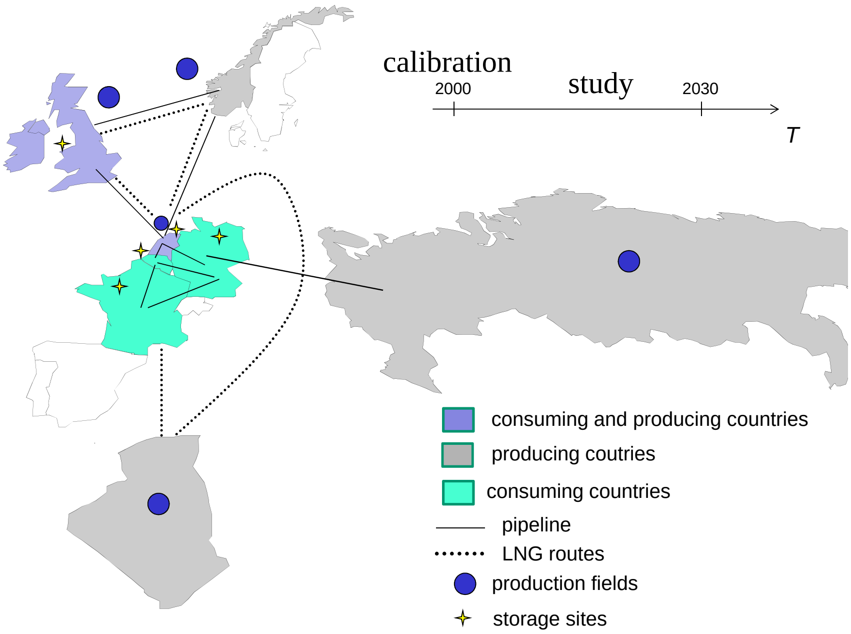

FIGURE 10.1 – *The northwestern European natural gas routes, production, and storage sites.*

Stochasticity is taken care of by the scenario tree presented in Section 9.2.4. The demand representation is given in Section 9.2.3. The model has been solved, in its extensive form, using the solver PATH (20) from GAMS. In order to have an algorithm convergence in a reasonable time, we used a five-year time-step resolution. We chose five years because it is the typical length of time needed to construct investments in production, infrastructure or storage. Also, the inverse demand function has been linearized.

10.2.2 The calibration

The calibration process has been carried out in order to best meet :
– the global natural gas consumption,

– the industrial sector gas price and
– the volumes produced by each gas producer,

at scenario node 0. We recall that this node corresponds to the time period between 2000 and 2004 (the first time period). Therefore, there is no randomness associated with this node.

The data for the market prices, consumed volumes, and imports is the publicly available set from IEA (36). We define a new variable $exch^t_{mpd}$ that represents the exported volume from producer p to market d. More precisely :

$$\forall t,\ m,\ p,\ d,\ exch^t_{mpd} = \sum_i B_{id}\ zp^t_{mpi} + x^t_{mpd}$$

The matrix B is such that $B_{id} = 1$ if the independent trader i is located in market d (e.g., GDF-SUEZ in France, E-On Ruhrgas in Germany) and $B_{id} = 0$ otherwise. Hence, one can notice that the exchanged volumes include both the spot and long-term contract trades.

The calibration elements we used are the inverse demand function parameters α^t_{md}, γ^t_{md}, pc^t_{md} and β^t_{md}. The idea is that the system dynamics model is run in order to calculate all the inverse demand function parameters, for all the markets and at each year and season of our study. The calibration technique slightly adjusts these values to make the model correctly describe the historical data (between 2000 and 2004).

In order to calibrate the produced volumes properly, we introduced security of supply parameters that link each pair of producer/consuming countries (p, d). A security of supply measure forces each country not to import from any producer more than a fixed percentage (denoted by SSP) of the overall imports.This property can be rewritten as follows :

$$\forall t,\ m,\ p,\ d,\ exch^t_{mpd} \leq SSP_{pd} \sum_p exch^t_{mpd}$$

The security of supply parameters are set exogenously.
The calibration tolerates a maximum error of 5% for the prices and consumed quantities and 10% for the imported/exported volumes. The tolerated error is higher for the exchanged volumes because they depend on the exports decided by the producers for all the targeted consumers, even those that are not in the scope of the model. As an example, the exported volumes from Russia to CIS (CEI) countries are exogenous to our model.

10.2.3 Results

We refer to (1) for a more detailed analysis of the results. This section focuses mainly on the theoretical results, in order to highlight the role of randomness in the model.

In order to estimate the demand function parameters, our model requests exogenous inputs : the markets' global energy demand and coal price evolution (scenario provided by the European

Commision (18)). As explained in Section 9.2.4, we assume that the coal price remains constant and that the oil price follows a Markov chain regime. The annual gross consumption growth per year is given in the following chart (starting from 2000).

Annual growth	Total gross consumption (in % per year)
France	0.46
Germany	0.06
United Kingdom	0.02
Belgium	0.06
The Netherlands	0.11

Figure 10.2 gives the evolution of the consumption and prices in the demand markets between 2000 and 2035, for the different scenarios given in Figure 9.4. We consider the different leaves of the tree, indexed by their scenario node number (between 24 and 31) and draw the evolution of the consumption, with time, following the path in the tree that leads to the corresponding leaf. It is important to remember that all the possible scenarios are solved simultaneously while respecting the non-anticipativity conditions. To analyze the actors' and markets' behavior, we also run S-GaMMES with two deterministic evolutions of the demand :

- A "High" demand case, where the oil price follows the path that leads to node 24. This corresponds to a deterministic increase of the oil price between 2000 and 2035 (a logarithmic change of $\lambda_1 \geq 0$).
- A "Low" demand case, where the oil price follows the path that leads to node 31. This corresponds to a deterministic decrease of the oil price between 2000 and 2035 (a logarithmic change of $\lambda_2 \leq 0$).

In the "High" and "Low" cases, it is assumed that the players know exactly, *a priori*, whether the oil prices are going to follow path 24 (constant and continuous increase of the oil price) or path 31 (constant and continuous decrease of the oil price).

One can notice that the evolution of the consumption and prices in all the scenarios is bounded by the deterministic "High" and "Low" scenarios. This result is intuitive because in both cases, it is assumed that the players have perfect foresight of the demand evolution. Besides, scenarios 24 and 31's results bind the other scenarios' consumption and prices evolution, because they correspond to a continuous a constant increase (scenario 24) or decrease (scenario 31) of the oil price.

The following table gives the consumption annual percentage growth (APG) mean value, between 2005 and 2035, in the European countries studied. The ratio betwen the standard deviation and the mean value of the consumption in 2035 (last period) is also given. The latter quantifies the impact of randomness on the spread of consumption with time. More particularly, we will define the spread by the following :

$$spread = \frac{\text{Standard deviation in 2035}}{\text{Mean value in 2035}} \tag{10.1}$$

The spread, a measure of the standard error, can be defined for all the parameters or market outcome that depend on the scenarios (consumption, prices, production, etc.). Intuitively, if the

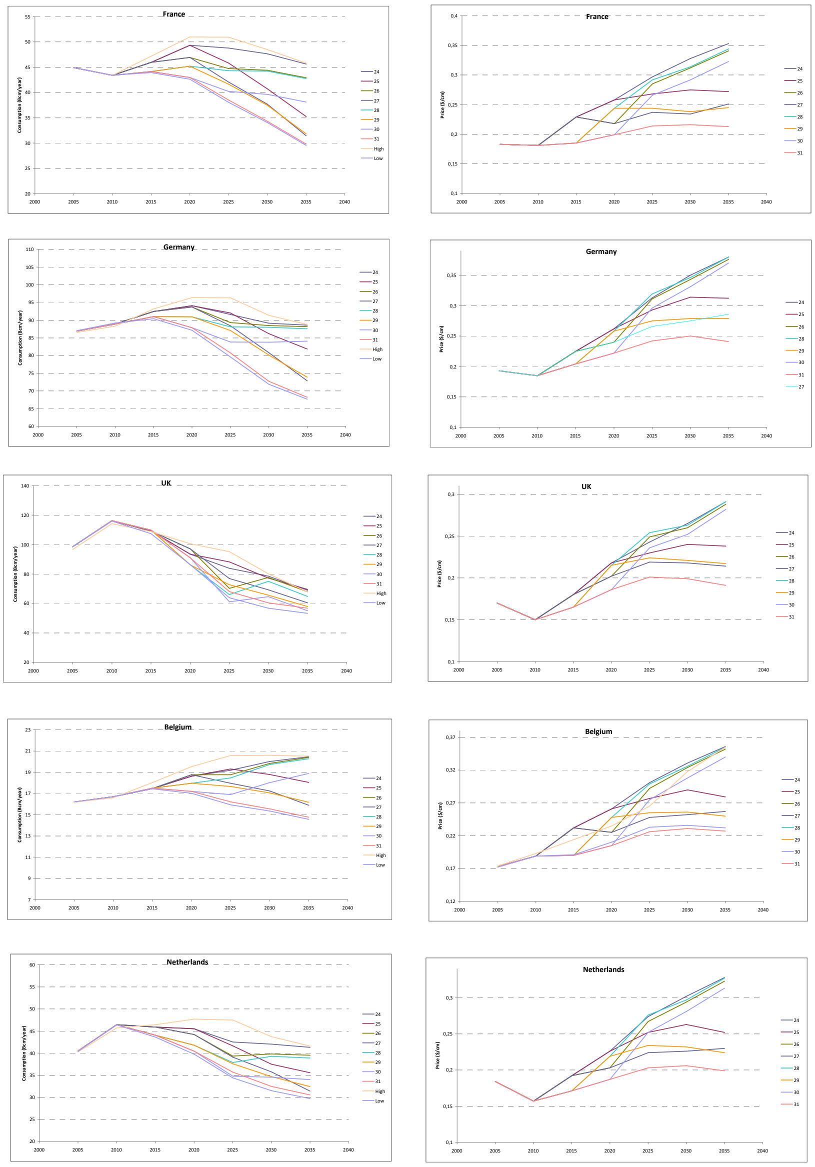

FIGURE 10.2 – *Consumption and prices in the different scenarios.*

spread is high, this means that randomness may play an important role in the decisions made by the actors that influence the market outcome. This situation indicates that a stochastic model is more accurate to represent the market behavior as compared to a deterministic one. This notion will be detailed later while introducing and calculating the value of the stochastic solution.

