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The economics and regulation of natural gas pipeline networks: four essays on the impact of demand uncertainty

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A mes parents et à ma famille.

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Summary

This PhD thesis is centered on the opportunities and impacts of demand uncertainty for the gas transmission network. We study the ability of various market designs to foster an efficient network allocation in liberalized gas markets when demand is variable or uncertain. We introduce and solve operation research models that bind an economic representation of the gas market and its associated regulation, to a technical representation of the gas network. The complex interactions at stake in liberalized gas markets, where shippers trade gas for its economic value and coordinate with system operators that allocate and operate the network, result in MCP or MPEC formulations. While a detailed network representation is necessary to assess the feasibility of gas flows under any market organization, the physics and engineering of gas transport networks adds non-linearities and non-convexities to those already challenging formulations. This thesis is divided in four contributions. We first introduce an approximated network representation of the Cobb-Douglas form and use it to study the impact of long-term demand uncertainty on investment problems in developing markets subject to rate-of-return regulation. We then study the effect of demand variability on daily gas dispatch in the European Entry-Exit system, using a linearized steady-state network representation. Finally, we assess the benefits of explicitly allocating network flexibility in gas locational marginal pricing auctions to handle intraday demand uncertainty. This requires the use of a linearized transient network formulation to account for linepack dynamics.

Keywords: Natural gas transmission network; Demand uncertainty; Rate-of-return regulation; Locational marginal pricing; Entry-Exit system; Transient gas flow.

Résumé

Cette thèse vise à développer les opportunités et conséquences d'une demande incertaine pour le réseau de transport de gaz. Ce sujet est décliné en quatre contributions. Les deux premières adoptent une perspective de long terme : on cherche à évaluer l'efficacité de la réglementation du taux rendement lorsqu'il s'agit d'inciter à la réalisation de projets d'infrastructures gazières dans des pays en développement. Une première contribution analytique présente le développement d'une représentation simplifiée du réseau de transport de gaz, de forme Cobb-Douglas. Inspiré par les projets d'acheminement de gaz naturel au Mozambique, celle-ci est ensuite utilisée pour évaluer dans quelles conditions il est possible pour une autorité de régulation de choisir un taux de rendement régulé qui améliore l'efficacité du système dans le cas où la demande réelle serait plus importante que la demande anticipée par la firme régulée. A moyen terme ensuite, l'efficacité face à une demande de plus en plus variable de la structure tarifaire actuelle dite « entrée-sortie » pour l'accès au réseau européen est évaluée. Après avoir démontré l'existence d'inefficacités dans un tel système, celles-ci sont évaluées numériquement. Enfin, la dernière contribution explore la possibilité d'offrir directement la flexibilité du réseau de transport de gaz à ses utilisateurs, dans le cadre d'enchères et du système de prix nodaux. Après avoir souligné la complexité d'un tel mécanisme, les limites à son efficacité sont présentées. A chaque fois, l'analyse repose sur la modélisation simultanée du réseau de transport de gaz (en régime statique ou transitoire) et des mécanismes économiques en jeu.

Mots-clés : Réseau de transport de gaz naturel ; Incertitude de la demande ; Régulation du taux de rendement ; Prix nodaux ; Système Entrée-Sortie ; Flux de gaz en régime transitoire.

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Chapter I. Introduction

I.1 Motivations

Being the fossil fuel with the lowest CO_2 emissions, natural gas is often seen as an important element of a successful energy transition. Natural gas resources are abundant worldwide, although often far from consumptions sites. Russian, Norwegian and Algerian fields, to name a few, are thousands of kilometres away from the main European regions where their gas is actually used. Thus, natural gas must often be transported over long distances, either in its gaseous form through pipelines, or in a liquified form (LNG) through tankers. When large recurring gas volumes are to be shipped over medium distances, pipeline gas transport remains the most efficient solution.

This infrastructure is highly capital-intensive though. New pipelines often represent investments of several billion euros. Moreover, once they are built, there transport capacity is more or less fixed. Thus, at the planning stage, the choice of the diameter of a pipeline almost completely defines its future maximum capacity. This irreversibility raises numerous challenges when the gas system faces an uncertain demand.

Uncertainty on demand is common for energy commodities and natural gas is no exception. On the long term, economic growth and cycles influence industrial consumption volumes. On the medium term, contingencies like seasonal temperature levels, also impact domestic gas use. On the short term, weather may locally induce an increase or a decrease in gas demand. The diversity of gas consumers, from industrial manufacturing facilities and gas-fired power plants to small industrial and domestic consumers, creates a variety of uncertainty sources for gas demand.

Additionally, the energy transition of the electrical industry relies on the massive introduction of renewable technologies, for instance in China or in Europe, as stated by the policy goals of the European Union. Intermittent renewable technologies, such as wind power turbines, and solar panels are already largely deployed and are likely to be ramped up even more in a near future. This raises important challenges for the proper balancing of the power system. For a number of reasons, renewables intermittency will be mainly backed-up by conventional thermal units, and especially combined cycle gas turbines, due to their lower CO_2 emissions, relatively low investment cost and most important, their high flexibility. As a result, the coupling between the gas and the electrical systems is likely to increase as intermittent renewables develop, transferring even more uncertainty to the gas system.

This raises issues at all stages of pipeline development and operation. At the design phase, sizing a pipeline is difficult. During the life of the pipeline, demand may grow (for instance in a country exploiting a new resources) or decrease (due to energy substitutions). When in operation then, the network must be shared between all users. While this is already a technically complex task for integrated utilities, it is also an economic challenge in liberalized markets. Network users must have the possibility to express their needs for gas transport in a clear manner, in coordination with gas trades.

The services provided through the pipeline infrastructures are diverse and inseparable though. Markets or utilities must allocate the different characteristics of the network such as flow and linepack storage. But gas flow physics makes injection and withdrawal decisions linked spatially and temporally, and a dispatch decision at given time may impact all other users for the following hours. Thus, network allocation must be made in a coordinated way, which is natural for integrated utilities but less easy to cast in a market context.

We believe that the questions of network allocation, pipeline investment and market organization in a context of uncertain demand can be analysed in depth only through modelling of the network and economic decisions simultaneously. This is what we try to do in this thesis, for a few specific questions tied to this broader economic topic. Those questions are of interest for all the stakeholders of the gas system, from gas producers to electricity suppliers, from transport system operators to final consumers, as well regulatory bodies and development agencies.

I.2 Contributions and organization of the dissertation

I.2.1 Chapter 1: Review of gas network formulations

In this first chapter, we briefly review the literature addressing physics formulations used for representing pipeline gas transport networks, and their application in operations research and economic models. We organize the review around three categories of formulations, from the simplest but less accurate fixed capacity model, to the more detailed stationary and transient formulations. After recalling the main physics equation ruling gas flow, we provide references in the technical and economic literature that employ these representations. When variants of these formulations exist, we offer suggestions regarding their domain of validity and their usefulness in market-oriented models.

I.2.2 Chapter 2: The technology of a natural gas pipeline, insights for costs and rate-of-return regulation

This chapter details a complete microeconomic characterization of the physical relationships between input use and the level of output of a simple point-to-point gas pipeline system and uses it to contribute to the public policy discussions pertaining to the regulation of natural gas pipelines. We show that the engineering equations governing the design and operations of that infrastructure can be approximated by a single production equation of the Cobb-Douglas type and use that result to inform three public policy debates. First, we prove that the long-run cost function of the infrastructure formally verifies the condition for a natural monopoly thereby justifying the need of regulatory intervention in that industry. Second, we examine the conditions for cost-recovery in the short-run and contribute to the emerging European discussions on the implementation of short-run marginal cost pricing on interconnector pipelines. Lastly, we document the performance of rate-of-return regulation in that industry and inform the regulatory policy debates on the selection of an appropriate rate-of-return. In particular, we highlight that, contrary to popular belief, the socially desirable rate of return can be larger than the market price of capital for that industry.

The simplified representation of pipeline gas transport presented in this chapter is then used to study the impact of long-term demand uncertainty on infrastructure development projects in Chapter IV and the influence of medium-term uncertainty on the performance of the European gas market regulatory framework in Chapter V.

This chapter is joint work with Olivier Massol. It has been presented at the student workshop of the French Association of Energy Economists (December 2016) and has been submitted to Utilities Policy in a short-communication format.

I.2.3 Chapter 3: Unlocking natural gas pipeline deployment in a LDC, a note on rate-of-return regulation

This third contribution addresses the impact of long-term demand uncertainty on the regulation of pipeline infrastructure projects. We examine the economics of the deployment of a natural gas pipeline in a developing region where no such infrastructures exist and rate-of-return regulation has been implemented. The situation studied, described as follows, is motivated by the case of a real pipeline project in northern Mozambique. At the planning stage (i.e., *ex ante*), we first briefly review the behavior of a foreign private firm that considers conservative demand assumptions and neglects the possible future emergence of currently-embryonic users to gauge the infrastructure. We then analyze the *ex post* behavior of the firm knowing that, once installed, the capital invested in the pipeline is fixed and cannot be adjusted if that larger demand materializes. Using the simplified representation of pipeline gas transport introduced in the previous chapter, we analytically prove how a regulatory agency can decide the allowed rate of return so as to leverage on the behavior of the regulated firm (i.e., the Averch-Johnson effect) to obtain *ex ante* the adapted degree of overcapitalization needed *ex post* to serve the demand in a cost-efficient manner. We finally discuss the conditions that make such a strategic use of rate-of-return regulation compatible with the traditional public policy objective of protecting society from monopoly prices.

This chapter is joint work with Olivier Massol. It has been presented at different development stages at the 10th and 11th Annual Trans-Atlantic Infraday conference (2015 & 2016, FERC, Washington), at the INFORMS conference (2015, Philadelphia), as well as at the student workshop of the French Association of Energy Economists (December 2016). It will soon be submitted to an international journal.

I.2.4 Chapter 4: Identifying inefficiencies in an Entry-Exit gas system

In this following chapter, we examine the efficiency of the current regulatory framework applied to the liberalized European gas market in a context of increased variability and uncertainty of demand.

The European Entry-Exit gas system regulates access to the gas network and organizes the commodity market. To do so, it separates gas trading from physical gas flows. While this fosters the liquidity of gas markets, it prevents from efficiently using the gas network. We present a bilevel model of a single-zone Entry-Exit gas system where a network operator sets tariffs and capacity limits and anticipates the economic decisions of gas shippers, which includes the stationary network representation introduced in 0. We compare it to the dispatch of an integrated utility and provide analytical and numerical results to document the sources and magnitudes of possible inefficiencies. We show that an Entry-Exit system is inefficient as soon as tariffs and capacity limits are set identically for more than two different demand instances. The enforcement of cost-reflective tariffs further hampers this prospect. Finally, we highlight that the choice of capacity limits based on technical grounds instead of economic ones is another distortion factor. Numerical results obtained on simple settings exhibit small inefficiencies, but also forcibly demonstrate the need to realize such case studies for existing Entry-Exit markets, where inefficiencies may be larger.

This work has been presented at different development stages at the EURO conference (2015, Glasgow), at the Young Energy Economists and Engineers Seminar (YEEES, June 2015, Paris), at the summer student seminar of the French Association of Energy Economists (2017, Paris) and at the conference « New Research Perspectives for a Rapidly-Changing World » (June 2017 Paris). It will also be soon submitted to an international journal.

I.2.5 Chapter 5: What short-term market design for efficient flexibility management in gas systems?

In this last chapter, we investigate the opportunity to better manage short-term demand uncertainty by explicitly offering the linepack flexibility of the gas network through locational marginal prices auctions.

As said before, with the increase of electric intermittent renewables, often backed-up by gas-fired power plants, variability has been transferred to the gas system. Balancing the network has become a technical and economic issue. The unique capability of gas transport networks to store gas, called linepack storage, is an advantage to deal with demand fluctuations, but its use raises a number of questions in a market environment. In currently implemented market designs, the valuation and control of linepack storage, and of the associated flexibility services, are solely in the hand of the system operator. Taking an opposite view, some authors have developed qualitative arguments in favor of a direct, market-based allocation of linepack flexibility. To clarify this debate, we discuss the opportunity of selling linepack flexibility services in a locational marginal pricing framework (LMP). We introduce an equilibrium model of day-ahead LMP auctions where network users face uncertainty and bid for firm as well as flexible products. To clear the market, the operator checks the feasibility of all network configurations resulting from the various possible real-time uses of the allocated flexibility services. We compare this design to the traditional LMP implementation with intraday markets. Using an approximate transient network representation, we compute numerical results on simple network cases. We show that while there is value in offering linepack flexibility to the market, successful implementation requires a careful and complex design of flexibility products, as well as solving difficult mathematical problems in real time. Moreover, benefits are conditioned to the specificities of the market, and especially to network topology and the structure of the uncertainty of consumers' demand profiles. Therefore, we conclude that the implementation of such a system should not be contemplated unless a thorough assessment of its benefits and costs has be conducted for each specific market.

This chapter has been presented at different development stages at the INFORMS conference (2015, Philadelphia), and at the EURO conference (2016, Poznan). It will soon be submitted as well to an international journal.

Chapter II. Review of gas network formulations

II.1 Introduction

A wide variety of problems, whether motivated by technical or economics concerns, require taking into account the physics of gas transport through pipelines. A vast literature develops mathematical models which can yield numerical solutions to answer such questions. When this path is chosen, the modeler must choose the equations that will represent gas transport. This is not an easy task, since gas flows are characterized by complex equations that can easily lead to intractable models when applied to real-life problems or when used in complex modeling framework such as MCP or MPEC. Consequently, a modeler must pick a flow formulation adapted to its particular questions. This choice will depend on the level of detail in representing gas transport that is needed to answer the question satisfyingly.

For each possible formulation of gas transport through pipelines, this review gives a few examples of its use in the literature. We provide references that are focused on technical, as well as economic questions when available.

Readers must be aware that three literature review on gas models with technical representation of gas transport already exist. They may well better suit their particular needs.

- (Rıos-Mercado and Borraz-Sánchez 2012) introduce the main technical modeling problems in the field of pipeline gas transportation: flow estimation, gas quality satisfaction, and fuel cost minimization. They review **the numerical methods** developed in the literature to solve such problems.
- (Zheng et al. 2010) review the optimization models proposed in the gas literature and **segment their review along the categories of application** of those models: gas production, gas transport through pipeline networks and gas markets.
- (Brouwer et al. 2011) go back to the physics equations for the transport of gas through a single pipe. They **derive various equations that represent transient gas flow** in different situations, depending on the assumptions made. Equations are proposed for non-isothermal models of gas transport as well as for the more classical isothermal models. Whenever available, works extracted from the technical literature on gas transport are associated to the formulation they use. Finally, numerical comparisons are proposed between isothermal and non-isothermal models.

Although those three reviews explore the broader field of gas system modeling and sometimes cite common references, they are not redundant and develop three different approaches. (Rıos-Mercado and Borraz-Sánchez 2012) mainly describe the numerical methods to solve gas problems, and therefore ignore models aimed at answering market-related questions. On the contrary, (Zheng et al. 2010) do review market-oriented models. However, recent works that appeal to transient flow representation to answer those questions are not included. Finally (Brouwer et al. 2011) give a broader and unified view on transient flow equations for gas transport in pipelines, but do not mention any economics-oriented applications of those equations.

In this review, our aim is to add a missing and recently developed part of the literature, namely economic models that include a transient representation of the gas network. We also structure it in an original way by exploring successively fixed-capacity, steady-state, and transient representation of gas flows. This may be of interest for certain readers, and especially to modelers interested in economic questions that require the use of transient flow models or who may not necessarily be familiar with the physics of gas transport.

II.2 General equations of gas transport through pipelines

In this first section, we briefly introduce the main physics equations necessary to model gas transport through pipelines, and how they fit in a more general description of pipeline gas transport. By referring to gas, we mean the various gaseous blends of methane, of diverse purity and composition, used around the world mostly for combustion usage in industrial and domestic context. This encompasses natural gas, biogas and manufactured gas.

II.2.1 Gas transport through a single pipeline

The main variables describing gas flow are ρ, $\vec{u}$, p and T, respectively the gas local density, velocity, pressure and temperature. Four equations are needed to describe the state of those variables in a flowing gas. Following (Brouwer et al. 2011), we detail them thereafter for a homogenous gas[1] flowing through a cylindrical pipe of diameter D, applying the one-dimensional assumption. Hence, each local variable is actually averaged over the cross-section of the pipe, and therefore depends only on time t and position along the pipe x. These equations remain valid as long as the length of the pipe is large against its diameter. Although we present them here to give an overview of the phenomena ruling gas pipeline transport, their full understanding is not a requirement to the next section. Interested readers can find more details in (Menon 2005).

State equation

The state equation is the first essential relationship for describing gas flow. It shows how gas temperature, density and pressure relate locally. We first present this equation for the *ideal gas*, and then for real gases.

[1] See (Selot 2008) for non-homogeneous flows.