Consumption	France	Germany	UK	Belgium	The Netherlands
(APG) mean value (%/year)	-1.15	-0.62	-1.76	-0.09	-0.81
Spread (in %)	11.1	8.5	4.9	10.4	6.5

One can notice that the spread has the highest value in France, which suggests that the fluctuations of the oil price influence the French gas consumption a great deal. On the contrary, the spread is relatively low in the United Kingdom. This country also has the highest decrease of consumption (in all scenarios). Indeed, the decrease of consumption is mainly due to the fact that the United Kingdom has low gas reserves (around 900 Bcm in 2000, (10).) and will have to rely on foreign (especially Russian and Algerian) supplies in the coming decades. Therefore, the evolution of the consumption in the UK is greatly dependent on the supply and less on the oil price fluctuations.
One can notice that the natural gas consumption is expected to decrease between 2000 and 2035, in most of the scenarios, even if the demand increases. This is mainly due to the fact that the initial natural gas reserves in Europe are relatively low compared to the demand. Since we do not represent explorations activities (for Shale gas for instance), the foreign dependence, especially toward Russia and Algeria, will grow with time, which will force the prices up mainly because of two reasons : first, the growing market power exerted by Russia and Algeria and second, the relatively high transportation costs.

The following table gives the price annual percentage growth (APG) mean value and the spread, between 2005 and 2035, in the European countries studied.

Price	France	Germany	UK	Belgium	The Netherlands
(APG) mean value (%/year)	0.89	1.14	0.73	1.25	0.65
Spread (in %)	17.2	17.1	15.5	16.3	18.4

The spread is higher for the prices than for the consumption. This is intuitive because the gas prices are more correlated to the oil price as compared to the consumption. Like for the consumption, France has the highest price spread in 2035.

Figure 10.3 shows the evolution of the production (dedicated to the consuming countries we studied) over time, in the different scenarios.

The Russian and Algerian shares in the European consumption is expected to grow, in all the possible scenarios. The Deutch and The UK production is expected to decrease with time, with a very small spread. This is mainly due to the limited reserves of gas initially present in these countries that reduces the correlation between the supply of these countries and the demand which is linked to the oil price that is random. The Norwegian production varies a lot with the scenario

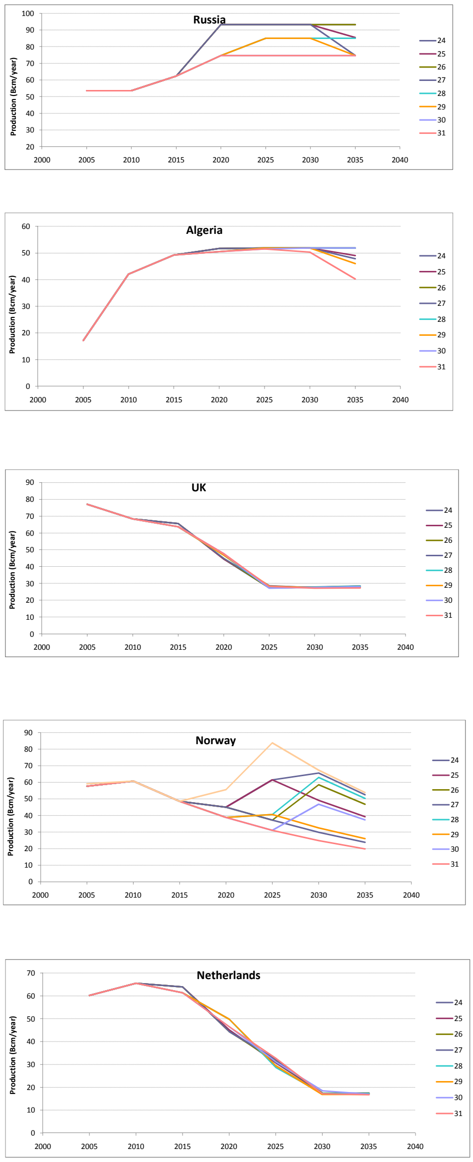

FIGURE 10.3 – *Production in the different scenarios.*

and has a relatively high spread.

Now it may be interesting to analyze the impact of randomness on the long-term contracts. The following table gives the different long-term contracts' (LTC) volumes and prices between the producers and the independent traders (stochastic). In order to compare with the deterministic version, we also report the LTC results in the High and Low cases. [1]

Stochastic

Volume(Bcm/year)	France	Germany	UK	Belgium	The Netherlands	Total
Russia	0.84	32.26	0.00	0.00	0.00	33.10
Algeria	4.03	0.00	0.00	8.80	0.00	12.84
The Netherlands	0.00	0.00	0.00	9.42	5.91	15.33
Norway	0.00	0.00	0.00	12.16	0.00	12.16
UK	0.00	0.00	24.55	0.00	0.00	24.55
Total	4.87	32.26	24.55	30.38	5.91	97.98

High

Volume(Bcm/year)	France	Germany	UK	Belgium	The Netherlands	Total
Russia	0.31	33.61	0.00	0.00	0.00	33.92
Algeria	4.04	0.00	0.00	8.79	0.00	12.83
The Netherlands	0.00	0.00	0.00	9.23	5.84	15.06
Norway	1.11	0.00	2.88	12.44	0.00	16.43
UK	0.00	0.00	23.64	0.00	0.00	23.64
Total	5.47	33.61	26.53	30.45	5.84	101.89

Low

Volume(Bcm/year)	France	Germany	UK	Belgium	The Netherlands	Total
Russia	0.86	30.95	0.00	0.00	0.00	31.80
Algeria	3.93	0.00	0.00	8.88	0.00	12.81
The Netherlands	0.00	0.00	0.00	9.58	5.58	15.16
Norway	0.00	0.00	0.00	12.02	0.00	12.02
UK	0.00	0.00	24.89	0.00	0.00	24.89
Total	4.78	30.95	24.89	30.48	5.58	96.69

Stochastic

Price($/cm)	France	Germany	UK	Belgium	The Netherlands
Russia	0.12	0.12	nc	nc	nc
Algeria	0.12	nc	nc	0.13	nc
The Netherlands	nc	nc	nc	0.13	0.14
Norway	nc	nc	nc	0.13	nc
UK	nc	nc	0.15	nc	nc

1. nc denotes a no-contract situation.

High Price($/cm)	France	Germany	UK	Belgium	The Netherlands
Russia	0.13	0.13	nc	nc	nc
Algeria	0.13	nc	nc	0.14	nc
The Netherlands	nc	nc	nc	0.14	0.16
Norway	0.13	0.16	0.14	nc	nc
UK	nc	nc	0.16	nc	nc

Low Price($/cm)	France	Germany	UK	Belgium	The Netherlands
Russia	0.12	0.12	nc	nc	nc
Algeria	0.12	nc	nc	0.12	nc
The Netherlands	nc	nc	nc	0.12	0.14
Norway	nc	nc	nc	0.12	nc
UK	nc	nc	0.15	nc	nc

One can check that the results of Theorem 13 are respected in our numerical study. Indeed, if a pair of producer-independent trader contract on the long-term, the corresponding LTC price is nonnegative. Theorem 14's results are also satisfied : the spot prices in the consuming countries, reported in Figure 10.2 are in general higher than the LTC prices. One can also notice that the Belgian trader is the one that diversifies the most his gas supplies (three sources). This is due to its geographical location, which is close to three producing countries : Norway, The Netherlands and Algeria (we remind that the Algerian production node is directly linked to Belgium via a LNG route). A consuming country, which produces natural gas, such as the UK or The Netherlands, sees the corresponding independent trader contract on the long-term exclusively with the local producer. This is quite intuitive because of the geographical proximity. Besides, for a particular trader, the LTC price is the same with respect to all the possible supply sources. This confirms Theorem 15's result and also suggests that the LTC prices are correlated to the spot prices. An independent trader may tolerate high supply marginal costs if his marginal revenue or the spot price in his spot market, which is the market where he has to support the least transportation costs, is high enough.

A comparison between the Stochastic, High and Low cases shows that the LTC volumes contracted are, on average, 1.5% higher in the Stochastic case than in the Low case. On the contrary, LTC volumes are, on average, 4% lower in the Stochastic case than in the High case. There are even some contracts in the High case that do not exist in the Stochastic case : Norway-UK (2.9 Bcm/year) and Norway-France (1.1 Bcm/year). Regarding the prices, the results are similar : the Stochastic LTC prices are, on average 2.5% higher than in the Low case and the Stochastic LTC prices are, on average 9% lower than in the High case.

These results are quite intuitive : in the stochastic model, the strategic players need to hedge their decisions on the long-term, against the oil price fluctuations. In a High scenario perfect foresight, the demand increases constantly with time and the independent traders need to contract more important volumes in order to ensure a sufficient supply. In that situation, the spot price is expected to be relatively high (see Figure 10.2) and therefore the independent traders can support higher supply costs, which benefits to the producers. This explanation holds for the Stochastic-Low cases LTC comparison.

10.2.4 The value of the stochastic solution

Now we define a measure that allows us to quantify the utility to introduce stochasticity in the S-GaMMES model. Following (54), we adapt the concept of the value of the stochastic solution, introduced by (7) to analyze the performance of stochastic programing, to the context of imperfect competition and Equilibrium problems.

For that purpose, we will compare the Stochastic Model (SM) and the Mean Value Model (MVM) results, which will be defined later. In our representation, at each time period, the oil price is modeled as a random variable whose mean value will be denoted by :

$$p_t = \langle p_t^{\omega} \rangle_{\omega} \tag{10.2}$$

The mean value is calculated by considering all the scenarios ω that correspond to time-step t.

The Mean Value Model is a deterministic model where, at each time step, we approximate the oil price by its mean value p_t. Figure 10.4 is a schematic description of both models.