At sufficiently low pressures, the properties of all gas blends tend toward those of the *ideal gas* model. In such a theoretical gas, particles can be described as points with no volume, interacting only through perfectly elastic collisions. Its properties are summed up in the state equation of ideal gas (1), where R is the universal gas constant.

$$p = \rho R T \tag{1}$$

Although the *ideal gas* is a good approximation for a number of real gases in low pressure conditions, the ideal gas law does not stand for gas transported in pipelines, given the interactions appearing at higher pressure and its inhomogeneous composition. Its behavior is better described by the state equation of real gases (2). This equation differs from the ideal gas state equation by the introduction of the compressibility factor z, which depends on the gas density and temperature.

$$pv = z(\rho, T) \cdot RT \tag{2}$$

The compressibility factor is empirically defined and varies depending on gas composition. See (Menon 2005) for a review of various formulations of the compressibility factor for gas.

Momentum equation

The dynamic behavior of gas is described by the momentum equations for viscous fluids, also called Navier-Stokes equations, or equations of motion. They link the variation in time of the momentum of a gas particle to the forces applied to this particle. For a flow through a pipe and in our one-dimensional approximation this comes down to equation (3). This equation gives essential information about the flow dynamics.

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} = -\rho g \frac{\partial h}{\partial x} - \frac{\partial p}{\partial x} - \frac{f}{2D}\, \rho\, u|u| \tag{3}$$

The terms, listed from left to right, represent the change of momentum in time at a given spot, the change of momentum in space at a given time (which is also referred to as the convective or kinetic term), gravitational forces (which matters if the pipe is sloping and where g is the gravitational constant), pressure forces and the effect of friction forces.

Although an analytical expression of the friction forces can be derived for mild flow conditions (when the flow is said to be laminar), this is not the case for the regime in which most of gas transport occurs, i.e. when the flow is partially or fully turbulent. In this case, a formulation based on the empirical Darcy friction factor[2] f is used.

[2] The Darcy friction factor is also sometimes called Moody friction factor. It can easily be confused with the very similar Fanning friction coefficient, which is exactly one fourth of Darcy friction factor.

Various empirical models have been proposed to describe the friction factor depending on the flow regime, that can be characterized mainly by the Reynolds number Re and the roughness of the pipe ε. The implicit Colebrook-White formulation (4), and the explicit Weymouth formulation (5) are historical examples[3] of friction factors formulations used for high pressure gas pipelines. However, many more exist and new friction factor definitions are regularly developed (Piggott et al. 2002).

$$\frac{1}{\sqrt{f}} = -2\, log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\,\sqrt{f}}\right) \tag{4}$$

$$\frac{1}{\sqrt{f}} = 10.3196\, D^{\frac{1}{6}} \tag{5}$$

Detailed reviews of the different flow conditions and discussions of existing friction factors can be found in (Coelho and Pinho 2007, Menon 2005). The Moody diagram is usually used to represent and compare the friction factors for different Reynolds number and pipe roughness.

Continuity equation

The third equation necessary to describe gas flow is the continuity equation (6). It can be viewed as a mass balance equation along the temporal and spatial dimensions of the flow. When integrated over a pipe section, it means that the change in the mass of gas in the section is equal to the difference between the mass entering the section and the mass of gas leaving it at a given time.

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \vec{u}) = 0 \tag{6}$$

Energy equation

Finally, the last equation is the energy equation (7). It shows how energy transfers occur in the fluid and at its border (i.e. with the pipe walls). It involves the specific heat capacity of gas c_v, which is defined as the energy needed to raise the temperature of the gas particles, and k_w, which characterizes how heat is exchanged between the pipe wall and the gas. Once again, depending on the accuracy required, various models exist for the specific heat capacity.

$$\frac{\partial}{\partial t}\left(c_v \rho\, T + \frac{\rho u^2}{2} + g\rho h\right) + \frac{\partial}{\partial x}\left(u\left(c_v \rho\, T + \frac{\rho u^2}{2} + g\rho h\right) + u\, p\right) = -\frac{k_w}{D}(T - T_w) \tag{7}$$

II.2.2 Gas transport through pipeline networks

The local equations presented above are the basic material used to describe gas flow through a single pipeline. However, to achieve transportation of gas from multiple sources to multiple delivery points, whole networks have been developed. They are made of a high number of pipes, mainly made of steel and with a diameter comprised between about 500 mm and 1200 mm, operated at pressures varying from 600 psi to 1500 psi. They move gas on distances usually of the order of several hundreds

[3] Hence not the most accurate.

to thousands of kilometres to directly supply gas to consumers (mainly industrials) or to deliver it to distribution networks. Those distribution networks, operated at much lower pressures are then in charge of supplying residential and smaller business consumers.

When modelling such networks made of multiple pipes, as well as other elements such as compressor stations, storages or LNG terminals, many other equations may be required. Since this review is focused on flow formulations, we just give a very brief description of those other elements and bibliographic references.

In a network made of multiple pipes, the previous equations can be used to describe the flow in each individual pipe. At the nodes connecting pipes, supplementary equations are necessary to describe how the flow variables at the end of the different pipes are linked. Two main assumptions are usually made: a) the mass is conserved at a node, b) the pressure is unique at a node. Other equations can be required depending on the formulation chosen for the flow equation, for example to characterize the uniqueness of the density at a node. See (Brouwer et al. 2011) for more details.

Before presenting a general version of those nodal equations, we introduce the mass flow q in equation (8), as the product of the gas density, the area of the pipe cross-section S. We use it from there on, as it is more convenient than velocity when considering industrial applications. Volumetric flow is also sometimes used (Coelho and Pinho 2007), as well as energy flow (Pepper et al. 2012), especially when changes in gas composition must be taken into account.

$$q = \rho S v \tag{8}$$

Concerning the network, we use the following notations. It is defined as in (De Wolf and Smeers 2000) by the pair (N, A), with N the set of nodes and $A \subseteq N \times N$ the set of pipes from and to those nodes. An individual pipe $a(i,j) \in A$ connects node $i \in N$ to node $j \in N$. We identify each flow variable of a particular node by the subscript i and of a particular pipe by the subscript $a(i,j)$. When flow variables can take different values at the entry and at the exit of pipe, we add subscript i to denote the value of the variable at the node i. Hence, $q_{i,a(i,j)}$ is the flow in pipe $a(i,j)$, at the end connected to node i. Finally, pipes are directed, meaning that gas is flowing from i to j (resp. from from j to i) when $q_{a(i,j)}$ is positive (resp. negative).

Using those notations, nodal conditions a) and b) can be written as in equation (9) and (10).

$$\sum_{j \in \mathcal{N} / a(j,i) \in \mathcal{A}} q_{j,a(j,i)} = \sum_{j \in \mathcal{N} / a(i,j) \in \mathcal{A}} q_{j,a(i,j)} \qquad \forall i \in \mathcal{N} \tag{9}$$

$$p_{i,a(i,j)} = p_i \qquad \forall i \in \mathcal{N}, \ \forall j \in \mathcal{N} / a(j,i) \in \mathcal{A} \tag{10}$$

At some nodes however, those assumptions may not hold, for instance when valves or pressure reducing capabilities are present in the network. Specific equations are then required (Pepper et al. 2012, Schmidt et al. 2012, p. 1).

Compressors stations are also an essential element when representing gas transport networks as they increase pressure and enable to increase the gas flow along a pipe. Describing compressor stations accurately is a particularly difficult task though, as each station may be composed of several compressor units, which can in turn use different technologies (e.g. reciprocal or centrifugal compressors), each involving non-linear equations and integer decision variables. While we concentrate here on the choice of a flow formula, interesting readers can find detailed accounts of different approaches to model compressor stations in (Carter 1996, Jenicek and Kralik 1995, Menon 2005, Odom and Muster 2009, Osiadacz 1980, Rose et al. 2016).

In this section, we have presented the main physics equations describing gas flow. To obtain simpler formulations adapted to modelling purposes, various assumptions can be made. Whatever the formula chosen, they all account for the facts that pressure decreases as gas moves along a pipe and, that increasing the pressure difference between the beginning and the end of a pipe is necessary to increase its throughput. In the next section we review the main existing flow formulations, their underlying assumptions as well as their uses that were documented in the literature.

II.3 Flow formula of gas transport used in OR models

Operation research (OR) models have proven to be extremely valuable tools to improve the technical operation of gas networks, as well as to gain insights on the economics of gas transport and gas market organization. However, given the complexity involved in modelling real-size networks and the mechanics of economic decisions, it is crucial to adapt the choice of a flow formula to the purpose of the model.

While it is obvious that models aimed at improving network operation require a detailed technical representation of the network, some market design questions are also heavily influenced by the dynamics of network. Introducing a set of equations representing the gas network to the model is then necessary as well. However, using the equations presented above as such, in large scale network models or in advanced economic formulations, can lead to intractable problems. Therefore, a compromise must be made between the tractability of the overall model and the accuracy of the network representation incorporated in it.

On the one hand, gas network *simulation* models have been developed for more than 40 years. Since many variables are fixed (such as flows entering and exiting the network, as well as compressors operation variables), the problems are less demanding in terms of computing power, although still quite challenging, and can be directly built upon the equations presented above. Hence, highly detailed simulation models are available and provide quite accurate descriptions of real-size gas networks (Kralik et al. 1988). After a necessary adaptation phase (Scott and Whaley 2000), models are now routinely used by gas network operators worldwide. On the other hand, *optimisation* models, which are the focus of this review, are much tougher problems: virtually all variables are decision variables and the objective of such models is not only feasibility anymore, but to minimize or maximize a function of those variables. Efforts to develop such models date back to the 1960s for steady-state flows (Wong and Larson 1968) and the 1980s for transient ones (Dupont and Rachford 1987, Mantri et al. 1985). The liberalization process, fundamentally changing the role of TSOs also tremendously increased the need for such models (Lloyd et al. 2006, Pfetsch et al. 2014, Rossi et al. 2014). Therefore, one must acknowledge the achievements of (Koch et al. 2015) in the recent years in steady-state optimization of gas networks. However, a lot remains to be done given the difficulties at play, especially regarding the impact of the network on market organization. We hope that this review will be a useful tool to map the existing literature and guide interested researchers new to the field of gas or gas network modelling interested in advancing this research.

We now review the various network representations used in the literature, from the simplest fixed capacity model, to the most common steady state model and up to the more complex transient models. We refer the reader to the references provided for detailed formulations.

II.3.1 Fixed capacity

The simplest way to represent gas flow through pipelines is to assume that pipes have a fixed maximum capacity K under which any amount of gas can be transported, as stated in equation (11). This capacity relates to the maximum pressure difference acceptable between the inlet and the outlet of the pipe.

$$q \leq K \tag{11}$$

This fixed capacity model is obviously not accurate, as it disregards the physics of gas transport, and especially the relationship between flow and pressure. However, it provides a simple assessment of gas flows when accuracy is not essential, such as economic models which focus on commercial gas trading rather than transport infrastructures.

Large scale models which describe market interactions between consumers, producers, storage operators and transporters are developed for the US in (Gabriel et al. 2005), or for the European market in (Abada et al. 2013, Egging et al. 2008). (Abrell and Weigt 2011) propose a model representing the European electricity and gas markets combined, where the network is also represented through fixed capacities. (Cremer et al. 2003, Lochner, Stefan 2009) use this simplistic network representation to derive nodal prices for gas.

II.3.2 Steady-state representation

While it goes without saying that using OR models to improve the technical operation of a pipeline infrastructure requires a certain degree of accuracy when modelling gas flow, (Midthun 2007, Midthun et al. 2009) has shown how important using a realistic gas flow model can be when studying economics oriented questions. The first step in this direction is to use a steady-state flow formulation. It is obtained from the initial physics equations by assuming that all the variables describing gas flow remain constant over time, i.e. that the flow has reached an equilibrium state.

Isothermal models

Assuming that the flow is isothermal, i.e. that the temperature profile of the flow does not change over time, and that friction forces are the main interactions determining the gas flow, a steady-state flow equation can be derived (Brouwer et al. 2011, Finch and Ko 1988, Schroeder Jr 2010). The main consequence to this representation is that the gas flow at the entry of the pipe is equal to the gas flow at its exit (otherwise gas would accumulate in the pipe, modifying the dynamics of the flow). While it is non-linear and non-convex, especially when flow direction is allowed to change, this formulation is simpler than the transient formulation, while it can be used to obtain rather accurate flow description. Consequently, it naturally appeals to both technical and market-oriented literature streams, when flows can be considered to change only slowly over time. The formulation of this equation largely depends on the friction factor used (Coelho and Pinho 2007, Menon 2005).

Example of stationary gas models oriented at technical network management can be found, among many others, in (Pfetsch et al. 2014), for validating the nominations of network users, and with different levels of detail (De Wolf and Smeers 2000, O'Neill et al. 1979, Schmidt et al. 2012, 2014) for network planning. First attempts to use this representation to gain insights on the economics of gas pipeline transmission dates back to the 1940s (Chenery 1949), and have been extended since (Massol 2011). Network investment and reinforcement problems have been addressed (André and Bonnans 2011, Babonneau et al. 2012, De Wolf and Smeers 1996, Maugis 1977, Yépez 2008). More recently, this formulation was used to study the opportunity of introducing interruptible capacities in the European gas market (Fodstad et al. 2015). Finally (An 2004, Arnold and Andersson 2008) integrate this network representation into multi-energy models, which compute a joint dispatch for gas and electricity flows.

Non-isothermal models

In non-isothermal representations, the temperature profile of the gas flow is no longer assumed to be constant. While taking into account the temperature dynamics in gas flows is important to get accurate results (Chaczykowski and Osiadacz 2012), it is likely not to change the nature of the interaction between the gas network and its market. Given the complexity it adds, no authors have yet used such models in models studying market organization. On the technical side, they were notably used for network planning (Steinbach 2007), or to assess network capacities (Koch et al. 2015).

II.3.3 Transient representation

Although the steady state approximation can be accurate enough for certain scenarios of pipeline operation, the gas flow through pipelines actually almost never reaches equilibrium under usual operating conditions. Consequently, it is important to model the dynamics of gas transport when a finer control of the network is required or when one tries to define a market organization that complies with the network constraints. However, as said before, using the equations previously presented as such leads to models highly difficult to solve for real life applications. Therefore, simpler, approximated transient models can be derived by observing that two main phenomena drive gas transport. One is the energy transfer between the molecules of the fluid themselves and between the fluid and the pipe walls. The other is the friction between the molecules and the pipe walls. Depending on the relative magnitude of those effects, some equations of the full physics model can be simplified. Contrarily to the fixed-capacity and the steady-state models, a single equation modelling only flow is not enough anymore. Other equations must account for the change in the mass of gas inside the pipe over time and space.

Non-isothermal models

When energy transfer phenomena are considered to be important, the equations presented in Section II.2 can be simplified to lead to non-isothermal, transient models of gas transport. (Brouwer et al. 2011) derive such non-isothermal models for pipeline gas transport. To our knowledge, these have not been used yet for gas transport applications. Although (Osiadacz and Chaczykowski 2001) have shown that modelling non-isothermal transient flows could be important, we believe that thermal considerations would not fundamentally change the impact of the gas transport network on economic organisation of the gas market.

Isothermal models

In the case of underground pipe networks for instance, energy transfer has not much impact on gas transport, as gas reaches the pipe wall temperature quickly, which remains mostly constant and equal to the soil temperature. In this example and others, friction plays a much more important role in the dynamics of gas transport than energy transfer. Based on this assumption, the energy equation states that the temperature profile of the fluid is time-independent. The equations presented in Section II.2 already become less complex under this assumption can be used as such (Mahlke et al. 2007). However, further assumptions on the timescale of flow variations allow to make them more tractable for market-oriented models. For the detailed formulations of each variant, we refer the reader to (Brouwer et al. 2011) or to the following references.

A first simplified formulation can be derived when the change of momentum in time and the convective term can be neglected. This is acceptable when flows variations happen at a relatively slow pace (such as hourly intervals). Given its better tractability, it has been used in technical models as well as economic ones. Technical works include (Correa-Posada and Sánchez-Martín 2014) who compare various solving methods for a linearized version of this model.

This formulation has recently been used in a number of works that investigate the impact of transient effects on the organization of the gas market. (Midthun et al. 2009) show on simple networks how these can influence prices and flows in a market context. (Keyaerts 2012, Keyaerts et al. 2014) use this formulation to analyse the efficiency of linepack management and balancing mechanisms in the European gas market. (Read et al. 2012) elaborate an auction design to implement nodal price allocation in a gas network, based on realistic transient flows. (Chaudry et al. 2008) propose a combined model of the electricity and gas market which takes into account transient gas flows and use it to study the impact of intermittent electric renewables on both the electric and the gas system (Qadrdan et al. 2010).