We will compare the different players' utilities in the Stochastic and Mean Value cases. We will also compare the Stochastic, the High demand and Low demand cases utilities. For a particular player, we will define :

- The SM utility Π_{SM} by the expected value of its global utility over the possible scenarios, in the SM output.
- The MVM utility Π_{MVM} by the expected value of its global utility over time, in the MVM output.
- The HM utility Π_{HM} by the expected value of its global utility over time, in the High case output.
- The LM utility Π_{LM} by the expected value of its global utility over time, in the Low case output.
- The value of the stochastic solution defined by :

$$VSS = \Pi_{SM} - \Pi_{MVM} \tag{10.3}$$

- The loss of the stochastic solution defined by :

$$LSS = \Pi_{SM} - \Pi_{HM} \tag{10.4}$$

- The gain of the stochastic solution defined by :

$$GSS = \Pi_{SM} - \Pi_{LM} \tag{10.5}$$

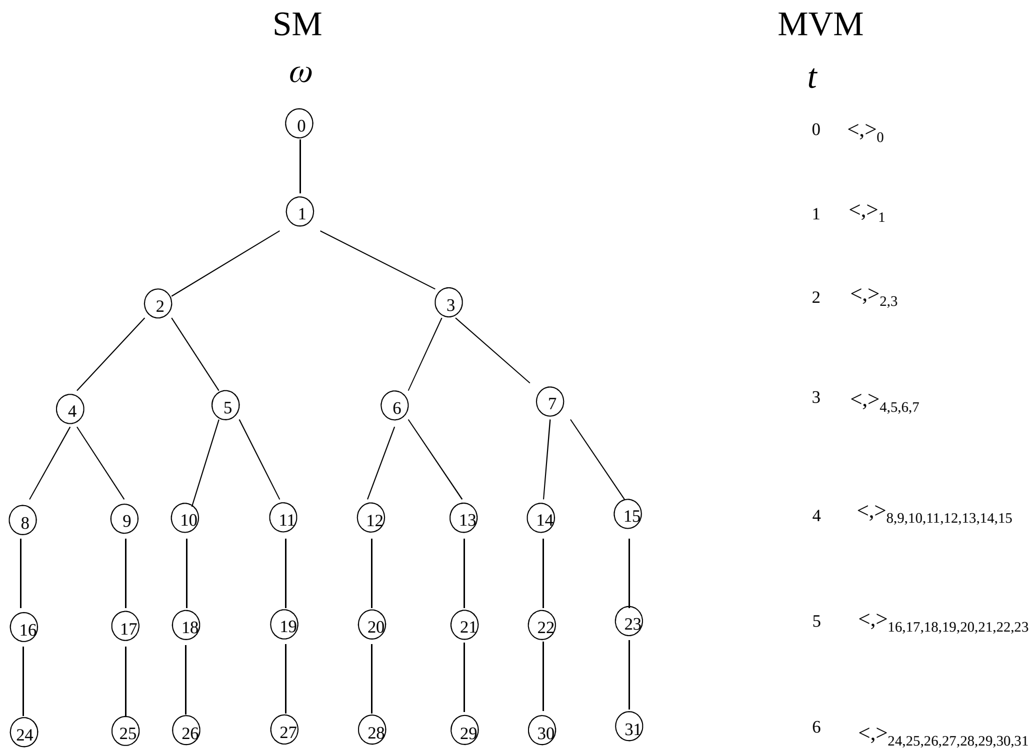

Figure 10.4 – *The stochastic Model (SM) and Mean Value Model (MVM).*

Note that the Mean Value Model, the High demand and Low demand cases are deterministic situations, where the players have a perfect foresight of the future. The VSS is a means to quantify the importance of using stochasticity in the model. The GSS measures the gain obtained by the players when taking into consideration stochasticity, as compared to a deterministic low demand regime. To the contrary, the LSS measures the loss supported by the players when taking into consideration stochasticity, as compared to a deterministic high demand regime. In linear and non-linear stochastic programing, Birge and Louveaux (7) demonstrated that the VSS is nonnegative. However, this result may not hold for MCPs or Equilibrium Stochastic Problems. Indeed, since we do not deal with a unique objective function to optimize, but rather with multiple correlated ones, it is not straightforward that each player would benefit from taking care of stochasticity. The same conclusions hold for the GSS and LSS. Indeed, in stochastic programing, it is intuitive that the GSS is nonnegative whereas the LSS is negative. However this may not be true when dealing with MCP formulations.

The following table gives the VSS, GSS and LSS for the producers. [2] A producer's utility is his profit.

	VSS (*10^9 \$)	VSS (%)	GSS (%)	LSS (%)
Russia	9.23	9.23	19.45	-19.85
Algeria	0.91	1.43	7.23	-17.32
The Netherlands	-0.87	-0.58	0.75	-4.69
Norway	-0.29	-0.29	2.4	-14.66
UK	-1.18	-0.72	0.8	-5.42
Total	7.80	1.35	4.80	-10.79

The average VSS for the producers is 1.35%, which is nonnegative. This means that on average, the producers benefit from the use of randomness in their optimization programs. There are some cases where the VSS is negative (The Netherlands, Norway and The UK). However, the corresponding values are relatively small. Russia is the producer that benefits the most from the use of stochasticity, with a VSS of 9%. Regarding the GSS and LSS, their average values are 4.8% and -10.8% respectively. All the producers have a nonnegative GSS and negative LSS.

The following table gives the VSS for the independent traders. An independent trader's utility is his profit.

2. The VSS in % is defined by $\frac{\Pi_{SM}-\Pi_{MVM}}{\Pi_{SM}}$. The definition is similar for the GSS and LSS in %.

	VSS (*10^9 \$)	VSS (%)	GSS (%)	LSS (%)
France	-0.03	-0.54	3.89	-27.08
Germany	0.15	0.37	4.09	-13.40
UK	0.17	1.28	1.01	-18.17
Belgium	-0.15	-0.51	1.24	-6.81
The Netherlands	0.55	14.29	12.08	2.55
Total	0.69	0.73	3.05	-12.17

The average VSS for the traders is 0.73%, which is nonnegative. This means that on average, the independent traders benefit from the use of randomness in their optimization programs. Nevertheless, there are some cases where the VSS is negative. The Dutch trader is the one that benefits the most from the use of stochasticity, with a VSS of 14%. The average GSS is 3.05% and the average LSS is -12.17%. The Dutch trader has a positive LSS. However, the value is relatively small compared to the GSS or the VSS.

The previous results concerned the strategic players, who directly take into consideration randomness in their profits. Now it may be interesting to measure the VSS for the non-strategic players.

The following table gives the VSS for the consumers. The consumers' utility is their surplus : if the inverse demand function is $p(q)$ and the consumed quantity is Q, then the utility is defined by $\int_0^Q (p(q) - p(Q))\, q\, dq$.

	VSS (*10^9 \$)	VSS (%)	GSS (%)	LSS (%)
France	0.76	0.28	2.43	-16.52
Germany	1.86	0.46	2.49	-9.28
UK	3.23	2.54	2.71	-2.93
Belgium	0.33	0.42	2.39	-11.1
The Netherlands	1.37	1.26	2.9	-12.5
Total	7.55	0.76	2.54	-10.88

The average VSS for the consuming countries is 0.76%, which is nonnegative. This means that in general, the consumers benefit from the use of randomness. Note that this result is not intuitive because the consumers' surplus is not taken into account in the producers or the traders payoff. However, this can be explained by the fact that in S-GaMMES, the strategic players have a better representation of the demand fluctuations that directly influences the consumers' surplus. The United Kingdom is the country that benefits the most from the use of stochasticity, with a VSS of 2.5%. The average GSS and LSS values are respectively 2.5% and -10.88%

The following table gives the VVS for the pipeline (TSO) and storage (SSO) operators. The utility is the opposite of the cost they minimize (see Section 9.2.5). The VSS and GSS are also nonnegative, whereas the LSS is negative.

	VSS ($*10^9$ \$)	VSS (%)	GSS(%)	LSS(%)
TSO	5.69	8.33	0.67	-38.67
SSO	0	0	0	0

In conclusion, the use of the Value of the Stochastic Solution allows us to quantify the gain earned by the players (strategic and non-strategic) when considering stochasticity in their decisions. When calculated in S-GaMMES, the results suggest that, on average, all the strategic and non-strategic players benefit from the use of randomness. However, since we are dealing with an Equilibrium Stochastic Problem, there are some players who may suffer from that situation, which may lead to their utility becoming smaller.

§ 10.3 Conclusion

This chapter applies S-GaMMES to the northwestern European gas trade. The model has been solved using the solver PATH on GAMS. After the calibration process, it was applied to understand the European natural gas trade forecast between 2000 and 2035 in terms of consumption, prices, production, and LTC prices and volumes, in the different possible scenarios allowed by the scenario tree. In particular, we have defined and calculated the spread of consumption and prices in the different countries in order to quantify the importance of taking into account stochasticity in the model. We have also compared LTC prices in the stochastic versus the deterministic situations, in order to understand how the producers hedge their investment-related risk when dealing with an uncertain demand.

More generally, we have defined the value of the stochastic solution, as well as the gain and loss of the stochastic solution. This can be carried out by comparing the stochastic model results and its deterministic equivalent. This also allows one to quantify the importance and usefulness of taking into consideration the demand randomness. The value of the stochastic solution can be calculated for all the players in order to identify those who benefit from the use of stochasticity from those who do not. On average, taking into account stochasticity benefits to all the players. However, since we are dealing with imperfect competition, some players may reduce their utility when dealing with stochastic programming. Such players have been identified in this chapter.

Bibliographie

[1] I. Abada, V. Briat, S. A. Gabriel & O. Massol, 2011. *A Generalized Nash-Cournot Model for the European Natural Gas markets with a Fuel Substitution Demand Function : The GaMMES Model.* Economix Working Paper, http ://economix.u-paris10.fr/.

[2] I. Abada & O. Massol, 2011. *Security of supply and retail competition in the European gas market. Some model-based insights.* Energy Policy 39 (2011), 4077-4088.

[3] I. Abada, V. Briat & O. Massol, 2011. *Construction of a fuel demand function portraying interfuel substitution, a system dynamics approach.* Economix Working Paper, available at http ://economix.fr/fr/dt/2011.php.