When the flow can still be considered as isothermal but the previously mentioned terms cannot be neglected, another formulation is obtained. It is more complex though. (Moritz 2007) developed a detailed transient optimization of the gas network based a linearization of this formulation and explored several solving methods (Domschke et al. 2011). (Banda and Herty 2008) use this formulation in conjunction with others, including stationary ones, to model a network using the most adapted gas equations for different types of network elements.

II.4 Linearization

Given the difficulty to solve those non-linear non-convex models, linearization techniques, among others, can be applied. In addition to (Correa-Posada and Sánchez-Martín 2014, Moritz 2007) for the transient case, we also refer interested readers to (van der Hoeven 2004, Möller 2004) for the stationary case.

II.5 Conclusion

While pipeline gas flow is a highly complex problem, advances in mathematics and programming have made it possible to approach it quite accurately through simulation and optimization models. Simplified formulations have also been developed to address both stationary and transient flows, and those can now be embedded into economic models, to answer market design and performance questions. Combining those technics to achieve the modelling of real networks in complex market environments could now unlock many more research opportunities.

II.6 References

Abada I, Gabriel S, Briat V, Massol O (2013) A Generalized Nash–Cournot Model for the Northwestern European Natural Gas Markets with a Fuel Substitution Demand Function: The GaMMES Model. *Networks and Spatial Economics* 13(1):1–42.

Abrell J, Weigt H (2011) Combining Energy Networks. *Netw Spat Econ* 12(3):377–401.

An S (2004) *Natural Gas and Electricity Optimal Power Flow*. (Oklahoma State University).

André J, Bonnans JF (2011) Optimal structure of gas transmission trunklines. *Optimization and Engineering* 12(1–2):175–198.

Arnold M, Andersson G (2008) Decomposed Electricity and Natural Gas Optimal Power Flow. (Glasgow).

Babonneau F, Nesterov Y, Vial JP (2012) Design and Operations of Gas Transmission Networks. *Operations Research* 60(1):34–47.

Banda MK, Herty M (2008) Multiscale modeling for gas flow in pipe networks. *Math. Meth. Appl. Sci.* 31(8):915–936.

Brouwer J, Gasser I, Herty M (2011) Gas Pipeline Models Revisited: Model Hierarchies, Nonisothermal Models, and Simulations of Networks. *Multiscale Model. Simul.* 9(2):601–623.

Carter RG (1996) Compressor Station Optimization: Computational Accuracy And Speed. (Pipeline Simulation Interest Group).

Chaczykowski M, Osiadacz AJ (2012) Comparative assesment of steady-state pipeline gas flow models / Analiza porównawcza modeli przepływu gazu w rurociągu w stanach ustalonych. *Archives of Mining Sciences* 57(1):23–38.

Chaudry M, Jenkins N, Strbac G (2008) Multi-time period combined gas and electricity network optimisation. *Electric Power Systems Research* 78(7):1265–1279.

Chenery HB (1949) Engineering Production Functions. *The Quarterly Journal of Economics* 63(4):507–531.

Coelho PM, Pinho C (2007) Considerations about equations for steady state flow in natural gas pipelines. *Journal of the Brazilian Society of Mechanical Sciences and Engineering* 29(3):262–273.

Correa-Posada C, Sánchez-Martín P (2014) Gas Network Optimization: A comparison of Piecewise Linear Models. *Optimization Online*.

Cremer H, Gasmi F, Laffont JJ (2003) Access to Pipelines in Competitive Gas Markets. *Journal of Regulatory Economics* 24(1):5–33.

De Wolf D, Smeers Y (1996) Optimal Dimensioning of Pipe Networks with Application to Gas Transmission Networks. *Operations Research* 44(4):596–608.

De Wolf D, Smeers Y (2000) The gas transmission problem solved by an extension of the simplex algorithm. *Management Science* 46(11):1454–1465.

Domschke P, Geißler B, Kolb O, Lang J, Martin A, Morsi A (2011) Combination of nonlinear and linear optimization of transient gas networks. *INFORMS Journal on Computing* 23(4):605–617.

Dupont TF, Rachford HH (1987) Optimization of Power Usage In Transient Gas Transmission Lines. *PSIG Annual Meeting*.

Egging R, Gabriel SA, Holz F, Zhuang J (2008) A complementarity model for the European natural gas market. *Energy Policy* 36(7):2385–2414.

Finch C, Ko DW (1988) Tutorial-Fluid Flow Formulas. *PSIG Annual Meeting*.

Fodstad M, Midthun KT, Tomasgard A (2015) Adding flexibility in a natural gas transportation network using interruptible transportation services. *European Journal of Operational Research* 243(2):647–657.

Gabriel SA, Zhuang J, Kiet S (2005) A large-scale linear complementarity model of the North American natural gas market. *Energy Economics* 27(4):639–665.

van der Hoeven T (2004) *Math in Gas and the art of linearization*. (Rijks Universiteit, Groningen, The Netherlands).

Jenicek T, Kralik J (1995) Optimized Control of Generalized Compressor Station. (Pipeline Simulation Interest Group).

Keyaerts N (2012) *Gas Balancing and Line-pack Flexibility. Concepts and Methodologies for Organizing and Regulating Gas Balancing in Liberalized and Integrated EU Gas Markets. (Gasbalancering en netwerkflexibiliteit. Concepten en methodologieën voor de organisatie en regulering van gasbalancering in vrijgemaakte en geïntegreerde EU gasmarkten.)*. (KU Leuven).

Keyaerts N, Delarue E, Rombauts Y, D'haeseleer W (2014) Impact of unpredictable renewables on gas-balancing design in Europe. *Applied Energy* 119:266–277.

Koch T, Hiller B, Pfetsch ME, Schewe L eds. (2015) *Evaluating Gas Network Capacities* (Society for Industrial and Applied Mathematics, Philadelphia, PA).

Kralik J, Stiegler P, Vostry Z, Zavorka J (1988) Simone - a Universal Dynamic Simulation Model of Gas Pipeline Networks.

Lloyd M, VanZelfden J, Brodsky A, Tsai M (2006) Tennessee Gas Pipeline's experience with optimization. *PSIG Annual Meeting*.

Lochner, Stefan (2009) Nodal prices, capacity valuation and investments in natural gas markets: Overview and analytical framework. *EWI working paper* (09 02).

Mahlke D, Martin A, Moritz S (2007) A simulated annealing algorithm for transient optimization in gas networks. *Math Meth Oper Res* 66(1):99–115.

Mantri VB, Preston LB, Pringle CS (1985) Transient Optimization Of A Natural Gas Pipeline System. *PSIG Annual Meeting*.

Massol O (2011) A Cost Function for the Natural Gas Transmission Industry: Further Considerations. *The Engineering Economist* 56(2):95–122.

Maugis JJ (1977) Etude de réseaux de transport et de distribution de fluide. *RAIRO-Operations Research-Recherche Opérationnelle* 11(2):243–248.

Menon ES (2005) *Gas Pipeline Hydraulics* (CRC Press).

Midthun KT (2007) *Optimization models for liberalized natural gas markets*. dissertation. (Norwegian University of Science and Technology).

Midthun KT, Bjorndal M, Tomasgard A (2009) Modeling optimal economic dispatch and system effects in natural gas networks. *The Energy Journal* 30(4):155–180.

Möller M (2004) *Mixed integer models for the optimisation of gas networks in the stationary case*. (TU Darmstadt).

Moritz S (2007) *A mixed integer approach for the transient case of gas network optimization*. (TU Darmstadt).

Odom FM, Muster GL (2009) Tutorial On Modeling of Gas Turbine Driven Centrifugal Compressors. (Pipeline Simulation Interest Group).

O'Neill RP, Williard M, Wilkins B, Pike R (1979) A Mathematical Programming Model for Allocation of Natural Gas. *Operations Research* 27(5):857–873.

Osiadacz A (1980) Nonlinear programming applied to the optimum control of a gas compressor station. *Int. J. Numer. Meth. Engng.* 15(9):1287–1301.

Osiadacz AJ, Chaczykowski M (2001) Comparison of isothermal and non-isothermal pipeline gas flow models. *Chemical Engineering Journal* 81(1):41–51.

Pepper W, Ring BJ, Read EG, Starkey SR (2012) Implementation of a Scheduling and Pricing Model for Natural Gas. Sorokin A, Rebennack S, Pardalos PM, Iliadis NA, Pereira MVF, eds. *Handbook of Networks in Power Systems II*. Energy Systems. (Springer Berlin Heidelberg), 3–35.

Pfetsch ME, Fügenschuh A, Geißler B, Geißler N, Gollmer R, Hiller B, Humpola J, et al. (2014) Validation of nominations in gas network optimization: models, methods, and solutions. *Optimization Methods and Software* 0(0):1–39.

Piggott J, Revell N, Kurschat T (2002) Taking the Rough With the Smooth-a New Look At Transmission Factor Formulae. *PSIG Annual Meeting*.

Qadrdan M, Chaudry M, Wu J, Jenkins N, Ekanayake J (2010) Impact of a large penetration of wind generation on the GB gas network. *Energy Policy* 38(10):5684–5695.

Read EG, Ring BJ, Starkey SR, Pepper W (2012) An LP Based Market Design for Natural Gas. Sorokin A, Rebennack S, Pardalos PM, Iliadis NA, Pereira MVF, eds. *Handbook of Networks in Power Systems II*. Energy Systems. (Springer Berlin Heidelberg), 77–113.

Rıos-Mercado RZ, Borraz-Sánchez C (2012) Optimization problems in natural gas transmission systems: a state-of-the-art survey.

Rose D, Schmidt M, Steinbach MC, Willert BM (2016) Computational optimization of gas compressor stations: MINLP models versus continuous reformulations. *Math Meth Oper Res* 83(3):409–444.

Rossi B, Djerourou F, de Carpentier F, Martin F (2014) Transient Optimization in Gas Transmission Networks a New Approach on GRTgaz Network. *PSIG Annual Meeting*.

Schmidt M, Steinbach MC, Willert BM (2012) High detail stationary optimization models for gas networks—Part 1: Model components. *IfAM Preprint* 94.

Schmidt M, Steinbach MC, Willert BM (2014) High detail stationary optimization models for gas networks—Part 2: Validation and results. *preparation. Zitiert auf Seiten iv und viii*.

Schroeder Jr DW (2010) A tutorial on pipe flow equations. *PSIG Annual Meeting*.

Scott DL, Whaley RS (2000) How to Get Gas Control to Accept And Believe In Simulation Models. (Pipeline Simulation Interest Group).

Selot A (2008) *Short-term supply chain management in upstream natural gas systems*. (Massachusetts Institute of Technology).

Steinbach MC (2007) On PDE solution in transient optimization of gas networks. *Journal of Computational and Applied Mathematics* 203(2):345–361.

Wong PJ, Larson RE (1968) Optimization of tree-structured natural-gas transmission networks. *Journal of Mathematical Analysis and Applications* 24(3):613–626.

Yépez RA (2008) A Cost Function for the Natural Gas Transmission Industry. *The Engineering Economist* 53(1):68–83.

Zheng QP, Rebennack S, Iliadis NA, Pardalos PM (2010) Optimization Models in the Natural Gas Industry. Pardalos PM, Rebennack S, Pereira MVF, Iliadis NA, eds. *Handbook of Power Systems I*. Energy Systems. (Springer Berlin Heidelberg), 121–148.

Chapter III. The technology of a natural gas pipeline: Insights for costs and rate-of-return regulation

III.1 Introduction

For the last 30 years, there has been an enduring interest in the construction of large-scale natural gas pipelines all around the world. Though an emerging literature has studied the market effects of a new pipeline project,[4] the examination of the technology and costs of these capital-intensive infrastructures has attracted less attention. Yet, that analysis is critically needed to inform policy decisions. Even in countries where liberalization reforms have been implemented, natural gas pipelines remain regulated (von Hirschhausen, 2008) and authorities must frequently deal with project-specific requests for adjustments of the regulatory framework.[5]

So far, two different methodological approaches have been considered to investigate the technology. The first is rooted in engineering and can be traced back to Chenery (1949). It aims at numerically determining the least-cost design of a given infrastructure using optimization techniques (Kabirian and Hemmati, 2007; Ruan et al., 2009; André and Bonnans, 2011). This approach is widely applied by planners and development agencies to assess the cost of a specific project (Yépez, 2008). Yet, because of its sophistication and its numerical nature, it is seldom considered in regulatory policy debates (Massol, 2011). The second approach involves the econometric estimation of a flexible functional form – usually a translog specification – to obtain an approximate cost function. This method has become popular in Northern America either to estimate the industry cost function using cross-section datasets (Ellig and Giberson, 1993) or to model the cost function of a single firm using a time series approach (Gordon et al., 2003). So far, data availability issues have hampered the application of this empirical approach in Continental Europe and Asia.

This research note develops a third approach: it proves that a production function of the Cobb-Douglas type captures the physical relationship between input use and the level of output of a simple

[4] Among others, Newbery (1987) assesses the trade opportunities generated by a new pipeline, Hubert and Ikonnikova (2011) evaluate the impacts on the relative bargaining powers of exporting and transit countries, and Rupérez Micola and Bunn (2007) and Massol and Banal-Estañol (2016) investigate the relation between pipeline utilization and the degree of spatial market integration between interconnected markets.

[5] For example, the augmented rate-of-return that was allocated to two new pipeline projects in France during the years 2009–16: the pipeline connecting the new Dunkerque LNG terminal to the national transportation network and the North-South Eridan project (CRE, 2012).

point-to-point pipeline infrastructure. More precisely, we show how that micro-founded model of the technology naturally emerges from the engineering equations governing the design of that infrastructure. One of the great merits of that approach is that it greatly facilitates the application of the standard theory of production to characterize the microeconomics of a natural gas pipeline system.

To explore the policy implications, we use that production function to successively examine the properties of the cost function in the long and in the short run. We also compare the market outcomes obtained under three alternative industrial organizations (a monopoly, average-cost pricing, and rate-of-return regulation). Our results: (i) document the presence of pronounced increasing returns to scale in the long run; (ii) confirm the natural monopolistic nature of a gas pipeline system and the need for regulatory intervention; (iii) clarify the conditions for cost-recovery if short-run marginal cost pricing is imposed on such an infrastructure; (iv) document the performance of rate-of-return regulation in that industry, and (v) reveal that the socially desirable rate-of-return is not necessarily equal to the market price of capital in that case.

III.2 Theoretical model of the technology

We consider a simple point-to-point pipeline infrastructure that consists of a compressor station injecting a pressurized flow of natural gas Q into a pipeline to transport it across a given distance l.

Following Chenery (1949) and Yépez (2008), designing such a system imposes to determine the value of three engineering variables: the compressor horsepower H, the inside diameter of the pipe D and τ the pipe thickness. These variables must verify three engineering equations presented in Table 1 (first column). The compressor equation gives the power required to compress the gas flow from a given inlet pressure p_0 to a predefined outlet pressure $p_0 + \Delta p$ where Δp is the net pressure rise. The Weymouth equation models the pressure drop between the inlet pressure $p_0 + \Delta p$ measured after the compressor station, and the outlet one p_1 which is assumed to be equal to p_0. Lastly, concerns about the mechanical stability of the pipe impose a relation between the thickness τ and the inside diameter D.

Table 1. Engineering equations

Exact engineering equations	Approximate engineering equations
Compressor equation: (a) $H = c_1.\left[\left(\frac{p_0+\Delta p}{p_0}\right)^b - 1\right]Q$	Approximate compressor equation: (a) $H = c_1 b \frac{\Delta p}{p_0} Q$
Weymouth flow equation: (b) $Q = \frac{c_2}{\sqrt{l}} D^{8/3} \sqrt{\left(p_0+\Delta p\right)^2 - p_1^{\,2}}$	Approximate flow equation: (b) $Q = \frac{c_2 p_0 \sqrt{2}}{\sqrt{l}} D^{8/3} \sqrt{\frac{\Delta p}{p_0}}$
Mechanical stability equation: (c) $\tau = c_3 D$	Mechanical stability equation: (c) $\tau = c_3 D$

Notes: (a) (b) the positive constant parameters c_1, c_2 and b (with $b<1$) are detailed in Yépez (2008) for the USCS unit system. Elevation changes along the pipeline are neglected in the flow equation. (c) This equation follows the industry-standard practice and assumes that the pipe thickness equals a predetermined fraction c_3 of the inside diameter (e.g., c_3=0.9% in Ruan et al. (2009 – p. 3044)).

We now combine these equations to construct an approximate production function. To our knowledge, the pressure rise Δp usually ranges between 1% and 30% of p_0 which leads to the first-order approximations detailed in Table 1 (second column). Combining them, one can eliminate the relative pressure rise $\Delta p/p_0$ and obtain the following relation between the output Q and two engineering variables H and D:

$$Q = \sqrt[3]{\frac{2\left(c_2 p_0\right)^2}{c_1 b l}} D^{16/9} H^{1/3} . \tag{1}$$

This relation can be reformulated as a production function that gives the output as a function of two inputs: energy and capital. First, we let E denote the total amount of energy consumed by the infrastructure to power the compressor. By definition, the total amount of energy E is directly proportional to the horsepower H. Second, we let K denote the replacement value of the pipeline. We assume that the capital stock K is directly proportional to the pipeline total weight of steel S and let P_S denote the unit cost of steel per unit of weight. Hence, $K = P_S S$. The total weight of steel S required to build that pipeline is obtained by multiplying the volume of steel in an open cylinder by the weight of steel per unit of volume W_S:

$$S = l\pi\left[\left(\frac{D}{2}+\tau\right)^2 - \left(\frac{D}{2}\right)^2\right]W_S\,, \tag{2}$$

where $\pi \approx 3.1416$ is the mathematical constant. Combining that equation with the mechanical stability equation in Table 1, the amount of capital expenditure related to the pipeline is as follows:

$$K = P_S l\pi D^2\left[c_3 + c_3{}^2\right]W_S\,. \tag{3}$$

This equation shows that the pipeline diameter is directly proportional to the square root of K, the amount of capital invested in the pipeline. So, the engineering equation (1) can readily be rewritten as a production function: $Q = B\,K^{8/9}E^{1/3}$, where B is a constant. To simplify, we rescale the output by dividing it by B and use this rescaled output thereafter. So, the Cobb-Douglas production function of a gas pipeline is:

$$Q^\beta = K^\alpha E^{1-\alpha}\,, \tag{4}$$

where the capital exponent parameter is $\alpha = 8/11$ and $\beta = 9/11$ is the inverse of the degree to which output is homogeneous in capital and energy. As $\beta < 1$, the technology exhibits increasing returns to scale.

III.3 Results and policy implications

In this section, we show how the technological model above can be applied to derive a collection of policy-relevant insights. Since natural gas pipelines are deemed as natural monopolies, we first examine whether that reputation is supported by the properties of the long-run cost function. Then, we examine the short-run cost function to gain insights on the performance of short-run marginal cost pricing. Lastly, we assess the performance of rate-of-return regulation for that industry.

III.3.1 Long-run cost

We let e denote the market price of the energy input and r the market price of capital faced by the firm. From the cost-minimizing combination of inputs needed to transport the output Q, one can derive the long-run total cost function (Cf., Appendix A):

$$C(Q) = \frac{r^\alpha e^{1-\alpha}}{\alpha^\alpha(1-\alpha)^{1-\alpha}}Q^\beta\,. \tag{5}$$

Three insights can be drawn from that specification. Firstly, the elasticity of the long-run cost with respect to output is $\beta = 9/11$ and lower than one. The cost function (5) also validates the empirical remarks in Chenery (1952) and Massol (2011) who suggested that this elasticity is almost constant over most of the output range. Secondly, the ratio of the long-run marginal cost to the long-run average cost is constant and also equals β. As $\beta < 1$, setting the price equal to the long-run marginal cost systematically yields a negative profit. Lastly, one can note that the univariate cost function (5) is concave and thus strictly subadditive (Sharkey, 1982 - Proposition 4.1). This property has important policy implications: it attests that a point-to-point gas pipeline system verifies the technological condition for a natural monopoly. As this particular industry structure may lead to a variety of economic performance problems (e.g., excessive prices, production inefficiencies, costly duplication of facilities), the implementation of price and entry regulation of some form can be justified to mitigate the social cost of these market failures (Joskow, 2007).

III.3.2 Short-run cost

We now examine how cost varies in the short-run. We consider an existing infrastructure that has been designed to transport the output Q_0 at minimum long-run cost by installing the amount of capital stock K_0. The short-run total cost function is obtained by holding K_0 constant and varying the output Q. Introducing the variable input requirements function $E(Q, K_0) = \sqrt[1-\alpha]{K_0^{-\alpha} Q^{\beta}}$ that gives the amount of energy needed to transport Q along that pipeline, the short-run total cost function is:

$$SRTC^{K_0}(Q) = rK_0 + eK_0^{\frac{-\alpha}{1-\alpha}} Q^{\frac{\beta}{1-\alpha}}. \tag{6}$$

The technical discussion presented in Appendix B confirms that the short-run average cost $SRAC^{K_0}$ curve is U-shaped and attains its minimum at $Q = \underline{Q}$, where $\underline{Q}$ is the unique output at which the short-run marginal cost curve intersects the $SRAC^{K_0}$ one. Solving, one can show that the output ratio $\underline{Q}/Q_0$ verifies:

$$\frac{\underline{Q}}{Q_0} = \left[\frac{\alpha}{\beta + \alpha - 1}\right]^{\frac{1-\alpha}{\beta}} = \sqrt[3]{\frac{4}{3}} \approx 1.1006. \tag{7}$$

It should be noted that this ratio is entirely determined by the technological parameters α and β and does not depend on the input prices or the capital stock K_0.

At the output level $Q = Q_0$, the short-run marginal cost is lower than the short-run average cost and expanding the output to $\underline{Q} = \sqrt[3]{4/3}Q_0$ occasions a reduction in the short-run average cost.

It follows that, for any output Q with $Q < \underline{Q}$, imposing the pipeline operator to charge a price equal to the short-run marginal does not allow that firm to break even. This last finding can usefully inform the contemporary European policy debates pertaining to the regular revision of the European Gas Target Model (ACER, 2015). In a recent policy proposal, Hecking (2015) advocates the application of short-run marginal-cost pricing for cross-border interconnector pipelines in Europe. Compared to the current ad-hoc pricing system, one of the main merits of this pricing arrangement is to favor an efficient use of these infrastructures in the short-run. Yet, it should be stressed that the capital costs bulk large as a percentage of the total cost of a gas pipeline system. Therefore, its application on an existing interconnector may generate a cost-recovery issue if the output is lower than the level $\underline{Q}$.[6] For new interconnector projects, this pricing scheme, when considered alone, can deter investment. It could thus adversely impact the feasibility of a series of major European projects aimed at fostering market integration across the continent (e.g., the MidCat project aimed at connecting the Iberian peninsula with France and the rest of Europe). This confirms the need to combine marginal-cost pricing of interconnectors with other cost-recovery instruments such as network tariffs to recover the remaining costs.

III.3.3 Rate-of-return regulation

The analysis above indicates that a pipeline has elements of a natural monopoly. As rate-of-return regulation[7] remains a popular instrument used by numerous authorities all over the world (e.g., in the U.S., Belgium, South-Africa), we now explore what insights our characterization of the technology can provide to regulators and practitioners.

[6] Arguably, a share of these capital costs could be considered as sunk which could trigger a discussion as to whether these costs have to be recouped or not.

[7] That form of regulation sees costs as exogenous and observable and forms prices on the basis of observed variable costs and a predetermined rate of return on invested capital s.

Following the literature (Klevorick, 1971: Callen et al., 1976), we assume the isoelastic inverse demand function $P(Q) = A\,Q^{-\varepsilon}$, where $1/\varepsilon$ is the absolute price elasticity with $\varepsilon < 1$ (so that the total revenue obtained by a firm producing zero output is zero) and $1-\beta < \varepsilon$ (to verify the second-order condition for a maximum in the regulated firm's optimization problem), and let s denote the allowed rate of return set by the regulatory authority. For concision, the solution of the profit-maximization problem of a regulated firm whose accounting profit (i.e., the total revenue $P(Q)Q$ minus $eE(Q,K)$ the cost of the variable input) cannot exceed the allowed return on invested capital sK is reviewed in Appendix C.

Callen et al. (1976) examine the problem of a regulator that sets the allowed rate of return s at the level s_R that maximizes the net social welfare given the regulated firm's reaction to that rate. They formally prove that this socially desirable rate is:

$$s_R = \max\left(r, \frac{\left[\beta-(1-\varepsilon)(1-\alpha)\right]^2}{\alpha\left[\beta-(1-\alpha)(1-\varepsilon)^2\right]} r \right). \tag{8}$$

We can use the values of α and β above to highlight two interesting results pertaining to the application of rate-of-return regulation in the gas pipeline sector. First, it is straightforward to verify that, whenever the demand parameter ε is in the open interval $\left(\left(2+4\sqrt{3}\right)/11,1\right)$, the condition $\left[\beta-(1-\varepsilon)(1-\alpha)\right]^2 > \alpha\left[\beta-(1-\alpha)(1-\varepsilon)^2\right]$ holds which indicates that the socially desirable rate of return is $s_R = \left[\beta-(1-\varepsilon)(1-\alpha)\right]^2 r \Big/ \left(\alpha\left[\beta-(1-\alpha)(1-\varepsilon)^2\right]\right)$ and thus verifies $s_R > r$. Hence, if the absolute price elasticity is low and in the range $1 < 1/\varepsilon < 1.232$, setting the allowed rate-of-return as close as possible to the market price of capital does not maximize the net social welfare. This is a noteworthy finding that contradicts a popular belief. Second, we can observe that the ratio s_R/r is bounded as the relation $(s_R/r) < (\beta/\alpha)$ holds for any value of ε in the assumed range $1-\beta < \varepsilon < 1$. This remark provides useful operational guidance for the selection of a rate of return: if the regulator has zero information on the value of the price elasticity of the demand and thus cannot exactly evaluate s_R, it should not implement a rate of return that is larger than $\beta r/\alpha$, that is $\beta/\alpha = 9/8 = 1.125$ times the market price of capital r.

It is also instructive to document the relative performance of rate-of-return regulation in the gas pipeline sector by comparing the market outcomes (subscripted with R) with the ones obtained in case of either a standard (unregulated) private monopoly (subscripted with M) or a benevolent social planner that maximizes the net social welfare while providing zero economic profit to the pipeline operator (subscripted with a as it sets the output at the level at which price equals the long-run average cost). To ease the comparisons, we simply tabulate the ratios presented in Callen et al. (1976) for a range of possible values for the demand elasticity (Cf., Table 2). These ratios are also detailed in Appendix C (cf., Table C-1) and respectively compare:

- the output levels decided by: a private monopoly Q_M, a social planner applying the average-cost-pricing rule Q_a and a regulated monopoly Q_R;
- the cost C_R incurred by the regulated firm subject to rate-of-return regulation and the cost $C(Q_R)$ that would have been incurred by a cost-minimizing firm producing the same output Q_R;
- the gain in net social welfare resulting from the regulation of a private monopoly $(W_R - W_M)$ and the gain in net social welfare $(W_a - W_M)$ that would be obtained by a social planner applying the average-cost-pricing rule to a previously monopolistically-controlled industry.

These ratios are invariant with the relative input prices and are entirely determined by: the demand and technology parameters, and the ratio s/r that relates s the allowed rate of return set by the regulator to r the market price of capital (Callen et al., 1976).

To begin with, we examine the case presented in Table 2 – Panel A of a regulatory agency that implements the socially desirable rate of return s_R in (8). If the absolute price elasticity of the demand is less than 1.30, we observe that: (i) the output level Q_R is substantially lower than the value Q_a obtained under the ideal case of a benevolent social planner imposing the long-run average cost pricing rule (it hardly attains the three quarters of that value); and (ii) the magnitude of the extra-cost caused by the overcapitalization effect pointed in Averch and Johnson (1962)[8] can be important (i.e., the cost increase is larger than 20% of the long-run total cost and attains 378.9% in case of a price

[8] The analysis of rate-of-return regulation in Averch and Johnson (1962) highlighted the tendency of the regulated firm to engage in excessive amounts of capital accumulation to expand the volume of its profits.

elasticity equal to 1.001). That said, it is worth noting that despite these two adverse effects, the application of rate-of-return regulation on an unregulated monopolistic operator induces a very large rise in the pipeline output level (cf., the large values of the output ratio Q_R/Q_M). Overall, that form of regulation generates substantial welfare gains: the net increase in social welfare $(W_R - W_M)$ attains more than 70% of the difference $(W_a - W_M)$ that measures the gains obtained under the theoretical benchmark of a benevolent social planner applying average-cost-pricing.

Table 2. Output, cost, and welfare ratios for alternative demand elasticities

Panel	Variable	$1/\varepsilon$ = 1.001	1.05	1.10	1.20	1.30	1.50	2.00
PANEL A The socially desirable case $s = s_R$	Ratio rate of return/price of capital $\frac{s}{r} = \frac{s_R}{r}$	1.124	1.090	1.061	1.013	1.000	1.000	1.000
	Output ratios $\frac{Q_R}{Q_a}$	0.127	0.448	0.551	0.678	0.746	0.815	0.895
	$\frac{Q_R}{Q_M}$	468.211	17.955	11.313	7.795	6.438	5.196	4.207
	Cost ratio $\frac{C_R}{C(Q_R)}$	4.789	1.705	1.456	1.273	1.187	1.104	1.036
	Welfare ratio $\frac{(W_R - W_M)}{(W_a - W_M)}$	0.729	0.724	0.725	0.735	0.750	0.781	0.850
PANEL B The case of a rate of return set at the upper bound $s = \beta r/\alpha$	Ratio rate of return/price of capital $\frac{s}{r} = \frac{\beta}{\alpha}$	1.125 *(+0.07%)*	1.125 *(+3.17%)*	1.125 *(+6.05%)*	1.125 *(+11.07%)*	1.125 *(+12.50%)*	1.125 *(+12.50%)*	1.125 *(+12.50%)*
	Output ratios $\frac{Q_R}{Q_a}$	0.127 *(-0.06%)*	0.435 *(-2.91%)*	0.520 *(-5.71%)*	0.603 *(-11.06%)*	0.645 *(-13.57%)*	0.683 *(-16.19%)*	0.684 *(-23.60%)*
	$\frac{Q_R}{Q_M}$	467.934 *(-0.06%)*	17.433 *(-2.91%)*	10.667 *(-5.71%)*	6.933 *(-11.06%)*	5.564 *(-13.57%)*	4.355 *(-16.19%)*	3.214 *(-23.60%)*
	Cost ratio $\frac{C_R}{C(Q_R)}$	4.788 *(-0.02%)*	1.691 *(-0.79%)*	1.436 *(-1.40%)*	1.244 *(-2.22%)*	1.161 *(-2.23%)*	1.084 *(-1.82%)*	1.024 *(-1.14%)*
	Welfare ratio $\frac{(W_R - W_M)}{(W_a - W_M)}$	0.729 *(+0.00%)*	0.724 *(-0.02%)*	0.724 *(-0.13%)*	0.730 *(-0.65%)*	0.738 *(-1.60%)*	0.748 *(-4.23%)*	0.731 *(-13.94%)*

Notes: In Panel B, the numbers in parentheses indicate the relative change (in percent) with respect to the ideal case of a regulator capable to set the regulated rate of return at the value s_R in equation (8).

As regulatory agencies do seldom have the knowledge of the demand price elasticity needed to evaluate the socially desirable rate of return s_R, Table 2– Panel B then examines the performance of rate-of-return regulation when the regulator simply sets $s = \beta r/\alpha$. By construction, the gains in social welfare values are lower than the ones detailed in Panel A. Yet, it is interesting to observe that the differences remain tolerable whenever the absolute price elasticity is less than 1.50, which is likely to be the case in the natural gas pipeline industry. Hence, that form of regulation remains a powerful regulatory instrument even when the regulator simply sets the allowed rate of return s within the range $r \le s \le \beta r / \alpha$.

III.4 Conclusion

The analysis presented in this concise paper shows how the complex engineering equations governing the functioning of a pipeline system can be combined in a single production equation of the Cobb-Douglas type that is commonly applied in microeconomics.

This characterization of the technology of a natural gas pipeline allows us to highlight the following points that should be pertinent to researchers and policymakers interested in understanding the economics of natural gas pipelines.

- First, the analysis documents the magnitude of the long-run economies of scale that exists on point-to-point pipeline systems, thereby confirming the natural monopolistic nature of such infrastructures and justifying the need to implement price and entry regulation of some form in that industry.
- Second, in the short-run, the analysis reveals that it is possible to monotonically lower the average transportation cost incurred on an existing pipeline infrastructure by expanding the output up to a threshold level that represents about 110% of the output that was considered at the time of the construction of that infrastructure. This finding has important implications for the applicability of short-run marginal-cost pricing: it highlights that this pricing scheme cannot allow to recover the capital costs incurred by the pipeline operator if the output is lower than that threshold level.
- Lastly, this paper combines the technological analysis above with the standard industrial organization literature to contribute to the understanding of the performance of rate-of-return regulation in that industry. It first reveals that, contrary to popular intuition, the rate of return that maximizes the net social welfare can be larger than the market price of capital if the price elasticity of demand is low. Then, it also assesses the magnitude of the Averch-Johnson distortions on both the output and the cost of the regulated firm. It should

be noted that, despite these distortions, the application of that simple form of regulation remains a valuable instrument to protect the community from monopolistic exploitation in that industry.

Future research could explore whether that methodology could be adapted and combined with the recent engineering literature on either hydrogen pipelines (André et al., 2013) or CO_2 pipelines (Massol et al., 2015) to inform the burgeoning policy discussions on the regulation of these future low-carbon technologies.