[4] F. R. Aune, K. E. Rosendahl, E. L. Sagen, 2009. *Globalisation of natural gas markets - effects on prices and trade patterns.* Special Issue of The Energy Journal 30, 39-54.

[5] J. F. Benders, 1962. *Partitioning procedures for solving mixed-variables programming problems.* Numerische Mathematik 4, 238-252.

[6] J. F. Benders, 1962. *Partitioning procedures for solving mixed-variables programming problems.* Computational Management Science 2, 3-19.

[7] J. R. Birge & F. Louveaux, 1997. *Introduction to Stochastic Programming.* Springer-Verlag, New York, Inc.

[8] J. R. Birge, 1982. *The value of the stochastic solution in stochastic linear programs with fixed recourse.* Mathematical Programming 24, 314-325.

[9] M. G. Boots & F. A. M. Rijkers & B. F. Hobbs, 2004. *Trading in the Downstream European Gas Market : A Successive Oligopoly Approach.* The Energy Journal, 25(3), 73-102.

[10] BP Statistical Review of World Energy, 2009. www.bp.com.

[11] W. A. Brock, W. D. Dechert, B. Lebaron & J. Scheinkman , 1995. *A Test for Independence Based on the Correlation Dimension.* Econ Working Papers http ://econpapers.repec.org

[12] J. Dupacov. Grwe-Kuska, W. Rmisch, 2003. *Scenario reduction in stochastic programming.* Mathematical Programming 95, 493-511.

[13] R. Egging, S. A. Gabriel, 2006. *Examining market power in the European natural gas market.* Energy Policy 34 (17), 2762-2778.

[14] R. Egging, S. A. Gabriel, F. Holtz, J. Zhuang, 2008a. *A complementarity model for the European natural gas market.* Energy Policy 36 (7), 2385-2414.

[15] R. Egging, F. Holz, S. A. Gabriel 2010. *The World Gas Model : A Multi-Period Mixed Complementarity Model for the Global Natural Gas Market.* Energy. 35 (10), pp. 4016-4029.

[16] R. Egging, 2010. *Multi-Period Natural Gas Market Modeling - Applications, Stochastic Extensions and Solution Approaches.* Ph.D. Dissertation, University of Maryland.

[17] European Commission, 2007. DG Competition Report on Energy Sector Inquiry 2007, Luxembourg : Office for Official Publ. of the Europ. Communities. *Aavailable at http ://ec.europa.eu/dgs/energy.*

[18] European Commission, 2008. European energy and transport : trends to 2030, update 2007, Luxembourg : Office for Official Publ. of the Europ. Communities. *Available at http ://ec.europa.eu/dgs/energy_ transport/figures/trends_ 2030_ update_ 2007/.*

[19] F. Facchinei & J. -S. Pang, 2003. *Finite-Dimensional Variational Inequalities and Complementarity Problems.* Springer, New York.

[20] M. C. Ferris & T. S. Munson, 1987. *The PATH solver.* Elsevier Science Publishers B.V. (North-Holland).

[21] S. A. Gabriel, S. Kiet, J. Zhuang, 2005a. *A Mixed Complementarity-Based Equilibrium Model of Natural Gas Markets.* Operations Research, 53(5), 799-818.

[22] S. A. Gabriel, J. Zhuang, S. Kiet, 2005b. *A Large-scale Complementarity Model of the North American Gas Market.* Energy Economics, 27, 639-665.

[23] S. A. Gabriel, K. E. Rosendahl, R. Egging, H. Avetisyan, S. Siddiqui, 2009. *Cartelization in Gas Markets : Studying the Potential for a 'Gas OPEC'.* In review.

[24] S. A. Gabriel, J. Zhuang & R. Egging, 2009. *Solving Stochastic Complementarity Problems in Energy Market Modelling Using Scenario Reduction.* European Journal of Operational Research, 197(3), 1028-1040.

[25] S. A. Gabriel & J. D. Fuller, 2010. *A Benders Decomposition Method for Solving Stochastic Complementarity Problems with an Application in Energy.* Computational economics, Volume 35, Number 4, 301-329.

[26] GDE-SUEZ Reference Document, 2009. *http ://www.gdfsuez.com/.*

[27] R. Golombek, E. Gjelsvik, K. E. Rosendahl, 1995. *Effects of liberalizing the natural gas markets in Western Europe.* Energy Journal 16, 85-111.

[28] R. Golombek, E. Gjelsvik, K. E. Rosendahl, 1998. *Increased competition on the supply side of the Western European natural gas market.* Energy Journal 19 (3), 1-18.

[29] P. T. Harker, 1991. *Generalized Nash games and quasi-variational inequalities.* European Journal of Operational Research, Vol.54 81-94.

[30] P. T. Harker & J. Pang, 1998. *Finite-dimensional variational inequality and nonlinear complementarity problems : A survey of theory, algorithms and applications.* Mathematical Programming 48 (1990) 161-220 (North-Holland).

[31] A. Haurie, G. Zaccour, J. Legrand & Y. Smeers, 1987. *A stochastic dynamic Nash Cournot model for the European gas market.* Technical Report, HEC Montreal.

[32] F. Holz, C. von Hirschhausen, C. Kemfert, 2008. *A Strategic Model of European Gas Supply (GASMOD).* Energy Economics 30, 766-788.

[33] F. Holz, 2009. *Modeling the European Natural Gas Market - Static and Dynamic Perspectives of an Oligopolistic Market.* Ph.D. Dissertation, Technische Universitat-Berlin.

[34] R. G. Hubbard & R. J. Weiner, 1986. *Regulation and Long-Term Contracting in U.S. Natural Gas Markets.* Journal of Industrial Economics, Vol. 35, No. 1, pp. 71-79

[35] IEA, 2004. *Natural Gas Information* .

[36] IEA, 2009. *Natural Gas Information* .

[37] International Energy Agency, 2007. *World Energy Outlook 2007.* OECD/IEA.

[38] International Energy Agency, 2008. *World Energy Outlook 2008.* OECD/IEA.

[39] International Energy Agency, 2009. *World Energy Outlook 2009.* OECD/IEA.

[40] International Gas Union, Gastech Conference Amsterdam March 2011, 2009. *New Trends in Gas Price Formation.*

[41] W. Lise & B. F. Hobbs, 2008. *Future evolution of the liberalised European gas market. Simulation results with the dynamic GASTALE model.* Energy Policy 36 (6), 1890-1906.

[42] W. Lise & B. F. Hobbs, 2009. *A Dynamic Simulation of Market Power in the Liberalised European Natural Gas Market.* The Energy Journal 30, 119-136.

[43] H. W. Lilliefors, 1967. *On The Kolmogorov-Smirnov Test For Normality With Mean And Variance Unknown.* Journal of the American Statistical Association, Vol. 62, No. 318. (Jun., 1967), p. 399-402.

[44] L. Mathiesen, K. Roland, K. Thonstad, 1987. *The European Natural Gas Mar- ket : Degrees of Market Power on the Selling Side.* In R. Golombek, M. Hoel, and J. Vislie (eds.) : Natural Gas Markets and Contracts, North-Holland.

[45] E. Moxnes, 1985. *Price of Oil, Gas, Coal and Electricity in four European Countries 1960-1983.* Christian Michelsen Institue, No. 852260-1, Bergen, Norway.

[46] E. Moxnes & E. Nesset, 1985. *Substitution between Oil, Gas and Coal in UK Industrial Steam Raising.* Christian Michelsen Institue, No. 842240-4, Bergen, Norway.

[47] E. Moxnes, 1987. *The dynamics of Interfuel Substitution in the OECD-Europe Industrial Sector.* Elsevier Science Publishers B.V. (North-Holland).

[48] Perner & Seeliger, 2004. *Prospects of gas supplies to the European market until 2030. Results from the simulation model EUGAS.* Utilities Policy 12 (4), 291-302.

[49] World Trade Gas Model, Rice, 2004. *www.rice.edu/energy/publications/docs/GSP_ WorldGasTradeModel_ Part1_ 05_ 26_ 04.pdf.*

[50] Rice, 2005. *http ://www.rice.edu/energy/publications/docs/GAS_ BIWGTM_ March2005.pdf.*

[51] Y. Smeers, 2008. *Gas Models and Three Difficult Objectives*, ECORE discussion paper available at http ://www.ecore.be/.

[52] P. Stevens, 2011. *The 'Shale Gas Revolution' : Hype and Reality.* A Chatham House Report.

[53] H. Xu & Z. Zhang, 2010. *A Trend Deduction Model of Fluctuating Oil Prices..* IDEAS Working Paper, http ://ideas.repec.org/.

[54] J. Zhuang & S. A. Gabriel, 2008. *A Complementarity Model for Solving Stochastic Natural Gas Market Equilibria.* Energy Economics 30(1), 113-147.

§ 10.4 Appendix 1

This appendix demonstrates the concavity of all the players' objective functions.
We will demonstrate that the production cost function is convex with respect to the quantity produced. The storage/withdrawal/investments costs are convex functions because they are linear.

Let's consider a producer p. First we demonstrate the convexity of the Golombek production cost function. We consider a production field f. To simplify the notation, let us denote by q the produced volume (a variable) and by Rf_f the reserve (a constant). We recall that the cost function Pc_f is as follows :

$$\begin{array}{rl} \frac{d\ Pc_f}{d\ q}: & [0, Rf_f) \longrightarrow R^+ \\ & q \longrightarrow a_f + b_f q + \ c_f \ \ln\left(\frac{Rf_f - q}{Rf_f}\right) \end{array}$$

where $c_f \leq 0$ and $b_f \geq 0$.

Theorem 17. *The Golombek production cost function Pc_f is convex.*

Démonstration. Pc_f is a $C^2([0, Rf_f))$ function (twice continuously differentiable) and we have :

$$\forall q \in [0, Rf_f)\ \frac{d^2 Pc_f}{d^2 q} = b_f - \frac{c_f}{Rf_f - q} \geq 0$$

Thus, Pc_f is convex because $c_f \leq 0$ and $b_f \geq 0$.