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III.6 References

ACER, 2015. European Gas Target Model - Review and Update. http://www.acer.europa.eu/Events/Presentation-of-ACER-Gas-Target-Model-/Documents/European%20Gas%20Target%20Model%20Review%20and%20Update.pdf.

André, J., & Bonnans J.F., 2011. Optimal structure of gas transmission trunklines. *Optimization and Engineering*, 12(1-2), 175–198.

André, J., Auray, S., Brac, J., De Wolf, D., Maisonnier, G., Ould-Sidi, M.M., Simonnet, A., 2013. Design and dimensioning of hydrogen transmission pipeline networks. *European Journal of Operational Research*, 229(1), 239–251.

Averch, H., Johnson, L., 1962. Behavior of the firm under regulatory constraint. *American Economic Review*, 52(5), 1052–1069.

Callen J.L., Mathewson G.F., Mohring H., 1976. The Benefits and Costs of Rate of Return Regulation. *American Economic Review*, 66(3), 290–297.

Chenery, H.B., 1949. Engineering production functions. *Quarterly Journal of Economics*, 63(4), 507–531.

Chenery, H.B., 1952. Overcapacity and the acceleration. *Econometrica*, 20(1), 1–28.

CRE, 2012. *Délibération du 13 décembre 2012 portant décision sur le tarif d'utilisation des réseaux de transport de gaz naturel*. Paris: Commission de Régulation de l'Energie.

Ellig J., Giberson, M., 1993. Scale, scope, and regulation in the Texas gas transmission industry. *Journal of Regulatory Economics*, 5(1): 79–90.

Gordon, D.V., Gunsch, K., Pawluk, C.V., 2003. A natural monopoly in natural gas transmission. *Energy Economics*, 25(5): 473–485

Hecking, H., 2015. Rethinking Entry-Exit: Two New Tariff Models to Foster Competition and Security of Supply in the EU Gas Market. *EWI Energy Research & Scenarios*, Cologne

Hubert, F., Ikonnikova, S., 2011. Investment options and bargaining power: the Eurasian supply chain for natural gas. *Journal of Industrial Economics*, 59(1): 85–116.

Joskow, P. L., 2007. 'Regulation of Natural Monopoly', in *Handbook of Law and Economics,* Volume 2, ed. A.M. Polinsky and S. Shavell, Elsevier.

Kabirian, A., Hemmati, M. R., 2007. A strategic planning model for natural gas transmission networks. *Energy Policy*, 35(11): 5656–5670.

Klevorick, A.K., 1971. The 'Optimal' Fair Rate of Return. *The Bell Journal of Economics and Management Science*, 2(1), 122–53.

Massol, O., 2011. A cost function for the natural gas industry: further considerations. *The Engineering Economist*, 56(2), 95–122.

Massol, O., Tchung-Ming, S., Banal-Estañol, A., 2015. Joining the CCS club! The economics of CO_2 pipeline projects. *European Journal of Operational Research*, 247(1), 259–275.

Massol, O. and Banal-Estañol, A., 2016. Market Power and Spatial Arbitrage between Interconnected Gas Hubs. *USAEE Working Paper* No. 16-281.

Newbery, D.M., 1987. 'The privatisation of British Gas and possible consequences for the international gas market', in *Natural Gas Markets and Contracts*, ed. R. Golombek, M. Hoel and J. Vislie, North-Holland.

Ruan, Y., Liu, Q., Zhou, W., Batty, B., Gao, W., Ren ,J. and Watanabe, T., 2009. A procedure to design the mainline system in natural gas networks. *Applied Mathematical Modelling*, 33(7), 3040–3051.

Rupérez Micola, A., Bunn, D.W., 2007. Two markets and a weak link. *Energy Economics*, 29(1): 79-93.

Sharkey, W.W., 1982. *The Theory of Natural Monopoly*, Cambridge University Press, Cambridge.

von Hirschhausen, C., 2008. Infrastructure, regulation, investment and security of supply: A case study of the restructured US natural gas market. *Utilities Policy*, 16(1): 1–10.

Yépez, R.A., 2008. A cost function for the natural gas industry. *The Engineering Economist*, 53(1), 68–83.

Appendix A – The long-run cost function

The long-run total cost function C to transport the output Q is the solution of the cost-minimization problem:

$$\min_{K,E} \quad C(Q) = r\,K + e\,E \tag{A.1}$$

$$\text{s.t.} \quad Q^{\beta} = K^{\alpha} E^{1-\alpha} \tag{A.2}$$

The first-order conditions for optimality indicate that the marginal rate of technical substitution of E for K has to equate the ratio of the input prices:

$$\frac{(1-\alpha)}{\alpha}\frac{K}{E} = \frac{e}{r}. \tag{A.3}$$

Using the variable input requirements function $E(Q,K) = \sqrt[1-\alpha]{K^{-\alpha} Q^{\beta}}$ that gives the amount of energy needed to transport Q along that pipeline, one can rearrange (A.3) to define a function that gives the long-run cost-minimizing amount of capital stock needed to transport the output Q :

$$K(Q) = \left(\frac{e\alpha}{r(1-\alpha)}\right)^{1-\alpha} Q^{\beta}, \tag{A.4}$$

The long-run total cost function is $C(Q) = rK(Q) + eE(Q, K(Q))$ and thus:

$$C(Q) = \frac{r^{\alpha} e^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} Q^{\beta}. \tag{A.5}$$

Appendix B – Short-run cost

<u>A review of short-run cost concepts</u>

Assuming a fixed amount of capital input K, the short-run total cost function is:

$$SRTC^{K}(Q) = rK + eE(Q,K), \tag{B.1}$$

where $E(Q,K) = \sqrt[1-\alpha]{K^{-\alpha} Q^{\beta}}$ is the variable input requirements function. As $\beta > 1-\alpha$ for the gas pipeline, this function is monotonically increasing and convex.

The short-run marginal cost function is:

$$SRMC^K\left(Q\right) = eE_Q\left(Q,K\right). \tag{B.2}$$

where $E_Q\left(Q,K\right)$ denote the derivative of the input requirement function with respect to the output variable.

The short-run average cost function is:

$$SRAC^K\left(Q\right) = \frac{rK}{Q} + e\frac{E\left(Q,K\right)}{Q}. \tag{B.3}$$

With $\alpha = 8/11$ and $\beta = 9/11$, this twice-differentiable function verifies $\lim_{Q\to 0^+} SRAC^K\left(Q\right) = +\infty$, $\lim_{Q\to +\infty} SRAC^K\left(Q\right) = +\infty$ and is strictly convex.[9] Hence, the short-run average cost curve has the usual U shape. Because of the strict convexity, the short-run average cost function has a unique minimum. At that output level, the short-run average cost equals the short-run marginal cost.[10] We let $\underline{Q}$ denote the output at which the short-run average cost is minimal, i.e. $\underline{Q} = \underset{Q>0}{\mathrm{Min}}\ SRAC^K\left(Q\right)$. For any output Q lower (respectively larger) than $\underline{Q}$, the short-run average cost $SRAC^K\left(Q\right)$ is larger (respectively lower) than the short-run marginal cost $SRMC^K\left(Q\right)$.

Discussion

We now consider the infrastructure that has been optimally designed to transport the output Q_0 at minimum long-run cost by installing the amount of capital stock $K_0 = K\left(Q_0\right)$, and aim at comparing the design output Q_0 and the average-cost-minimizing output $\underline{Q}$ on that specific pipeline system.

Recall that $\underline{Q}$ is such that the short-run average cost $SRAC^{K_0}\left(\underline{Q}\right)$ equals the short-run marginal cost $SRMC^{K_0}\left(\underline{Q}\right)$, that is:

$$\frac{rK_0}{\underline{Q}} + e\frac{E\left(\underline{Q},K_0\right)}{\underline{Q}} = eE_Q\left(\underline{Q},K_0\right). \tag{B.4}$$

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[9] Remark that its second derivative equals $2\left(rKQ^{-3} + eK^{-8/3}\right)$ which is positive for any $Q > 0$.

[10] Proof: The gradient of $SRAC^K$ w.r.t. Q equals $\left[-SRAC^K\left(Q\right) + eE_Q\left(Q,K\right)\right]/Q$, that is using (B.2) $\left[-SRAC^K\left(Q\right) + SRMC^K\left(Q\right)\right]/Q$.

Using $E(Q,K) = \sqrt[1-\alpha]{K^{-\alpha}Q^{\beta}}$ and simplifying, one obtains:

$$\underline{Q} = \left(\frac{r(1-\alpha)}{e(\beta+\alpha-1)} \right)^{\frac{1-\alpha}{\beta}} K_0^{\frac{1}{\beta}}. \tag{B.5}$$

Using (A.4), one can directly obtains the design output Q_0 as a function of the capital stock that has been installed:

$$Q_0 = \left(\frac{r(1-\alpha)}{e\alpha} \right)^{\frac{1-\alpha}{\beta}} K_0^{\frac{1}{\beta}}. \tag{B.6}$$

Equations (B.5) and (B.6) together indicate that the ratio $\underline{Q}/Q_0$ is entirely determined by the technological parameters α and β :

$$\frac{\underline{Q}}{Q_0} = \left[\frac{\alpha}{\beta+\alpha-1} \right]^{\frac{1-\alpha}{\beta}}. \tag{B.7}$$

With $\alpha = 8/11$ and $\beta = 9/11$, this ratio indicates that $\underline{Q} = \sqrt[3]{4/3}Q_0 \approx 1.1006Q_0$. It should be noted that for any output lower than $\underline{Q}$, the short-run average cost is larger than the short-run marginal cost.

Appendix C – Supplementary document

This Appendix is aimed at being disseminated as a supplementary document and is organized as follows. Section III.6.1 summarizes the assumptions and introduces the notation. Section III.6.2 reviews the standard cases of a monopoly and a social planner. Section III.6.3 examines the case of rate-of-return regulation and gives a concise presentation of Klevorick (1971) and Callen et al. (1976) who were the first to analytically examine the economics of rate-of-return regulation for a Cobb-Douglas technology. Section III.6.4 details the ratios presented in the paper.

III.6.1 Assumptions and notations

Technology

We consider the simple point-to-point pipeline infrastructure studied in our paper and assume the Cobb-Douglas production function: $Q^{\beta} = K^{\alpha}E^{1-\alpha}$, where $\alpha = 8/11$ is the capital exponent parameter and $\beta = 9/11$ is the scale coefficient.

From that production function, one can define $E(Q,K)=\sqrt[1-\alpha]{K^{-\alpha}Q^{\beta}}$ the variable input requirements function that gives the amount of energy needed to transport the output Q on a pipeline infrastructure that has a given fixed amount of capital input K. We let $E_Q(Q,K)$ (respectively, $E_K(Q,K)$) denote the derivative of the input requirement function with respect to the output (respectively, the capital) variable. With our technology parameters, $E_Q(Q,K)>0$ and $E_K(Q,K)<0$.

Input prices

We let e denote the market price of the energy input and r denote the market cost of capital faced by the firm.

Cost function

Following the argumentation presented in Appendix A, the long-run cost-minimizing amount of capital stock needed to transport the flow Q is:

$$K(Q)=\left(\frac{e\alpha}{r(1-\alpha)}\right)^{1-\alpha}Q^{\beta}, \tag{C.1}$$

The long-run total cost function is $C(Q)=rK(Q)+eE(Q,K(Q))$ and thus:

$$C(Q)=\frac{r^{\alpha}e^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}Q^{\beta}. \tag{C.2}$$

Demand

The inverse demand function is: $P(Q)=A\,Q^{-\varepsilon}$, where A is a constant and $1/\varepsilon$ is the absolute value of the price elasticity of demand. Here, it is assumed that: $\varepsilon<1$ so that the total revenue obtained by a monopolist producing zero output is zero and that $\varepsilon>1-\beta$ so that the demand schedule always intersects the marginal cost schedule from above.[11]

For notational convenience, we follow Callen et al. (1976) and introduce three parameters: (i) $\gamma\equiv\beta+\varepsilon-1$, (ii) $\delta\equiv e\beta/\left[A(1-\varepsilon)(1-\alpha)\right]$, and (iii) $\eta\equiv\beta-(1-\varepsilon)(1-\alpha)$.

III.6.2 The cases of a monopoly and of a social planner

This section briefly reviews the standard outcomes obtained under two polar cases: (i) the profit-maximizing unregulated monopoly that charges a non-discriminatory price; and (ii) a welfare-

[11] These restrictions together impose that $1/\varepsilon$ is in the range (1,5.5) which is not a concern in our application.

maximizing social planner that behaves so as to maximize the sum of the producers' and consumers' surpluses (i.e., the net social welfare) while ensuring that the firm obtains zero economic profit.[12]

These two cases can be modeled using the optimization problems presented in Table C-1. For concision, we omit the straightforward derivations of the first-order conditions and simply report the optimal decisions.

Note that in both cases: (i) the optimal amount of capital stock equals the cost-minimizing amount, that is, $K_M = K(Q_M)$ and $K_a = K(Q_a)$; and (ii) production is cost efficient as the equations $C(Q_M) = rK_M + eE(Q_M, K_M)$ and $C(Q_a) = rK_a + eE(Q_a, K_a)$ hold. Note also that, for the social planner, substitution of the optimal decisions Q_a and K_a in the zero profit condition (26) gives $P(Q_a)Q_a - C(Q_a) = 0$ which means that the output is set at a level such that the price equals the long-run average cost.

Table C-1. The optimal decisions taken by a profit-maximizing unregulated monopoly and a welfare-maximizing social planner providing zero profit to the firm

	The unregulated monopoly	**The welfare-maximizing planner that provides zero-profits to the firm**
Optimization program	$\underset{K,Q}{\text{Max}}$ $\Pi_M(Q) = P(Q)Q - rK - eE(Q,K)$ (C.3)	$\underset{K,Q}{\text{Max}}\ W(Q) = \int_0^Q P(q)dq - rK - eE(Q,K)$ (C.4) s.t. $P(Q)Q - rK - eE(Q,K) = 0$
Solution:		
Output	$Q_M = \left[\dfrac{A(1-\varepsilon)}{\beta}\left(\dfrac{\alpha}{r}\right)^{\alpha}\left(\dfrac{1-\alpha}{e}\right)^{1-\alpha}\right]^{\frac{1}{\gamma}}$ (C.5)	$Q_a = \left[A\left(\dfrac{\alpha}{r}\right)^{\alpha}\left(\dfrac{1-\alpha}{e}\right)^{1-\alpha}\right]^{\frac{1}{\gamma}}$ (C.6)
Capital	$K_M = \left(\dfrac{e\alpha}{r(1-\alpha)}\right)^{1-\alpha}(Q_M)^{\beta}$ (C.7)	$K_a = \left(\dfrac{e\alpha}{r(1-\alpha)}\right)^{1-\alpha}(Q_a)^{\beta}$ (C.8)

Note: The objective function (C.3) is the firm's profit, i.e.: the difference between the total revenue $P(Q)Q$ and the sum of the capital cost rK and the energy cost $eE(Q,K)$. The objective function (C.4) is the net social welfare defined as the sum of the consumer surplus $\int_0^Q P(q)dq - P(Q)Q$ and the producer's surplus $P(Q)Q - rK - eE(Q,K)$. The constraint (26) states that the firm is compelled to obtain zero economic profit.

[12] For concision, we omit the first-best solution that consists of solely maximizing the sum of the producers' and consumers' surpluses without paying attention to the firm's profitability. As this first-best solution entails establishing an output level for which price equals the long-run marginal cost, it compels the pipeline operator to operate at a loss, which is not realistic.

Callen et al. (1976) define s_M the monopolist's rate of return on invested capital which is the ratio of: the accounting profit derived from the production of the output Q_M (that is: $P(Q_M)Q_M - eE(Q_M, K_M)$), and K_M the profit-maximizing capital stock: $s_M \equiv \left[\beta/(1-\varepsilon)-(1-\alpha)\right]r/\alpha$.

III.6.3 Rate-of-return regulation

We now assume that the infrastructure is provided by a private monopoly that is subject to rate-of-return regulation. This section briefly presents the theoretical literature on rate-of-return regulation for the special case of a Cobb-Douglas technology (Klevorick, 1971; Callen et al., 1976). It first reviews the behavior of the regulated monopoly before discussing the identification of a socially desirable rate of return.

The behavior of the regulated monopoly

The regulated monopoly is allowed to earn a fixed and exogenously-determined rate of return s that is lower than the rate of return s_M obtained by an unregulated monopolist (i.e., $s < s_M$).

The rate-of-return constraint stipulates that the monopoly's accounting profit (i.e., the total revenue $P(Q)Q$ minus $eE(Q,K)$ the cost of the variable input) cannot exceed the allowed return on invested capital sK. As the condition $s < s_M$ holds, the rate-of-return constraint is binding:

$$P(Q)Q - e\,E(Q,K) = s\,K, \tag{C.9}$$

The regulated firm is allowed to choose any combination of inputs (K and E) and output (Q) that jointly verifies the production function equation, and the rate-of-return constraint. Assuming profit maximization, the behavior of the regulated monopoly is thus determined by the following program:

$$\begin{aligned} &\max_{K,Q} \quad \Pi(Q) = P(Q)Q - r\,K - e\,E(Q,K) \\ &\text{s.t.} \quad P(Q)Q - e\,E(Q,K) = s\,K \\ &\qquad K \geq 0\ ,\ Q \geq 0\,. \end{aligned} \tag{C.10}$$

If the allowed rate of return is lower than the market cost of capital (i.e., $s < r$), profit maximization involves a corner solution: the firm's optimal decision is to withdraw from the market.

One must thus concentrate on the situation $s \geq r$. As shown in Klevorick (1971), the firm's optimal decisions must jointly verify the rate-of-return constraint (C.9) and the condition:

$$(s-r)\left[P'(Q)Q + P(Q) - e\,E_Q(Q,K)\right] = 0, \tag{C.11}$$

One can first examine the case $s > r$ where the allowed rate of return is larger than the market price of capital. The condition (C.11) indicates that the marginal revenue $P'(Q)Q + P(Q)$ must equal the regulated marginal cost $eE_Q(Q,K)$ which is the marginal cost of producing an additional unit of output when K is set at the level required to satisfy the rate-of-return constraint (C.9). Using that condition and the rate-of-return constraint (C.9), Callen et al., (1976) obtain the optimal decisions (K_R, Q_R) for a Cobb-Douglas production function and then evaluate: C_R the cost incurred by the regulated operator and W_R the net social welfare. Their results are summarized in Table C-2.

Table C-2. The optimal decisions taken by a regulated monopoly (case $s > r$)

Output	$Q_R = \left[\dfrac{A\delta - e}{s\delta^{1/\alpha}}\right]^{\alpha/\gamma}$	(C.12)
Capital	$K_R = \delta^{(1-\alpha)/\alpha} Q_R^{\eta/\alpha}$	(C.13)
Cost	$C_R = r\delta^{(1-\alpha)/\alpha} Q_R^{\eta/\alpha} + \dfrac{e}{\delta} Q_R^{1-\varepsilon}$	(C.14)
Net social welfare	$W_R = \dfrac{1}{1-\varepsilon} P(Q_R) Q_R - C_R$	(C.15)

In the specific case $s = r$, the allowed rate of return equals the market price of capital and the regulated firm is constrained to make at most zero economic profit. Klevorick (1971) highlights that the behavior of the regulated monopoly is indeterminate: the three combinations $(0,0)$, (K_a, Q_a), and (K_R, Q_R) evaluated with $s = r$ yield zero economic profit. To avoid that indeterminacy, we assume hereafter that the rate effectively implemented by the regulatory authority will be no less than r plus an infinitesimally small and positive increment. This rule imposes the choice of the combination (K_R, Q_R).

The socially desirable rate or return

Klevorick (1971) and Callen et al. (1976) both examine the determination by a regulator of the fair rate of return s that maximizes the net social welfare given the regulated firm's reactions to that rate. They consider the two-level optimization problem:

$$\max_{s} \; W(s) = \int_0^Q P(q)\,dq \; - \; r\,K \; - \; e\,E(Q,K) \qquad \text{(C.16)}$$

$$\text{s.t.} \quad \left[\begin{array}{ll} \displaystyle\max_{K,Q} & \Pi(Q) = P(Q)Q \; - \; r\,K \; - \; e\,E(Q,K) \\ \text{s.t.} & P(Q)Q \; - \; e\,E(Q,K) \; = \; s\,K \\ & K \geq 0 \;,\; Q \geq 0\,. \end{array}\right.$$

We let s_R denote the solution to that program. The discussion above has shown that for a given rate of return s with $s_M > s > r$, the unique solution to the lower-level problem is the pair (K_R, Q_R) defined in Table C-2 which is parameterized by s. Callen et al. (1976) thus reformulate the problem as a single-variable optimization problem:[13]

$$\underset{s}{\text{Max}} \quad W(s) = \int_0^{Q_R(s)} P(q)dq \; - \; r\,K_R(s) \; - \; e\,E\big(Q_R(s), K_R(s)\big). \tag{C.17}$$

The first-order condition for optimality yields the optimum value of the allowable rate of return s_R :

$$s_R = \frac{\eta^2 r}{\alpha\left[\beta - (1-\alpha)(1-\varepsilon)^2\right]}. \tag{C.18}$$

Note that, by assumption, the condition $0 < \varepsilon < 1$ holds, so the socially desirable rate of return s_R is lower than s_M the one obtained by the unregulated monopolist.

The rate s_R in (C.18) is valid if and only if, it verifies $s_R > r$, that is, if the elasticity and technological parameters are such that $\eta^2 > \alpha\left[\beta - (1-\alpha)(1-\varepsilon)^2\right]$. If that is not the case, the authority's best decision is to set s_R equal to r (plus an infinitesimally small and positive increment).

III.6.4 Static comparisons

To assess the performance of rate-of-return regulation, Callen et al. (1976) propose a series of ratios that are detailed in Table C-3. These ratios are entirely determined by the ratio s/r, the demand elasticity and the technology parameters.

These ratios respectively compare:

- the output levels decided by: a private monopoly Q_M, a social planner applying the average-cost-pricing rule Q_a and a regulated monopoly Q_R;
- the cost C_R incurred by the regulated firm and $C(Q_R)$ the cost that would have been incurred by a cost-minimizing firm producing the same output Q_R to assess the magnitude of the cost-increases caused by the Averch-Johnson effect (Averch and Johnson, 1962).
- the gain in net social welfare resulting from the regulation of a private monopoly $(W_R - W_M)$ versus $(W_a - W_M)$ the gain in net social welfare resulting from the implementation of a social planner applying the average-cost-pricing rule in a monopolistically-controlled industry.

[13] Note that this reformulation is rendered possible by their derivation of an analytical solution of the lower-level problem for the specific case of a Cobb-Douglas specification for the production function.

Table C-3. The performance ratios

Output	$\dfrac{Q_R}{Q_a}=\left(\dfrac{r}{s}\dfrac{\eta}{(1-\varepsilon)\alpha}\right)^{\alpha/\gamma}\left(\dfrac{(1-\varepsilon)}{\beta}\right)^{\frac{1}{\gamma}}$
Output	$\dfrac{Q_R}{Q_M}=\left(\dfrac{r}{s}\dfrac{\eta}{(1-\varepsilon)\alpha}\right)^{\alpha/\gamma}$
Cost	$\dfrac{C_R}{C(Q_R)}=(1-\alpha)\left[\dfrac{s}{r}\dfrac{(1-\varepsilon)\alpha}{\eta}\right]^{\alpha}+\alpha\left[\dfrac{r}{s}\dfrac{\eta}{(1-\varepsilon)\alpha}\right]^{1-\alpha}$
Net social welfare	$\dfrac{(W_R-W_M)}{(W_a-W_M)}=\dfrac{A}{B}$ Where $A\equiv\begin{cases}\left[\dfrac{1}{1-\varepsilon}\left(\dfrac{Q_R}{Q_M}\right)^{1-\varepsilon}\left(\dfrac{1-\varepsilon}{\beta}\right)^{\frac{1-\varepsilon}{\gamma}}-\dfrac{C_R}{C(Q_R)}\left(\dfrac{Q_R}{Q_M}\right)^{\beta}\left(\dfrac{1-\varepsilon}{\beta}\right)^{\frac{\beta}{\gamma}}\right]\\ -\left[\dfrac{1}{1-\varepsilon}\left(\dfrac{1-\varepsilon}{\beta}\right)^{\frac{1-\varepsilon}{\gamma}}-\left(\dfrac{1-\varepsilon}{\beta}\right)^{\frac{\beta}{\gamma}}\right]\end{cases}$ and $B\equiv\dfrac{\varepsilon}{1-\varepsilon}-\left[\dfrac{1}{1-\varepsilon}\left(\dfrac{1-\varepsilon}{\beta}\right)^{\frac{1-\varepsilon}{\gamma}}-\left(\dfrac{1-\varepsilon}{\beta}\right)^{\frac{\beta}{\gamma}}\right]$

Note: *As the derivation of the ratio* $(W_R-W_M)/(W_a-W_M)$ *is not detailed in Callen et al. (1976), we briefly explain how it can be reconstructed. The net social welfare* W_M *and* W_a *are obtained using the formula:* $W=\left[A/(1-\varepsilon)\right]Q^{1-\varepsilon}-C(Q)$.

Recall that Q_a *is the output such that price equals the average cost:* $A(Q_a)^{1-\varepsilon}=C(Q_a)$. *So, the net social welfare is:* $W_a=P(Q_a)Q_a\left[\varepsilon/(1-\varepsilon)\right]$.

Remarking that $Q_M=\sqrt[\gamma]{((1-\varepsilon)/\beta)}Q_a$ *and using the relation* $A(Q_a)^{1-\varepsilon}=C(Q_a)$, *the net social welfare obtained in case of a monopoly is:*

$$W_M=P(Q_a)Q_a.\left(\frac{1}{1-\varepsilon}\left(\frac{1-\varepsilon}{\beta}\right)^{\frac{1-\varepsilon}{\gamma}}-\left(\frac{1-\varepsilon}{\beta}\right)^{\frac{\beta}{\gamma}}\right), \tag{C.19}$$

Under rate-of-return regulation, the net social welfare W_R is defined in (C.15) and can be rearranged as follows:

$$W_R = \frac{A}{1-\varepsilon}\left(\frac{Q_R}{Q_M}\times\frac{Q_M}{Q_a}\times Q_a\right)^{1-\varepsilon} - \frac{C_R}{C\left(Q_R\right)}C\left(\frac{Q_R}{Q_M}\times\frac{Q_M}{Q_a}\times Q_a\right). \qquad \text{(C.20)}$$

As the output Q_a is such that $A\left(Q_a\right)^{1-\varepsilon} = C\left(Q_a\right)$, the net social welfare W_R can be rewritten so as to be directly proportional to the total revenue $P\left(Q_a\right)Q_a$ obtained by the firm if average cost pricing is implemented:

$$W_R = P\left(Q_a\right)Q_a \cdot \left[\frac{1}{1-\varepsilon}\left(\frac{Q_R}{Q_M}\right)^{1-\varepsilon}\left(\frac{1-\varepsilon}{\beta}\right)^{\frac{1-\varepsilon}{\gamma}} - \frac{C_R}{C\left(Q_R\right)}\left(\frac{Q_R}{Q_M}\right)^{\beta}\left(\frac{1-\varepsilon}{\beta}\right)^{\frac{\beta}{\gamma}}\right]. \qquad \text{(C.21)}$$

III.6.5 References

Averch, H., Johnson, L., 1962. Behavior of the firm under regulatory constraint. *American Economic Review*, 52(5), 1052–1069.

Callen J., Mathewson G.F., Mohring H., 1976. The Benefits and Costs of Rate of Return Regulation. *The American Economic Review*, 66(3), 290–97.

Klevorick, A.K., 1971. The ‘Optimal’ Fair Rate of Return. *The Bell Journal of Economics and Management Science*, 2(1), 122–53.

Chapter IV. Unlocking natural gas pipeline deployment in a LDC: a note on rate-of-return regulation

IV.1 Introduction

In a small developing economy, the discovery of large natural gas deposits is usually depicted as a bonanza as it is expected that the rise of gas extraction activities will catapult the country away from a low development trap. Yet, translating a mineral resource wealth into developmental achievements that are perceptible by the greater population is a challenging task with possibly dramatic consequences if their absence can trigger regional tensions or jeopardize the country's political stability. While pursuing their quests for inclusive growth and development strategies, governments are often ambitioning to leverage on the nation's resource endowment and use a share of the gas extracted to fuel the country's growing energy needs (e.g., by promoting the domestic use of natural gas in the power sector or in small and medium businesses). Yet, the feasibility of such a strategy necessitates the construction of a capital intensive domestic pipeline infrastructure. Hence, understanding how regulatory tools can be applied to this complex technology in developing economies is key.

The purpose of this paper is to contribute to the analysis of the economics of the *ex-nihilo* construction of a new infrastructure project in a LDC. We consider the case of a sizeable natural gas pipeline system, aimed at supplying gas to a developing region where gas consumption is limited at present. Development planners envision that this infrastructure could trigger possible future developments of the domestic natural gas sector. Because of an assumed capital scarcity in that country, the infrastructure is provided by a foreign private company that is subject to a traditional rate-of-return (RoR) regulation. The firm is presumed to be reluctant to build ahead of proven demand. Using a representation of gas pipeline technology adapted to economic studies, we examine whether RoR regulation be tuned to accommodate for future growing demand.

Based on the example of recent natural gas discoveries and related development projects in Mozambique, we detail in a first section the roles and goals of agents involved in pipeline investment in a developing country. We then turn to analytical modeling to describe the interactions between those players and conclude on the benefits of tuning regulation. We start by combining a simplified gas pipeline representation and classical economic literature examining rate-of-return regulation to analyze the behavior of the investor at planning stage. Then, we derive the potential reaction of the operator to new demand conditions and compare it to the hypothetical optimal combination of capital and energy factors. This leads us to analytical prove that when the demand anticipated by the investor and the demand that actually materialize differ moderately, the parameters of the initial regulation can be tuned to influence the investors planning decisions and lead to optimal management under actual demand conditions. We conclude in a final section by the illustrating those results numerically on simple point to point infrastructure and for the case of Mozambique.

Useful policy recommendations for LDCs investing in gas infrastructure projects can be derived from this work, while it proves to be a valuable tool for regulatory agencies looking for a better understanding of RoR regulation when applied to gas infrastructure.

IV.2 Background

In this section, we use a real pipeline project in Northern Mozambique to review the specificities of the provision of natural gas infrastructures in developing countries. We first detail the economic and institutional context. We then examine the opportunity of building ahead of demand such a pipeline infrastructure when domestic demand is initially limited. Finally, we discuss the regulatory framework imposed on pipeline operators for such projects and highlight its consequences on infrastructure design in the specific case of rate-of-return regulation.

IV.2.1 The Mozambican natural gas scene

Emergence of a gas-fired economy

Mozambique recently joined the club of gas-rich nations as a series of large discoveries offshore the country's northeastern coast, in the Rovuma basin, radically changed the country's resource endowment.[14] If monetized, these resources have the potential to fundamentally transform this economy which is currently one of the world's least developed (Melina and Xiong, 2013). To generate the flows of export revenues needed to finance the costly extraction equipment of the Rovuma fields, the government and the oil companies are currently advancing an ambitious investment plan in Liquefied Natural Gas (LNG) manufacturing which could make the country the third largest LNG exporter in the world after Qatar and Australia (IMF, 2016). Yet, LNG exports alone are unlikely to solve all the economic problems of Mozambique as they promise few permanent jobs and generate little forward linkages with the rest of the economy. To improve the wellbeing of the population, the government also adopted a national gas master plan aimed at leveraging on the nation's subsoil wealth to leap forward by accelerating both industrialization and the emergence of domestic gas uses (ICF, 2012; Ministério da Planificação E Desenvolvimento, 2014).

The first aspect of the Mozambican plan consists in the installation of export-oriented, large-scale gas-based industries to manufacture fertilizers (ammonia, urea), petrochemicals (methanol, olefins) or direct reduced iron. Following the Rovuma discoveries, a number of foreign investors companies have expressed interest in developing such "mega-projects" in Mozambique. These projects are financed by foreign direct investment (FDI) and makes it possible to organize the emergence of an industrial sector without having to imitate the public-sector driven and commercially inadequate industrialization strategies experienced in hydrocarbon-rich countries during the early 1980's (Auty and Gelb, 1986; Auty, 1988a, 1988b). The expected benefits of these industries include the creation of linkages à la Hirschman (1958)[15] and a moderation of the variance of the country's export revenues by diversifying commodity price risk away from gas (Massol and Banal-Estañol, 2014).

The second aspect of the governmental strategy echoes both the strong expectation from the population that the benefits of gas developments should be felt directly and the rise of the country's energy needs recently highlighted in the long-run scenarios discussed in Mahumane and Mulder

[14] According to the 2014 Oil & Gas Journal annual survey, Mozambique's proved natural gas reserves now amounts to 100 trillion cubic feet (Tcf) – compared to 4.5 Tcf the previous year – indicating that the country's endowment is now the third largest in Africa (Xu and Bell, 2013).

[15] For example, a fertilizer industry is expected to foster the modernization of the country's agricultural sector which is currently dominated by subsistence farming with a scant use of fertilizers (Franza, 2013).