□

Producer p's objective function is :

$$\begin{aligned} &\sum_{\omega,m,f,i} \pi(\omega)\delta^{t(\omega)}\eta_{pi}(zp^{\omega}_{mfpi}) \\ &+ \sum_{\omega,m,f,d} \pi(\omega)\delta^{t(\omega)} \left(p^{\omega}_{md}(x^{\omega}_{mfpd} + \overline{x^{\omega}_{mfpd}})\right) x^{\omega}_{mfpd} \\ &- \sum_{\omega,f} \delta^{t(\omega)} \left(Pc_f\left(\sum_{\omega' \leq \omega} \sum_m q^{\omega'}_{mfp}, Rf_f\right) - Pc_f\left(\sum_{\omega' < \omega} \sum_m q^{\omega'}_{mfp}, Rf_f\right)\right) \\ &- \sum_{\omega,f} \delta^{t(\omega)} Ip_f(ip^{\omega}_{fp}) \\ &- \sum_{\omega,m,p,a} \delta^{t(\omega)}((Tc_a + \tau^{\omega}_{m,a}) fp^{\omega}_{m,p,a}) \end{aligned}$$

Theorem 18. *Producer p's objective function is concave with respect to his decision variables.*

Démonstration. As mentioned before, the inverse demand function has been linearized. Let's write the natural gas price in market d, season m and node ω as follows :

$$p^{\omega}_{md} = a^{\omega}_{md} - b^{\omega}_{md}(x^{\omega}_{mfpd} + \overline{x^{\omega}_{mfpd}})$$

where $b^{\omega}_{md} > 0$. The function $\sum_{\omega,m,f,d} \pi(\omega)\delta^{t(\omega)} \left(p^{\omega}_{md}(x^{\omega}_{mfpd} + \overline{x^{\omega}_{mfpd}})\right) x^{\omega}_{mfpd}$ is therefore a concave function of the variables x^{ω}_{mfpd}. Indeed, the Hessian matrix H^{ω}_{md} associated with the spot market profit is diagonal and such that the diagonal terms are $H^{\omega}_{md} = -2b^{\omega}_{md} < 0$. Hence, the Hessian matrix is negative definite.

Let us consider the global cost function GP :
$q^{\omega}_{mfp} \longrightarrow GP(q^{\omega}_{mfp}) = \sum_{\omega,f} \delta^{t(\omega)} \left(Pc_f \left(\sum_{\omega' \leq \omega} \sum_m q^{\omega'}_{mfp}, Rf_f \right) - Pc_f \left(\sum_{\omega' < \omega} \sum_m q^{\omega'}_{mfp}, Rf_f \right) \right)$.
And let's demonstrate that GP is convex. Let us consider two variable vectors $q1^{\omega}_{md}$ and $q2^{\omega}_{md}$ and $\lambda \in [0,1]$.
We denote by Ω_l the subset of Ω that contains all the leaves of the tree.

$$
\begin{aligned}
& GP(\lambda q1^{\omega}_{md} + (1-\lambda) q2^{\omega}_{md}) \\
& = \\
& \textstyle\sum_{\omega,f} \delta^{t(\omega)} \left(Pc_f \left(\sum_{\omega' \leq \omega} \sum_m (\lambda q1^{\omega'}_{md} + (1-\lambda) q2^{\omega'}_{md}), Rf_f \right) \right) \\
& - \textstyle\sum_{\omega,f} \delta^{t(\omega)} \left(Pc_f \left(\sum_{\omega' < \omega} \sum_m (\lambda q1^{\omega'}_{md} + (1-\lambda) q2^{\omega'}_{md}), Rf_f \right) \right) \\
& = \\
& \textstyle\sum_f \sum_{\omega \in \Omega} \delta^{t(\omega)} \left(Pc_f \left(\sum_{\omega' \leq \omega} \sum_m (\lambda q1^{\omega'}_{md} + (1-\lambda) q2^{\omega'}_{md}), Rf_f \right) \right) \\
& - \textstyle\sum_f \sum_{\omega \notin \Omega_l} \delta^{t(\omega)+1} \left(Pc_f \left(\sum_{\omega' \leq \omega} \sum_m (\lambda q1^{\omega'}_{md} + (1-\lambda) q2^{\omega'}_{md}), Rf_f \right) \right) \\
& = \\
& \textstyle\sum_f \sum_{\omega \notin \Omega_l} (\delta^{t(\omega)} - \delta^{t(\omega)+1}) \left(Pc_f \left(\sum_{\omega' \leq \omega} \sum_m (\lambda q1^{\omega'}_{md} + (1-\lambda) q2^{\omega'}_{md}), Rf_f \right) \right) \\
& + \textstyle\sum_f \sum_{\omega \in \Omega_l} \delta^{Num} \left(Pc_f \left(\sum_{\omega' \leq \omega} \sum_m (\lambda q1^{\omega'}_{md} + (1-\lambda) q2^{\omega'}_{md}), Rf_f \right) \right) \\
& = \\
& \textstyle\sum_f \sum_{\omega \notin \Omega_l} \delta^{t(\omega)} (1-\delta) \left(Pc_f \left(\sum_{\omega' \leq \omega} \sum_m (\lambda q1^{\omega'}_{md} + (1-\lambda) q2^{\omega'}_{md}), Rf_f \right) \right) \\
& + \textstyle\sum_f \sum_{\omega \in \Omega_l} \delta^{Num} \left(Pc_f \left(\sum_{\omega' \leq \omega} \sum_m (\lambda q1^{\omega'}_{md} + (1-\lambda) q2^{\omega'}_{md}), Rf_f \right) \right)
\end{aligned}
$$

Since $0 \leq \delta \leq 1$ and Pc_f is convex, we can write :

$$
\begin{aligned}
& \textstyle\sum_f \sum_{\omega \notin \Omega_l} \delta^{t(\omega)} (1-\delta) \left(Pc_f \left(\sum_{\omega' \leq \omega} \sum_m (\lambda q1^{\omega'}_{md} + (1-\lambda) q2^{\omega'}_{md}), Rf_f \right) \right) \\
& + \textstyle\sum_f \sum_{\omega \in \Omega_l} \delta^{Num} \left(Pc_f \left(\sum_{\omega' \leq \omega} \sum_m (\lambda q1^{\omega'}_{md} + (1-\lambda) q2^{\omega'}_{md}), Rf_f \right) \right) \\
& \leq \\
& \lambda \textstyle\sum_f \sum_{\omega \notin \Omega_l} \delta^{t(\omega)} (1-\delta) \left(Pc_f \left(\sum_{\omega' \leq t} \sum_m q1^{\omega'}_{md}, Rf_f \right) \right) \\
& + (1-\lambda) \textstyle\sum_f \sum_{\omega \notin \Omega_l} \delta^{t(\omega)} (1-\delta) \left(Pc_f \left(\sum_{\omega' \leq t} \sum_m q2^{\omega'}_{md}, Rf_f \right) \right) \\
& + \lambda \textstyle\sum_f \sum_{\omega \in \Omega_l} \delta^{Num} \left(Pc_f \left(\sum_{\omega' \leq \omega} \sum_m q1^{\omega'}_{md}, Rf_f \right) \right) \\
& + (1-\lambda) \textstyle\sum_f \delta^{Num} \left(Pc_f \left(\sum_{\omega' \leq \omega} \sum_m q2^{\omega'}_{md}, Rf_f \right) \right) \\
& = \\
& \lambda GP(q1^{\omega}_{md}) + (1-\lambda) GP(q2^{\omega}_{md})
\end{aligned}
$$

Hence, the cost function is convex. The rest of the profit is made of linear functions of the decision variables. The concavity of the producers' objective function is thus demonstrated. □

Theorem 19. *The independent traders' objective function is concave with respect to his decision variables.*

Theorem 20. *The pipeline and storage operators objective functions are convex.*

Theorem 21. *All the players' constraint sets are convex.*

Démonstration. The proof of the independent traders' concavity of their objective function is similar to the previous proof. Like for the producers, the spot market benefit is in particular concave.
The pipeline and storage operators objective functions are convex because they are linear.
The feasibility sets are all convex due to linearity of the constraint functions. □

§ 10.5 Appendix 2

This appendix presents the KKT conditions derived from S-GaMMES. Once the KKT conditions written, we get the Mixed Complementarity Problem (MCP) given below.

The producers' KKT conditions

$$\forall \omega,\ m,\ f,\ p,\ i, \quad 0 \leq zp^{\omega}_{mfpi} \quad \perp \quad \pi(\omega)\delta^{t(\omega)}\eta_{pi} - \gamma^{\omega}_{mfp} - \epsilon 2^{\omega}_{mfpi} - \eta p^{\omega}_{pi} - \sum_n M2_{in}\alpha p^{\omega}_{m,p,n} \leq 0 \tag{10.6a}$$

$$\forall \omega,\ m,\ f,\ p,\ d, \quad 0 \leq x^{\omega}_{mfpd} \quad \perp \quad \pi(\omega)\delta^{t(\omega)}p^{\omega}_{md}(x^{\omega}_{mfpd} + \overline{x^{\omega}_{mfpd}}) + \pi(\omega)\delta^{t(\omega)}\frac{\partial p^{\omega}_{md}}{\partial x^{\omega}_{mfpd}}(x^{\omega}_{mfpd} + \overline{x^{\omega}_{mfpd}})x^{\omega}_{mfpd} - \gamma^{\omega}_{mfp} - \epsilon 1^{\omega}_{mfpd} - \sum_n M3_{dn}\alpha p^{\omega}_{m,p,n} \leq 0 \tag{10.6b}$$

$$\forall \omega,\ m,\ f,\ p, \quad 0 \leq q^{\omega}_{mfp} \quad \perp \quad -\sum_{\omega' \geq \omega}\pi(\omega')\delta^{t(\omega')}\frac{\partial Pc_f}{\partial q}(\sum_{\omega'' \leq \omega'}\sum_m q^{\omega''}_{mfp}, Rf_f) + \sum_{\omega' > \omega}\pi(\omega')\delta^{t(\omega')}\frac{\partial Pc_f}{\partial q}(\sum_{\omega'' < \omega'}\sum_m q^{\omega''}_{mfp}, Rf_f) - \sum_{\omega' \geq \omega}\phi^{\omega'}_f - \chi^{\omega}_{mf} + \gamma^{\omega}_{mfp} - (-1)^m(\vartheta 1^{\omega}_{fp} - \vartheta 2^{\omega}_{fp}) - \epsilon 3^{\omega}_{mfp} + \sum_n M1_{fn}\alpha p^{\omega}_{m,p,n} \leq 0 \tag{10.6c}$$