(2016). It emphasizes the supply of natural gas to domestic-oriented uses (e.g., local businesses, cement manufactures, households) to promote job-creation and allow the substitution of expensive and imported oil products (ICF, 2012). It also envisions some potential in the power sector as the country's electrification rate is low[16] and the extension of the national grid remains an important policy objective (IEA, 2014). In the Northern provinces, the installation of thermal generators fueled by gas extracted from the Rovuma fields could support the government's electrification plans (ICF, 2012). That said, it should be underlined that the domestic gas-consuming sectors have to be built from a very low existing base (IEA, 2014).

Mozambique's infrastructure deficit and pipeline ambitions

Geography rapidly comes into play when assessing the feasibility of the governmental aspirations related to the domestic-oriented uses. The Rovuma fields are located offshore the rural and little densely populated districts of the Cabo Delgado province which is one the country's least developed. Because of their remote location, the construction of a pipeline system connecting the fields to the country's main population and industrial centers is needed. However, implementing a gas pipeline infrastructure is a classic instance of a "chicken and egg" problem. It is not worth building an expensive pipeline system without a critical mass of consumers capable to support the construction of the infrastructure and, without the pipeline, the potential demand from users is unlikely to materialize. In a LDC, the problem is even trickier as neither the power sector nor the other domestic sectors[17] will attain that critical size in the foreseeable future.

To overcome it, it has been suggested to leverage on the FDI-financed mega-projects to facilitate the deployment of this pipeline infrastructure (ICF, 2012; IEA, 2014). Rather than allowing their constructions in Palma in the immediate neighborhood of the LNG plant, it is envisioned to strategically locate them in the deep-port city of Nacala that provides a larger development potential. The city is the marine terminal of a favorable agricultural production area which is home to approximately 10 million people: the Nacala Development Corridor that reaches westward from Nacala to landlocked Malawi and Zambia (Figure 1). Locating the mega-projects there would provide

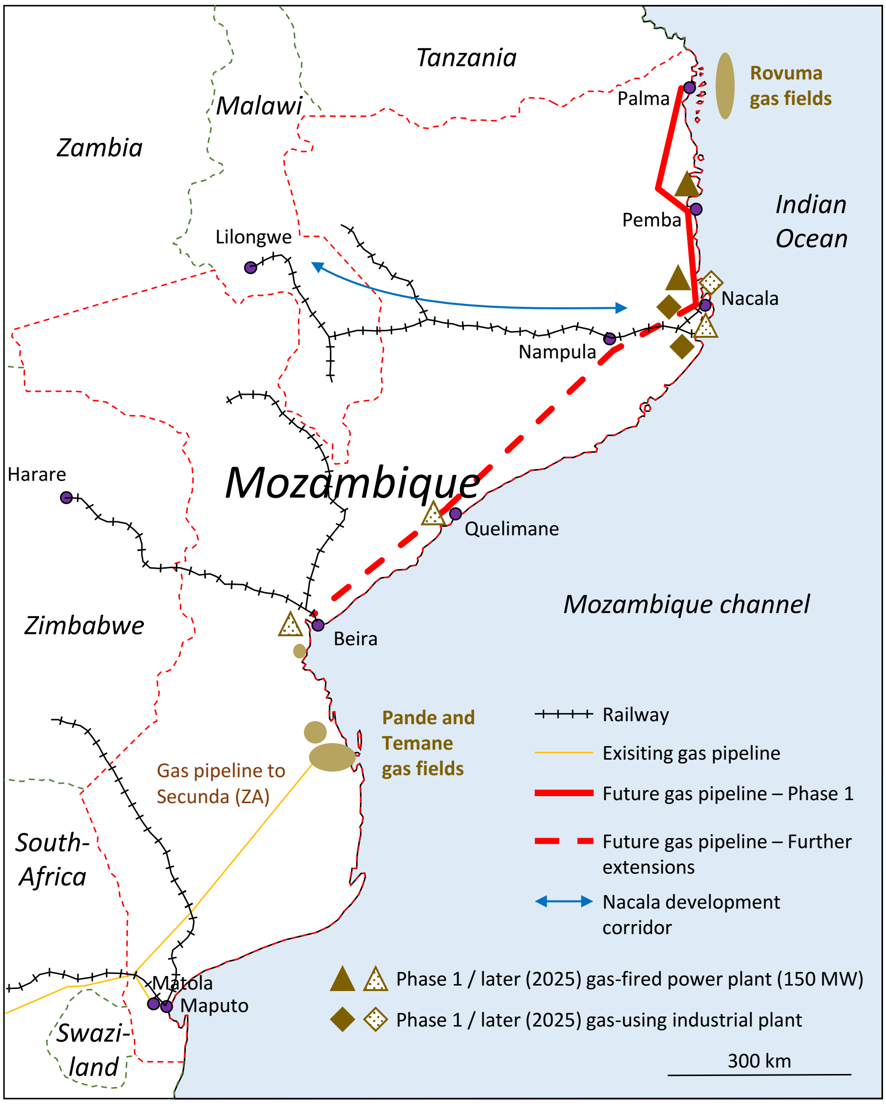
[16] The current electrification rate is 39% and the government's goal is to attain 85% by 2035 (IEA, 2014).

[17] To the authors' knowledge, the list of existing local industries that could potentially adopt natural gas along the pipeline is limited to three cement works operating in the Nacala area.

the "anchor" load needed to justify the construction of a first pipeline system running along the northern coastline from Palma to Nacala (ICF, 2012). In recognition of this, the government is actively trying to promote FDI in Nacala and has created a special economic zone aimed at providing incentives and guarantees to foreign investors.

This Palma-to-Nacala pipeline unlocks the potential for the future infrastructure development phases envisioned in ICF (2012). The route is aimed at being integrated within a longer pipeline system reaching first the cities of Nampula, Quelimane and Beira and ultimately the capital Maputo and the South African market. It could also be integrated within a broader transnational pipeline infrastructure such as the one recently examined by researchers from the Earth Institute at Columbia University (Demierre et al., 2015).

Figure 1. Map of Mozambique's northern pipeline deployment.



IV.2.2 Building a pipeline ahead of demand?

While the possible route of any future gas pipeline is mostly determined by geographic and economic considerations, as in our Mozambican example, important engineering aspects come into play when calibrating the size of the future infrastructure. In particular, the cost-engineering literature on natural gas pipeline systems provides two interesting insights. First, the technology of a gas pipeline system exhibits pronounced increasing return to scale in the long-run (Kahn, 1988, vol. II, p. 153; Yépez, 2008). The long-run cost function is strictly subadditive and verifies the technological condition for a natural monopoly (Perrotton and Massol, 2017). Second, investment in a natural gas pipeline conveys some irreversibility. *Ex ante*, during the planning phase, investors can use any combination of pipe diameter and compressors horsepower, as long as the corresponding engineering constraints are observed. However, once installed, the diameter of a pipeline – and thus the capital stock that has been immobilized – can no longer be modified without incurring prohibitive costs.[18] Because of this irreversibility, any *ex post* rise in output must be accommodated by adjustments in the compression horsepower. The joint presence of irreversibility and pronounced economies of scale has important implications for investment planning in a pipeline. The standard cost-engineering literature (e.g., Chenery, 1952; Manne, 1961) indicates that, in case of an output level that will rise in the future, it is rationale to "build ahead of demand" and install an optimum degree of overcapacity (in the form of a larger pipeline diameter than the one installed in case of zero future increment in output) to minimize the present value of the infrastructure's total cost. In our Mozambican case, a Palma-to-Nacala pipeline would initially serve a low demand that is expected to grow as years go by and it can be justified to install a certain level of excess capacity on that pipeline system to anticipate the envisioned phased pipeline deployment.

Yet, applying that strategy a developing country deserves a pragmatic examination. The optimal degree of overcapacity to be installed is directly related to the timing and the magnitude of the future increments in output (Massol, 2011). Given the currently embryonic nature of the domestic-oriented uses in Mozambique and the difficulty to predict their future trajectory, it can be hard for a pipeline operator to *ex ante* invest in a capacity that largely overshoots the predictable demand levels (i.e., those from the mega-projects). In Mozambique, it is expected that the capital needs of the pipeline operator will be financed by its cash flow stream. Hence, before the expansion of the domestic sector (or the second phase) will materialize, the infrastructure will have to operate at a low degree of capacity utilization and the extra-cost of that overcapacity will thus have to be recouped from the FDI-financed mega-projects through higher prices. Yet, the promoters of these mega-projects are unlikely to decide the installation of immobile gas-processing assets in Nacala without having before: (i) signed supply contracts ensuring that a certain volume of natural gas will be delivered at the gate of their plant and (ii) verified that the price provisions stipulated in these contracts allow their projects to break-even. Moreover, even if extra capacity cost does not hamper the construction of the mega-

[18] The diameter is thus reputed to give an index of the size of the infrastructure (Chenery, 1952).

projects, the higher prices may also negatively influence the emergence of a domestic-oriented gas consuming sector. All in all, this discussion highlights an important limitation of the standard literature supporting the policy recommendation to "build ahead of demand": it implicitly posits that the infrastructure's output levels are price inelastic. As the price sensitivity of the demand can hardly be neglected in developing regions lacking an existing domestic gas sector like northern Mozambique, that standard literature has to be extended to incorporate both the pricing behavior of the pipeline operator and the demand response to its price.

IV.2.3 Pipeline infrastructure provision in Mozambique

The preceding discussion calls for a clarification of the status of the pipeline operator and the price setting mechanism likely to be at play. As gas pipelines can be identified as natural monopolies, a regulatory framework is necessary to protect the society from monopoly prices. Therefore, we now review the institutional context governing the provision of infrastructures in a LDC and how these considerations have shaped the Mozambican regulatory framework applied to natural gas pipelines.

During the 1960s, an international consensus emphasized the provision of energy infrastructures by public enterprises because their objectives (e.g., welfare maximization, cost minimization) were purportedly aligned with the ones pursued by development planners. However, in the early 1990s, sentiment had changed: the poor performance of some state-owned firms motivated a shift toward liberalization and privatization leading to an increased participation of foreign firms (Parker and Kirkpatrick, 2005). In a comprehensive review focused on LDCs, Joskow (1999) examines how effective regulatory institutions can be established in a developing economy. He first stresses that the regulatory framework should be adapted to take into account the presence of weak institutions and the lack of regulatory and antitrust expertise. He then highlights that, in case of a nascent infrastructure sector, it is preferable to implement simple rules and procedures. Lastly, the ideal regulatory framework should be adapted to account for industry-specific economic attributes and public policy goals. On that later point, Joskow also underlines that, depending on the local context, the public policy objectives (e.g., attracting investment, increasing sector productivity, bringing prices in line with costs) can be conflicting goals and strongly recommends policy makers to prioritize them.

In the early 2000s, Mozambique already had to face these discussions and clarify its objectives. At that time, the country was planning the construction of the southern gas pipeline system connecting the Pande and Temane fields to large users located in South-Africa (see Figure 1). In light of the pressing development needs faced by the state, it was then decided that the Mozambican pipeline infrastructures must be FDI-financed which makes investment attraction the chief objective assigned to the country's regulatory framework. Mozambique thus opted for a simple and proven form of rate-of-return (RoR) regulation for the gas pipeline infrastructures.[19] RoR regulation sees costs as exogenous and observable and forms prices on the basis of observed costs and a predetermined appropriate rate of return on the investments. This form of regulation thus provides foreign investors with a reasonable opportunity to recover investment and operating costs as well as a return on capital.

The shortcomings of RoR regulation are extensively discussed in the literature though and were first presented in Averch and Johnson (1962). The main reservations against this approach are that it does not provide incentives for cost savings and efficiency improvements, and that it rewards an excessive investment in fixed assets. This so-called Averch-Johnson effect calls for a condemnation of the tendency of regulated firms to engage in excessive amounts of durable capital accumulation to expand the volume of their profits.[20]

For the Palma-to-Nacala pipeline and similar projects, this effect could play a positive role though. Our discussion in section 2.2 suggests that, while governmental planners could wish to encourage some degree of "building ahead of proven demand" to cost-efficiently supply the future flows of gas consumed in Mozambique, they may have a hard time convincing the foreign investors in the pipeline system to immobilize the extra amount of capital needed to serve an embryonic domestic market whose future take-off is far from being granted. Interestingly, the Averch-Johnson effect suggests that a myopic (or conservative) profit-maximizing operator subject to RoR regulation (i.e., a firm that totally ignores the evolution of the domestic-oriented uses and solely bases its decisions on the natural

[19] This form of regulation has been extensively used to regulate privately owned pipeline infrastructures in the US and is also implemented for natural gas pipelines in a neighboring country: South Africa.

[20] A possible remedy to this effect can consist in the adoption of a regulatory control over the input choice (Laffont and Tirole 1993). Yet, that control can be difficult to organize in the context of a LDC where the newly created regulatory agency may face a shortage of skilled personnel.

gas demand from the FDI-financed mega-projects) can rationally decide some degree of overcapitalization. In the next section, we will thus show that under certain conditions planners could leverage on the behavior of the regulated firm to induce the installation of an appropriate degree of overcapacity.

IV.3 Model

In this section, we present our modeling framework and prove that it is possible to adapt the parameters of rate-of-return regulation to a potential demand growth so as to encourage building ahead of demand and foster efficient pipeline operation. We first introduce our representation of the technology of a gas pipeline. We then examine the investment planning decisions taken by a regulated operator who adopts a conservative attitude with respect to the demand estimates. We then derive the firm's reaction to an *ex post* demand growth and prove that under certain conditions, it is possible to induce *ex ante* an efficient degree of building ahead of demand by adjusting the allowed rate of return.

For the sake of concision, all the mathematical proofs are presented in Appendix B.

IV.3.1 Assumptions and notations

Technology

We consider the installation of a simple point-to-point pipeline system that consists of a compressor station injecting a pressurized flow of natural gas Q into a pipeline to transport it across a given distance. Building on a rich engineering-based literature (e.g., Chenery 1949; Kabirian and Hemmati, 2007; Yépez, 2008; André and Bonnans, 2011), Perrotton and Massol (2017) recently proved that the engineering equations governing the design and operations of such an infrastructure can be approximated by a single production equation of the Cobb-Douglas type that relates three economic variables: Q the flow of gas transported along the pipeline, E the amount of energy consumed by the infrastructure to power the compressor, and K the capital stock employed by the pipeline operator which is directly related to the diameter of the pipeline. This production function is:

$$Q^{\beta} = K^{\alpha} E^{1-\alpha}, \tag{1}$$

where the capital exponent parameter is $\alpha = 8/11$ and $\beta = 9/11$ is the inverse of the degree to which output is homogeneous in capital and energy. As $\beta < 1$, the technology exhibits increasing returns to scale.

From that production function, one can define $E(Q,K) = \sqrt[1-\alpha]{K^{-\alpha}Q^{\beta}}$ the variable input requirements function that gives the amount of energy needed to transport the output Q on a pipeline infrastructure that has a given fixed amount of capital input K. We let $E_Q(Q,K)$ (respectively, $E_K(Q,K)$) denote the derivative of the input requirement function with respect to the output (respectively, the capital) variable. With our technology parameters, $E_Q(Q,K) > 0$ and $E_K(Q,K) < 0$.

Input prices

We let e denote the market price of the energy input and r the market price of capital faced by the firm.

Costs

The long-run cost-minimizing amount of capital stock needed to transport the flow Q is (Perrotton and Massol, 2017):

$$K(Q) = \left(\frac{e\alpha}{r(1-\alpha)}\right)^{1-\alpha} Q^{\beta}, \tag{2}$$

and the long-run total cost function is $C(Q) = rK(Q) + eE(Q,K(Q))$ which after simplification is:

$$C(Q) = \frac{r^{\alpha}e^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} Q^{\beta}. \tag{3}$$

This presentation naturally leads to the following definition.

Definition: *The capital-output combination (K,Q) is cost-efficient if the capital stock K equals the long-run cost-minimizing amount of capital stock needed to transport the flow Q, that is $K = K(Q)$.*

IV.3.2 The ex ante behavior of the regulated firm

As that pipeline system has elements of a natural monopoly, we assume that the infrastructure is provided by a monopolistic private operator that is subject to rate-of-return regulation and examine the *ex ante* behavior of that regulated firm at the planning stage (i.e., before the construction of the infrastructure). Consistent with the discussion in Section 2, we assume that the regulated firm is prudent and only considers *ex ante* a conservative demand schedule that only represents the aggregate consumption emanating from a limited list of existing large industrial users that could start utilizing natural gas after the opening of the infrastructure. To examine that situation, we briefly review the standard contributions of Klevorick (1971) and Callen et al. (1976) that clarify the behavior of a regulated monopoly that serves a known demand schedule. We suppose that this inverse aggregate demand function is:

$$P(Q) = A\,Q^{-\varepsilon}, \tag{4}$$

where A is a constant and the constant $1/\varepsilon$ is positive and denote the absolute value of the price elasticity of demand. We assume, as did Klevorick (1971) and Callen et al. (1976), that: $\varepsilon < 1$ so that the total revenue obtained by a monopolist producing zero output is zero and that $\varepsilon > 1-\beta$ so that the demand schedule always intersects the marginal cost schedule from above.[21] For notational convenience, we follow the convention in Callen et al. (1976) and introduce three parameters: (i) $\gamma \equiv \beta + \varepsilon - 1$, (ii) $\delta \equiv e\beta / \left[A(1-\varepsilon)(1-\alpha)\right]$, and (iii) $\eta \equiv \beta - (1-\varepsilon)(1-\alpha)$.