$$\forall \omega,\ f,\ p, \quad 0 \leq ip^{\omega}_{fp} \quad \perp \quad -\pi(\omega)\delta^{t(\omega)}Ip_f - \epsilon 4^{\omega}_{fp} + \sum_m\sum_{\omega' > \omega}\chi^{\omega'}_{mf}(1 - dep_f)^{t(\omega') - t(\omega)} - \iota p^{\omega}_f + Lf_f\sum_{\omega' > \omega}\iota p^{\omega'}_f(1 - dep_f)^{t(\omega') - t(\omega)} \leq 0 \tag{10.6d}$$

$$\forall\ p,\ i, \quad 0 \leq up_{pi} \quad \perp \quad \sum_{\omega}\eta p^{\omega}_{pi} - \eta_{pi} \leq 0 \tag{10.6e}$$

$$\forall \omega,\ f, \quad 0 \leq \phi^{\omega}_f \quad \perp \quad \sum_p\sum_{\omega' \leq \omega}\sum_m q^{\omega'}_{mfp} - Rf_f \leq 0 \tag{10.6f}$$

$$\forall \omega,\ m,\ f, \qquad 0 \leq \chi^{\omega}_{mf} \quad \perp \quad \sum_{p} q^{\omega}_{mfp} - Tk_f(1-dep_f)^{t(\omega)} - \sum_{p}\sum_{\omega'<\omega} ip^{\omega'}_{fp}(1-dep_f)^{t(\omega)-t(\omega')} \leq 0 \tag{10.7a}$$

$$\forall \omega,\ m,\ f,\ p, \qquad 0 \leq \gamma^{\omega}_{mfp} \quad \perp \quad -q^{\omega}_{mfp} + \sum_{i} zp^{\omega}_{mfpi} + \sum_{d} x^{\omega}_{mfpd} \leq 0 \tag{10.7b}$$

$$\forall \omega,\ f,\ p, \qquad 0 \leq \vartheta 1^{\omega}_{fp} \quad \perp \quad \sum_{m}(-1)^m q^{\omega}_{mfp} - fl_f \leq 0 \tag{10.7c}$$

$$\forall \omega,\ f,\ p, \qquad 0 \leq \vartheta 2^{\omega}_{fp} \quad \perp \quad -\sum_{m}(-1)^m q^{\omega}_{mfp} - fl_f \leq 0 \tag{10.7d}$$

$$\forall t,\ f, \qquad 0 \leq \iota p^{\omega}_{f} \quad \perp \quad \sum_{p} ip^{\omega}_{fp} - Lf_f Kf_f(1-dep_f)^{t(\omega)} - Lf_f \sum_{p}\sum_{\omega'<\omega} ip^{t(\omega')}_{fp}(1-dep_f)^{t(\omega)-t(\omega')} \leq 0 \tag{10.7e}$$

$$\forall \omega,\ f, m,\ p,\ d, \qquad 0 \leq \epsilon 1^{\omega}_{mfpd} \quad \perp \quad x^{\omega}_{mfpd} - O_{fp}H \leq 0 \tag{10.7f}$$

$$\forall \omega,\ m, f,\ p,\ i, \qquad 0 \leq \epsilon 2^{\omega}_{mfpi} \quad \perp \quad zp^{\omega}_{mfpi} - O_{fp}H \leq 0 \tag{10.7g}$$

$$\forall \omega,\ m, f,\ p, \qquad 0 \leq \epsilon 3^{\omega}_{mfp} \quad \perp \quad q^{\omega}_{mfp} - O_{fp}H \leq 0 \tag{10.7h}$$

$$\forall \omega,\ f,\ p, \qquad 0 \leq \epsilon 4^{\omega}_{fp} \quad \perp \quad ip^{\omega}_{fp} - O_{fp}H \leq 0 \tag{10.8a}$$

$$\forall \omega,\ p,\ i, \qquad \text{free} \quad \eta p^{\omega}_{pi} \qquad up_{pi} - \sum_{f,m} zp^{\omega}_{mfpi} = 0 \tag{10.8b}$$

$$\forall\ p,\ i, \qquad \text{free} \quad \eta_{pi} \qquad ui_{pi} - up_{pi} = 0 \tag{10.8c}$$

The independent traders' KKT conditions

$$\begin{aligned} \forall \omega,\ m,\ p,\ i, \qquad 0 \leq zi^{\omega}_{mpi} \quad \perp \quad & -\pi(\omega)\delta^{t(\omega)}\eta_{pi} - \eta i^{\omega}_{pi} \\ & + \psi^{\omega}_{mi} \\ & + \sum_{n} M2_{in}\alpha i^{\omega}_{min} \\ & + (1 - min_{pi})\upsilon^{\omega}_{mpi} \leq 0 \end{aligned} \tag{10.9a}$$

$$\begin{aligned} \forall \omega,\ m,\ i,\ d, \qquad 0 \leq y^{\omega}_{mid} \quad \perp \quad & \pi(\omega)\delta^{t(\omega)}p^{\omega}_{md}(y^{\omega}_{mid} + \overline{y^{\omega}_{mid}}) \\ & + \pi(\omega)\delta^{t(\omega)}\frac{\partial p^{\omega}_{md}}{\partial y^{\omega}_{mid}}(y^{\omega}_{mid} + \overline{y^{\omega}_{mid}})y^{\omega}_{mid} \\ & - \psi^{\omega}_{mi} - \sum_{n} M3_{dn}\alpha i^{\omega}_{m,i,n} \leq 0 \end{aligned} \tag{10.9b}$$

$$\forall \omega,\ i,\ s, \qquad 0 \leq r^{\omega}_{is} \quad \perp \quad -\pi(\omega)\delta^{t(\omega)}Rc_s + \mu^{\omega}_{is} - \beta s^{\omega}_{s} \leq 0 \tag{10.9c}$$

$$\begin{aligned} \forall \omega,\ i,\ s, \qquad 0 \leq in^{\omega}_{is} \quad \perp \quad & -\pi(\omega)\delta^{t(\omega)}(Ic_s + Wc_s) \\ & - \mu^{\omega}_{is} - \sum_{m}(-1)^{m}\psi^{\omega}_{mi} \\ & - \sum_{n} M4_{sn}\alpha i^{\omega}_{m,i,n}(-1)^{m} \leq 0 \end{aligned} \tag{10.9d}$$

$$\forall p,\ i, \qquad 0 \leq ui_{pi} \quad \perp \quad \sum_{\omega} \eta i^{\omega}_{pi} + \eta_{pi} \leq 0 \tag{10.9e}$$

$$
\begin{array}{llllll}
\forall \omega,\ m,\ i, & \text{free} & \psi^{\omega}_{mi} & & \sum_{p} zi^{\omega}_{mpi} - \sum_{d} y^{\omega}_{mid} + (-1)^{m} \sum_{s} in^{\omega}_{is} & = 0 \quad (10.10\text{a}) \\
\forall \omega,\ i,\ s, & & 0 \leq \mu^{\omega}_{is} & \perp & in^{\omega}_{is} - r^{\omega}_{is} & \leq 0 \quad (10.10\text{b}) \\
\forall \omega,\ p,\ i, & \text{free} & \eta i^{\omega}_{pi} & & ui_{pi} - \sum_{m} zi^{\omega}_{mpi} & = 0 \quad (10.10\text{c}) \\
\forall p,\ i, & \text{free} & \eta_{pi} & & ui_{pi} - up_{pi} & = 0 \quad (10.10\text{d}) \\
\forall \omega,\ m,\ p,\ i, & & 0 \leq \upsilon^{\omega}_{mpi} & \perp & - zi^{\omega}_{mpi} + min_{pi} \sum_{m} zi^{\omega}_{mpi} & \leq 0 \quad (10.10\text{e})
\end{array}
$$

The pipeline operator's KKT conditions

$$\forall \omega,\ m,\ p,\ a, \qquad 0 \le fp^{\omega}_{m,p,a} \quad \perp \quad -\pi(\omega)\delta^{t(\omega)}(Tc_a + \tau^{\omega}_{m,a}) - \tau^{\omega}_{m,a} + \sum_n M6_{an}\alpha p^{\omega}_{p,m,n}(1 - loss_a) - \sum_n M5_{an}\alpha p^{\omega}_{p,m,n} \le 0 \tag{10.11a}$$

$$\forall \omega,\ m,\ i,\ a, \qquad 0 \le fi^{\omega}_{m,i,a} \quad \perp \quad -\pi(\omega)\delta^{t(\omega)}(Tc_a + \tau^{\omega}_{m,a}) - \tau^{\omega}_{m,a} + \sum_n M6_{an}\alpha i^{\omega}_{i,m,n}(1 - loss_a) - \sum_n M5_{an}\alpha i^{\omega}_{i,m,n} \le 0 \tag{10.11b}$$

$$\forall \omega,\ a, \qquad 0 \le ik^{\omega}_a \quad \perp \quad -\pi(\omega)\delta^{t(\omega)} Ik_a + \sum_{\omega' > \omega} \tau^{\omega'}_{m,a} - \iota a^{\omega}_a + La_a \sum_{\omega' > \omega} \iota a^{\omega'}_a \le 0 \tag{10.11c}$$

$$\forall \omega,\ m,\ a, \qquad 0 \le \tau^{\omega}_{m,a} \quad \perp \quad \sum_p fp^{\omega}_{m,p,a} + \sum_i fi^{\omega}_{m,i,a} - Tk_a - \sum_{\omega' < \omega} ik^{\omega'}_a \le 0 \tag{10.11d}$$

$$\forall \omega,\ a, \qquad 0 \le \iota a^{\omega}_a \quad \perp \quad ik^{\omega}_a - Tk_a - \sum_{\omega' < \omega} ik^{\omega}_a \le 0 \tag{10.11e}$$

$$\forall \omega,\ m,\ p,\ n, \qquad \text{free} \quad \alpha p^{\omega}_{m,p,n} \qquad \sum_a M_6(a,n) fp^{\omega}_{m,p,a}(1 - loss_a) - \sum_a M5_{an} fp^{\omega}_{m,p,a} + \sum_f q^{\omega}_{mpf} M1_{fn} - \sum_d \sum_f x^{\omega}_{mfpd} M3_{dn} - \sum_i \sum_f zp^{\omega}_{mfpi} M2_{in} = 0 \tag{10.11f}$$

$$\forall \omega,\ m,\ i,\ n, \qquad \text{free} \quad \alpha i^{\omega}_{m,i,n} \qquad \sum_a M6_{an} f i^{\omega}_{m,i,a}(1 - loss_a) - \sum_a M5_{an} f i^{\omega}_{m,i,a} - \sum_d y^{\omega}_{mpd} M3_{dn} + \sum_p z i^{\omega}_{mpi} M2_{in} - (-1)^m \sum_s \sum_i in^{\omega}_{is} M4_{sn} = 0 \tag{10.12a}$$

The storage operator's KKT conditions

$$\forall \omega,\ s, \qquad 0 \le is^{\omega}_s \quad \perp \quad -\pi(\omega)\delta^{t(\omega)} Is_s + \sum_{\omega' > \omega} \beta s^{\omega'}_s - \iota s^{\omega}_s + Ls_s \sum_{\omega' > \omega} \iota s^{\omega'}_s \le 0 \tag{10.13a}$$

$$\forall \omega,\ s, \qquad 0 \le \beta s^{\omega}_s \quad \perp \quad \sum_i r^{\omega}_{is} - Ks_s - \sum_{\omega' < \omega} is^{\omega'}_s \le 0 \tag{10.13b}$$

$$\forall \omega,\ s, \qquad 0 \le \iota s^{\omega}_s \quad \perp \quad is^{\omega}_s - Ls_s Ks_s - Ls_s \sum_{\omega' < \omega} is^{\omega'}_s \le 0 \tag{10.13c}$$

Septième partie

Conclusion

Conclusion

Dans ce travail de thèse, nous avons proposé une représentation de l'évolution de l'économie du gaz naturel en Europe jusqu'en 2035. Pour ce faire, nous nous appuyons sur des outils de modélisation permettant de décrire le comportement stratégique de long-terme des différents acteurs de la chaîne gazière : producteurs, traders, consommateurs et opérateurs de transport et de stockage. Les marchés sont modélisés comme des oligopoles dissymétriques où les traders et producteurs exercent un pouvoir de marché inégal auprès des consommateurs finaux. Les contrats long-terme ainsi que les augmentations de capacités de transport, de stockage et de production sont des variables endogènes au modèle. Les effets de substitution énergétique au niveau de la consommation sont pris en considération grâce à un modèle de type systèmes dynamiques. Le modèle GaMMES ainsi développé consiste en un problème dynamique de Nash-Cournot généralisé dont nous avons donné une extension stochastique qui permet de rendre compte de l'influence du prix du pétrole sur l'évolution des marchés gaziers.

Dans cette thèse, les réponses apportées aux différentes questions posées en introduction sont les suivantes :

– La question de l'insécurité d'approvisionnement en Europe a été traitée en prenant en compte la structure actuelle des marchés européens. Les traders sont considérés comme des acteurs stratégiques pouvant exercer un pouvoir de marché au niveau aval de la chaîne. Ces traders doivent choisir leurs sources d'importations en connaissant le risque de rupture associé à chaque producteur. Les consommateurs quant à eux sont décrits grâce à leur fonction de demande de long et de court-terme. Cette dernière permet d'estimer leur perte de bien-être en cas de rupture d'approvisionnement. Grâce à cette modélisation, nous pouvons proposer une explication des choix d'approvisionnement en gaz naturel de l'Allemagne dans les années 1980 entre importations de la Russie (producteur peu fiable) ou de la Norvège (producteur sûr). Nous déduisons notamment l'existence d'un effet seuil sur la probabilité de rupture qui imposerait à l'Allemagne de choisir exclusivement le gaz norvégien. Par ailleurs, notre approche nous permet également d'étudier la situation actuelle de la Bulgarie qui est fortement dépendante du gaz russe. Nous arrivons ainsi à analyser l'évolution des paramètres du marché (prix, consommation, etc.) en fonction du risque de rupture. Nous déduisons, en particulier, que les traders bénéficient de deux marges dans leur profit. La première est la marge oligopolistique inhérente à la structure économique du marché. Elle est due au fait que les traders ont intérêt à créer de la rareté dans le but de faire monter le prix du gaz. La deuxième consiste en une marge de sécurité : les traders préfèrent hausser les prix dans l'objectif de se couvrir face au risque de rupture. Nous avons établi que la marge de sécurité augmente avec la probabilité de rupture. Nous proposons une régulation du marché, utilisant un contrôle des volumes importés, valable dans toute économie gazière particulièrement vulnérable. L'avantage de notre régulation est qu'elle assure l'optimalité du bien-être social. Nous étudions notamment les situations dans lesquelles elle s'applique, en fonction de la probabilité de rupture et des réserves stratégiques de gaz naturel. Finalement, nous appliquons notre modèle au marché espagnol du gaz naturel afin de comparer entre deux modes de régulation du marché : la première est celle qui est actuellement appliquée dans ce pays et qui force les traders à diversifier les origines de leurs importations. La seconde (que nous proposons) les autorise à choisir leur mix librement. En contrepartie, les traders devront compenser les usagers en cas de crise. En fonction du risque de rupture, nous sommes ainsi capables de préconiser l'application de la première ou de la seconde régulation, à des fins d'optimisation du bien-être social.

– Le modèle GaMMES permet d'étudier les scénarii d'évolution des marchés du gaz naturel en Europe, jusqu'en 2040. Nous avons ainsi pu constater que la production européenne va décroître dans le temps, à cause des faibles réserves de gaz naturel en Europe, forçant les producteurs locaux à un rationnement des ressources. Au contraire, les parts des exportations russes et algériennes vers l'Europe vont augmenter progressivement, ce qui aura pour conséquence une hausse des prix due aux coûts de transport. Cette hausse est également le fruit d'une augmentation de la demande et par conséquent de la consommation européenne.

– La décroissance de la production européenne aura pour conséquence directe une augmentation de la dépendance énergétique. La Russie, l'Algérie et la Norvège (dans une moindre mesure) sont les trois producteurs qui augmenteront leur production dans les décennies à venir. Bien entendu, cela s'accompagnera d'une augmentation des prix, mue par les forts coûts d'investissements en production et en transport.

– La substitution énergétique a été prise en compte dans la fonction de demande en gaz naturel grâce à un modèle de type systèmes dynamiques. Ce modèle permet de représenter le comportement des consommateurs qui doivent choisir leur mix optimal en fonction des prix de marché des énergies dont ils disposent. Cette modélisation du comportement est fondée sur les investissements en technologies et considère la concurrence entre les produits pétroliers, le charbon et le gaz naturel. Plus spécifiquement, nous considérons le développement du parc des brûleurs (ou technologies) à même d'utiliser les trois formes de sources d'énergie citées ainsi que sa dépréciation dans le temps. Le modèle se formule grâce à un ensemble d'équations différentielles partielles corrélées que nous résolvons numériquement. Il a été appliqué avec succès, après un processus de calibration, pour représenter le comportement des consommateurs dans plusieurs pays, pour le secteur industriel ainsi que pour la demande primaire totale en gaz naturel. Une fois le modèle validé, nous l'avons utilisé afin d'estimer la fonction de demande en gaz naturel. Le résultat obtenu est particulièrement intéressant : la forme fonctionnelle que nous déduisons tient compte de la substitution entre combustibles. Ainsi, nous incluons au sein même de la fonction de demande en gaz naturel, à la fois le prix du pétrole et celui du charbon. Par ailleurs, nous avons pu appréhender les effets d'élasticités de court/long-terme dans la demande en gaz naturel ainsi que les inerties de consommation.

– Nous avons réussi à rendre les contrats long-terme liant chaque paire de producteur/trader endogènes à GaMMES. Pour ce faire, nous avons modélisé certaines clauses de TOP dans un marché virtuel de contrats long-terme où les producteurs décident de l'offre et les traders de la demande. Une contrainte d'égalisation de l'offre et de la demande permet d'estimer le prix du contrat long-terme. Celui-ci est déduit de la variable duale associée à cette contrainte. Cette particularité de notre représentation fait de GaMMES un problème de type Nash-Cournot généralisé. L'application de notre modèle au marché européen a permis d'avoir une estimation des volumes et des prix des différents contrats long-terme, que nous avons confrontée à la réalité du marché.

– La structure économique (à double niveau) producteurs/traders/consommateurs a été prise en compte dans notre modèle de marché. Elle permet, en particulier, de doter les producteurs et les traders de pouvoirs de marché non symétriques. En effet, les producteurs possèdent un avantage par rapport aux traders dans la mesure où ils ont la possibilité de choisir entre les ventes aux traders sur les contrats long-terme (LTC) et les ventes aux consommateurs finaux sur les marchés spot. Par conséquent, on constate qu'ils acquièrent une plus grande part de marché, dans la consommation européenne, en comparaison avec les traders. Ainsi, ce fort pouvoir de marché de la part des producteurs se fera sentir de plus en plus au niveau des prix spot, dès que la production européenne aura commencé à décliner, à partir de 2015.

– Le modèle GaMMES montre que les prix des contrats sont en général inférieurs aux prix spot. Cela s'explique de manière assez simple : d'un point de vue économique, les contrats long-terme représentent le seul moyen pour les traders de s'approvisionner auprès des producteurs. Ainsi, les prix LTC peuvent être considérés comme des coûts marginaux d'approvisionnement de long-terme pour les traders. Puisque le prix spot est directement relié au profit des traders, on déduit qu'une situation favorable de vente sur un marché de consommation se présente dès que le prix spot est supérieur au coût marginal d'approvisionnement, soit le prix LTC. En outre, le modèle GaMMES nous indique qu'un trader s'entend généralement avec les producteurs sur un même prix d'approvisionnement. Ceci se justifie par le fait que les producteurs n'exercent pas un pouvoir de marché explicite au niveau intermédiaire de l'échange long-terme.

– Le modèle S-GaMMES est une extension stochastique du modèle GaMMES où l'aléa est porté par le prix du pétrole. Etant donné les fortes fluctuations de ce dernier, nous l'avons modélisé comme une variable aléatoire dont la loi de probabilité a été estimée par une étude économétrique. L'avantage de S-GaMMES est qu'il permet de quantifier l'impact de telles fluctuations sur les marchés gaziers. Ainsi, il nous a été possible de réaliser des scénarii d'évolution des paramètres du marché en fonction de la trajectoire suivie par le prix du pétrole. Par ailleurs, l'impact des fluctuations sur les contrats long-terme a été appréhendé et quantifié. En effet, dans la mesure où les contrats long-terme permettent aux producteurs de couvrir leur risque d'investissement et aux traders d'assurer une fourniture sûre du gaz, la prise en compte du caractère aléatoire de la demande modifie sensiblement les paramètres des contrats. [3] Par conséquent, une comparaison entre des modélisations déterministe et stochastique au niveau des LTC nous a permis de comprendre comment l'incertitude (au niveau du prix du pétrole) modifie le comportement de couverture des producteurs. Finalement, nous avons défini et estimé une mesure de l'effet de la prise en compte des fluctuations du prix du pétrole dans les programmes d'optimisation des acteurs stratégiques et non stratégiques. Cette mesure (*value of the stochastic solution*) permet d'identifier les acteurs qui bénéficient de la prise en compte de l'aléa et ceux qui voient leur utilité décroître lorsqu'ils l'intègrent dans leurs décisions (ou subissent des pertes).

Bien entendu, les différents modèles que nous avons élaborés dans le cadre de cette thèse présentent quelques limites. La plus importante est sans doute la simplification de la réalité qu'ils engendrent dans la plupart des cas. Ainsi, l'hypothèse de rationalité des acteurs peut être remise en question puisque dans la réalité, elle peut être altérée notamment par des considérations d'ordre géopolitique dont il est difficile de rendre compte. Aussi, notre représentation de la structure économique et du fonctionnement du marché présente quelques difficultés. Au niveau des contrats long-terme, l'indexation directe de leurs prix sur le prix du pétrole ou du charbon n'est pas modélisée. Concernant les volumes, leur détermination est le fruit d'une négociation complexe et directe entre les producteurs et les traders. Dans le modèle GaMMES, nous laissons le marché décider à la fois des quantités et des prix grâce à une contrainte d'égalisation de l'offre et de la demande au niveau des LTC. Par ailleurs, les consommateurs sont représentés par leur fonction de demande inverse, ce qui ne leur accorde aucun pouvoir de déterminer directement leur consommation. Cependant, cette simplification semble nécessaire dans les problèmes de type concurrence à la Cournot. Au niveau du transport et du stockage, l'hypothèse d'absence d'exercice de pouvoir de marché en Europe reste discutable mais cette simplification permet de mieux comprendre les interactions stratégiques des autres agents.

Concernant notre étude de la sécurité d'approvisionnement, l'hypothèse la plus forte est sans doute celle qui conduit chaque producteur à vendre son gaz aux différents traders au même prix, et à interrompre son approvisionnement à l'ensemble de ses clients, en cas de rupture (ceci implique que nous considérons seulement les ruptures d'ordre technique). Cette hypothèse est néanmoins nécessaire si l'on souhaite résoudre analytiquement les équations du modèle.

Notre représentation des fluctuations du prix du pétrole utilise un arbre de scénarii issu d'une évolution du prix déterminée par une chaîne de Markov. Cette approche est bien entendu simplificatrice dans la mesure où elle implique une absence de "mémoire" (au-delà de deux périodes) dans la formation du prix du pétrole. Aussi, nous avons dû négliger d'autres facteurs nécessitant

3. Nous rappelons que le prix du pétrole influence la demande en gaz naturel à cause de la substitution énergétique.

une modélisation stochastique de la demande, tels que la saisonnalité. En outre, le caractère incertain ne concerne évidemment pas uniquement la demande. Ainsi, l'offre présente également une dimension aléatoire. A titre d'exemple, on peut évoquer les nouveaux producteurs de la mer Caspienne qui sont susceptibles de fournir l'Europe dans les prochaines années. A cet effet, le pipeline NABUCCO, dimensionné pour transporter 30 Bcm/an, sera opérationnel en 2017 et desservira plusieurs pays dépendant du gaz russe, tels que la Hongrie. En outre, la récente guerre en Lybie souligne à quel point la production du gaz naturel peut être incertaine, puisque beaucoup de pays assurant l'offre au niveau mondial sont politiquement instables.

En amont de l'industrie du gaz naturel, l'intuition économique nous indique que les producteurs exercent intrinsèquement plus de pouvoir de marché que les traders. A cet égard, il est possible de considérer les producteurs comme leaders du jeu et les traders comme suiveurs. Toutefois, le fait d'introduire un jeu dynamique à la Stackelberg complique fortement l'analyse de la détermination des ventes optimales. C'est pour cette raison que dans le modèle GaMMES, nous ne considérons que le cas d'un jeu simultané entre producteurs et traders.

Enfin, la représentation de la demande se concentre sur la substitution énergétique entre les trois énergies fossiles (gaz naturel, pétrole et charbon) et néglige les interactions avec d'autres formes de production énergétique telle que les renouvelables et le nucléaire. En réalité, concernant cette dernière, il nous a été très difficile d'obtenir des données publiques d'estimation directe du coût de l'utilisation du nucléaire (investissement + utilisation + démantèlement).

Au vu des différentes limites et difficultés reoncontrées dans notre étude, quelques extensions de ce travail de thèse sont possibles.

Au niveau de la structure économique des marchés, il serait intéressant de modéliser l'interaction stratégique entre les producteurs et les traders comme une concurrence à la Stackelberg où les producteurs seraient leaders et les traders suiveurs. Bien entendu, cette représentation doterait les producteurs d'une pouvoir de marché plus important que celui qui est pris en compte dans GaMMES. Toutefois, cette modélisation pourrait présenter un inconvénient majeur, à savoir une incohérence temporelle, qui constitue une particularité notoire des modèles de type Stackelberg. Cette dernière intervient lorsque la trajectoire optimale décidée par un joueur est susceptible de changer avec l'origine temporelle du programme d'optimisation. Par ailleurs, il est à noter que les techniques numériques de résolution de tels problèmes sont très peu nombreuses et font l'objet d'une part importante de la recherche actuelle en mathématiques appliquées. [4]

Sur la base de notre modèle GaMMES, il est possible d'intégrer de nouveaux gisements de gaz naturel afin d'analyser des situations de géopolitique européenne actuelle. Par exemple, il permettrait de connaître l'impact de la pénétration des gaz non conventionnels, en particulier le gaz de schiste sur l'évolution des marchés. Ce gaz non conventionnel a été découvert dans plusieurs pays de l'Union Européenne il y a quelques années. Les réserves les plus importantes seraient localisées

4. Une modélisation complexe de type Stackelberg fait intervenir des problèmes de type MPEC (Maths Programming with Equilibrium Constraints) ou EPEC (Equilibrium Problem with Equilibrium Constraints).

en Pologne, en Allemagne, en France et au Royaume-Uni. Aujourd'hui, l'exploitation du gaz de schiste en Europe se heurte à une forte contestation de l'opinion publique, essentiellement due aux techniques usuelles d'extraction. A l'instar des Etats-Unis, la production de gaz de schiste en Europe emploierait principalement la *fracturation hydraulique*, méthode présentant un impact très négatif sur l'environnement notamment à cause de sa forte pollution des nappes phréatiques. Toutefois, les motivations politiques et économiques restent très fortes pour certains pays désirant limiter leur dépendance énergétique vis-à-vis des producteurs usuels. Ainsi, une potentielle exploitation du gaz de schiste permettrait à la Pologne de produire la totalité de sa consommation, voire de devenir un nouvel exportateur. Cette situation changerait certainement la donne énergétique au niveau de l'Europe et aurait tendance à diminuer les prix et augmenter la consommation du gaz naturel, en favorisant la concurrence en amont. Le modèle GaMMES permettrait ainsi d'étudier et surtout de quantifier les divers effets relatifs à la possible pénétration du gaz de schiste en Europe. Cela serait possible si le modèle était calibré à une échelle plus large, incluant plus de pays consommateurs (tels que la Pologne). Cette étude nous paraît constituer une extension intéressante de notre modèle.

La substitution énergétique pourrait être élargie en considérant les énergies renouvelables ainsi que l'énergie nucléaire. En effet, une calibration de notre approche de type systèmes dynamiques incluant plus de types d'énergies constituerait également une extension importante de notre travail. Par ailleurs, la dépréciation anticipée des brûleurs dans le cas où le prix de marché associé devient très élevé peut être prise en compte directement dans le cadre de notre modélisation.

Notre traitement de la problématique de sécurité d'approvisionnement peut également être approfondi. Ainsi, l'introduction des risques de rupture à caractère politique, ainsi que la possibilité de mieux cibler les victimes de la crise constituent une extension intéressante de notre travail.

Enfin, nous avons résolu le modèle stochastique S-GaMMES dans sa forme extensive, sans chercher à utiliser des moyens numériques de réduction du temps de calcul. Par conséquent, une extension du périmètre géographique ou temporel de notre étude conduirait à un problème de très grande taille qu'il n'est pas aisé de résoudre directement. Ainsi, conviendrait-il d'employer des méthodes spécifiques de résolution, utilisant par exemple la décomposition (décomposition de Benders) ou la réduction de scénarii. L'application de ces techniques constituerait incontestablement une augmentation importante des possibilités d'utilisation de notre étude.

Le travail accompli lors de cette thèse constitue en définitive une base de modélisation, d'analyse et de réflexion permettant une meilleure compréhension du fonctionnement du marché du gaz naturel et ouvre la voie à un grand nombre d'extensions ou de raffinements.