The regulated monopoly is allowed to earn an exogenously-determined rate of return s which is not greater than the rate of return $s_M \equiv \eta r / \left[\alpha(1-\varepsilon)\right]$ that would have been obtained by an unregulated monopolist.[22]

[21] These restrictions together impose the absolute value of the demand elasticity to be in the range (1,11/2) which should not be a concern in most applications. For example, in the empirical study in Maddala et al. (1997), the estimated the absolute value of the long-run price elasticity of natural gas demand in the U.S. residential sector is about 1.358.

[22] For concision, the derivation of s_M is detailed in Appendix A.

The rate-of-return constraint stipulates that the monopoly's accounting profit which is defined as the difference between: $P(Q)Q$ the revenue derived from the output level Q, and $eE(Q,K)$ the cost of the variable input used in producing it cannot exceed the allowed return on invested capital sK. As the condition $s \leq s_M$ holds, the rate-of-return constraint is binding and can be formulated as an equality constraint:

$$P(Q)Q - e\,E(Q,K) = s\,K\,, \tag{5}$$

Assuming profit maximization, the behavior of the regulated monopoly amounts to decide the combination of capital K and output Q that solves the optimization program (6) presented in Table 1 (Panel 1) where $P(Q)Q$ is the total revenue derived from selling Q units of products, rK is the total cost of capital, $eE(Q,K)$ is the total cost of energy.

Table 1. The *ex ante* behavior of the regulated firm (Callen et al. , 1976)

	The regulated monopoly
Panel 1: **Optimization program if** $s \le s_M$	$$\underset{K,Q}{\text{Max}} \quad \Pi(Q) = P(Q)Q - rK - eE(Q,K)$$ $$\text{s.t.} \quad P(Q)Q - eE(Q,K) = sK \qquad (6)$$ $$K \ge 0 \ , \ Q \ge 0 .$$
Panel 2: Solution if $r \le s \le s_M$	
Output	$$Q^* = \left[\frac{A\delta - e}{s\delta^{1/\alpha}}\right]^{\alpha/\gamma} \qquad (7)$$
Capital	$$K^* = \delta^{(1-\alpha)/\alpha} Q^{*\eta/\alpha} \qquad (8)$$
Panel 3: Implications	
Cost	$$C^* = rK^* + eE\left(Q^*, K^*\right) = r\delta^{(1-\alpha)/\alpha} Q^{*\eta/\alpha} + \frac{e}{\delta} Q^{*1-\varepsilon} \qquad (9)$$
Net Social Welfare	$$W^* = \int_0^{Q^*} P(q)\,dq - C^* = \frac{1}{1-\varepsilon} P\left(Q^*\right)Q^* - C^* \qquad (10)$$
Output ratio [(a)]	$$\frac{Q^*}{Q_M} = \left(\frac{r}{s}\frac{\eta}{(1-\varepsilon)\alpha}\right)^{\alpha/\gamma} \qquad (11)$$
Overcapitalization ratio	$$\frac{K^*}{K\left(Q^*\right)} = \left(\frac{r}{s}\frac{\eta}{(1-\varepsilon)\alpha}\right)^{1-\alpha} \qquad (12)$$
Cost ratio	$$\frac{C^*}{C\left(Q^*\right)} = (1-\alpha)\left[\frac{s}{r}\frac{(1-\varepsilon)\alpha}{\eta}\right]^{\alpha} + \alpha\left[\frac{r}{s}\frac{\eta}{(1-\varepsilon)\alpha}\right]^{1-\alpha} \qquad (13)$$

Note: If $s = r$ *, the pair* $\left(K^*, Q^*\right)$ *is not the unique solution to the optimization program (Klevorick, 1971).* (a) *For concision, the derivation of the output* Q_M *decided by an unregulated private monopoly is detailed in Appendix A.*

If the allowed rate of return is lower than the market price of capital (i.e., $s < r$), profit maximization involves a corner solution: the firm's optimal decision is to withdraw from the market. We thus concentrate on the more interesting case whereby the allowed rate of return is not lower than the cost of capital (i.e., $s \ge r$ and $s \le s_M$).

As shown in Klevorick (1971), the first order optimality conditions of the problem (6) are such that the firm's optimal decisions must jointly verify the rate-of-return constraint (C.9) and the following condition:

$$\left(s-r\right)\left[P'\left(Q\right)Q+P\left(Q\right)\ -e\,E_Q\left(Q,K\right)\right]\ =0\,, \tag{14}$$

where $E_Q\left(Q,K\right)$ is the derivative of the input requirement function with respect to the output variable.

If $s>r$ and $s\leq s_M$, the condition (C.11) is equivalent to $P'\left(Q\right)Q+P\left(Q\right)-eE_Q\left(Q,K\right)=0$ which is the analogue for a regulated monopoly of the standard condition for profit maximization: the marginal revenue $P'\left(Q\right)Q+P\left(Q\right)$ has to be equal to the regulated marginal cost $eE_Q\left(Q,K\right)$ which is the marginal cost to produce an additional unit of output when K is set at the level required to satisfy the rate-of-return constraint (C.9). Callen et al. (1976) detail the unique capital and output combination $\left(K^*,Q^*\right)$ that solves the optimization program (cf., Table 1– Panel 2).

If the allowed rate of return is set equal to the market price of capital ($s=r$), the behavior of the regulated monopoly is indeterminate as any capital and output combination that verifies the rate-of-return constraint (C.9) – i.e., which yields zero profit – can be considered by the firm (Klevorick, 1971).[23] As that model does not allow to prefer one of them, we thereafter prohibit setting s equal to r and only consider that s is in the range $r<s\leq s_M$.

To examine the implications, Callen et al. (1976) also detail: (i) the total cost C^* incurred by the regulated firm; (ii) the net social welfare W^*; (iii) the output ratio Q^*/Q_M that measures the relative increase in output effected by imposing a regulated rate of return on an unregulated monopoly; (iv) the capital ratio $K^*/K\left(Q^*\right)$ that provides a relative measure of the capital stock employed by the regulated firm K^* to $K\left(Q^*\right)$ obtained using (2) the capital stock that would have been installed by a cost-minimizing firm producing the same output; and (v) the cost ratio $C^*/C\left(Q^*\right)$ that compares C^* the total cost incurred by the regulated firm and $C\left(Q^*\right)$ the total cost that would have been incurred to

[23] If $s=r$, each of the pairs: $\left(0,0\right)$, $\left(K_a,Q_a\right)$ corresponding to the capital and output chosen by welfare-maximizing social planner subject to zero loss condition described in Appendix A, and $\left(K^*,Q^*\right)$ in Table 1, yields zero economic profit.

serve the output Q^* if production used a cost-minimizing combination of inputs. These values are presented in Table 1 (Panel 3).

The following lemma indicates that the capital ratio is inversely related to the allowed rate of return.

Lemma 1*: *The capital ratio $K^*/K(Q^*)$, is a smooth and monotonically decreasing function of s/r the ratio of the allowable rate of return to the cost of capital indicating that there is a one-to-one mapping between $s \in (r, s_M]$ the range of admissible values for the allowed rate of return and $\left[1, \lim_{s \to r} K^*/K(Q^*)\right) = \left[1, \left(\eta/\left[(1-\varepsilon)\alpha\right]\right)^{1-\alpha}\right)$ the range of feasible values for the overcapitalization ratio $K^*/K(Q^*)$.

For the sake of concision, we omit the straightforward proof of this lemma but rather emphasize the economic implications. If the allowed rate of return s (with $s > r$) is lower than the rate of return obtained by an unregulated monopoly (i.e., $s < s_M$), the value of the capital ratio is greater than one. Hence, the capital-output combination (K^*, Q^*) is not cost-efficient as the profit-maximizing behavior of the regulated firm causes the famous overcapitalization distortion pointed in Averch and Johnson (1962). This lemma also suggests that the allowed rate of return s is a control variable for that degree of overcapitalization.

IV.3.3 The ex post behavior of the regulated firm

Having observed the fair rate of return s with $r < s < s_M$, the regulated firm has signed supply contracts for a total pipeline throughput equal to Q^* and has installed the capital stock K^*. We now provide an original contribution to examine the *ex post* situation after the opening of the infrastructure.

Following the discussion in Section 2, we assume that an expanded demand is observed *ex post*. For example, investors could decide to locate new gas-based industrial facilities along the pipeline and existing small users that were overlooked at the planning stage (e.g., small and medium enterprises) may substitute expensive and polluting fuels (e.g., heating oil, coal) for natural gas once it becomes available.[24] So, the pipeline operator now faces the *ex post* inverse demand function:

$$P_{\lambda}(Q) = (1+\lambda)\, A\, Q^{-\varepsilon}, \tag{15}$$

where λ is a positive parameter reflecting the market enlargement unlocked by the pipeline.

Two important features have to be noted. First, the regulatory authority cannot renege on the fair rate of return s with $r < s < s_M$ that has been set before the construction of the pipeline. Second, investment in a pipeline has an irreversible nature: once installed, the diameter of a pipeline can no longer be modified without incurring prohibitive costs. The capital stock employed by the firm is thus fixed and maintained at the *ex ante* value K^*. Hence, any *ex post* change in output is solely accommodated by adjustments in the variable input: energy.

The firm now must verify the *ex post* rate-of-return constraint:

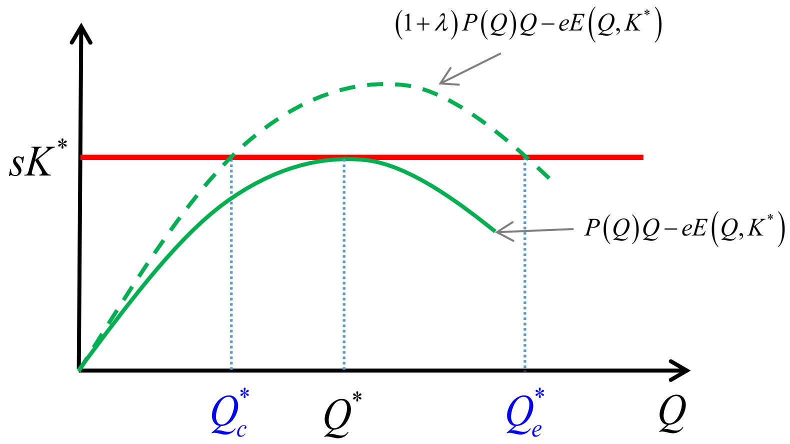
$$(1+\lambda) P(Q) Q - eE(Q, K^*) = sK^*. \tag{16}$$

As $\lambda > 0$, the output level Q^* chosen *ex ante* does not verify the *ex post* rate-of-return constraint. (Inserting Q^* in (16) and using the *ex ante* rate-of-return constraint (C.9) yields the equation $\lambda P(Q^*) Q^* = 0$ which cannot hold because Q^* is positive). So, the regulated firm has to adjust its output level *ex post*. To overcome that problem, the following proposition indicates that the firm can either consider a contraction of its output down to the level Q_c^* or an expansion up to Q_e^*.

Proposition 1: *If* $\lambda > 0$, *there exists exactly two output levels:* Q_c^* *and* Q_e^*, *such that the ex post rate-of-return constraint* (16) *is verified. These two output levels verify* $Q_c^* < Q^* < Q_e^*$.

[24] In a policy-oriented article, Sovacool (2009) also discusses the possibility to observe an enlarged demand ex post.

Figure 2. The *ex post* behaviour of the regulated firm with $r < s < s_M$



An illustration is presented in Figure 2. It shows the value of the regulatory constraint sK^* which is constant as the capital stock K^* is fixed and two curves. The solid one represents the *ex ante* situation and shows how the firm's accounting profit, obtained as the difference between the total revenue minus the total variable cost, that is $P(Q)Q - eE(Q,K^*)$, varies with the firm's output level. The developments above have shown that the unique output such that the *ex ante* accounting profit equals the allowed value sK^* is Q^*. The dotted curve illustrates the *ex post* case as it represents the *ex post* accounting profit $(1+\lambda)P(Q)Q - eE(Q,K^*)$. In that case, two output levels verify the *ex post* rate-of-return constraint (16).

Unless a secondary objective (e.g., sales maximization) is assumed, the *ex post* behaviour of the regulated firm is indeterminate. It is thus instructive to confront the two candidate solutions with the context presented in Section 2. In a developing country, the supply relationships between the gas producers connected to the pipeline system, the users served by the pipeline and the pipeline operator are governed by specific long-term bilateral contracts. These contracts are signed *ex ante* (i.e., before the construction of the infrastructure)[25] and traditionally include minimum "take-or-pay" obligations that: (i) compel the buyer to purchase at least the contracted quantity, and (ii) commit the producer and

[25] Recall that a pipeline has elements of a relationship-specific asset which, once investment in that infrastructure is sunk, generates appropriable specialized quasi rents (Klein et al., 1978). If transactions are governed by "simple" short-term contracts, asset-specific investments and uncertainty imply high transaction costs that can jeopardize the feasibility of the transaction and thus the construction of the infrastructure. In such situations and if full vertical integration is not feasible, transaction costs can be reduced by signing ex ante long-term contracts (Williamson, 1983) that includes requirement clauses, price indexation, liquidation damages, arbitration, and other provisions.

the pipeline operator to supply at least that quantity. Because of these contractual arrangements, a contraction of the output below the level Q^* is unlikely. Against this backdrop, an expansion of the output up to the level Q_e^* represents the preferred option.[26]

In general, it is not possible to determine a closed-form expression for Q_e^* as a function the technology and demand parameters. Nevertheless, the following corollary clarifies how the output level Q_e^* varies with the *ex post* demand expansion coefficient λ.

Corollary 1: *The output Q_e^* (respectively Q_c^*) is monotonically increasing (respectively decreasing) with the demand parameter λ.*

It should be noted that Corollary 1 and Proposition 1 jointly provide a characterization of the output level Q_e^* as the unique output level that both verifies the *ex post* rate-of-return constraint (16) and is monotonically increasing with the demand parameter λ.

IV.3.4 Installing ex ante an appropriate degree of overcapitalization

The discussion in Section 3.2 has highlighted the tendency of a myopic regulated firm to engage in excessive amounts of capital accumulation at the planning stage and the preceding subsection has just shown that, once the infrastructure is in place and if a larger demand materializes, this firm has no choice but to expand its output beyond the planned level. One may thus wonder whether the *ex post* expansion of the output could be large enough to "absorb" the larger-than-needed amount of capital stock immobilized *ex ante*. In other words, one could wonder whether, in case of an initial demand underestimation, the *ex ante* overcapitalization could provide an opportunity to optimally install the amount of capital stock needed to transport the *ex post* output in a cost-efficient manner.

[26] Of course, in case of a very large λ, one could question the feasibility of the output expansion without taking into consideration the technical constraints that govern the mechanical stability of a pressurized pipeline. Yet, a series of discussions with industry representatives and technical experts have convinced us that the influence of these technical considerations can be omitted for the range of λ considered in the present analysis.

To examine this, we first derive a closed-form expression for the capital-output combination that is cost-efficient and verifies the *ex post* rate-of-return constraint. Then, we clarify the conditions under which the regulated firm would install *ex ante* that desired level of capital stock.

The cost-efficient capital-output combination that verifies the *ex post* rate-of-return constraint

We consider a cost-efficient capital-output combination (K_{ce}, Q_{ce}) that also verifies the *ex post* rate-of-return constraint:

$$(1+\lambda)P(Q_{ce})Q_{ce} - eE(Q_{ce}, K_{ce}) = sK_{ce}, \tag{17}$$

where s is the given fair rate of return with $r < s < s_M$.

Replacing the capital stock K_{ce} by $K(Q_{ce})$ where the value of $K(Q_{ce})$ is defined in (2), subtracting the total cost of capital $rK(Q_{ce})$ on both sides of that equation and remarking that the total cost $rK(Q_{ce})+E(Q_{ce},K(Q_{ce}))$ equals the long-run total cost to transport the flow Q_{ce}, that is $C(Q_{ce}) = rK(Q_{ce})+E(Q_{ce},K(Q_{ce}))$, we obtain:

$$(1+\lambda)P(Q_{ce})Q_{ce} - C(Q_{ce}) = (s-r)K(Q_{ce}). \tag{18}$$

Substituting equations (2), (3) and (4) into (18) and solving that single-variable equation yields the output level Q_{ce} such that the capital-output combination $(K(Q_{ce}), Q_{ce})$ is cost-efficient and verifies the *ex post* rate-of-return constraint:

$$Q_{ce} = \left[\frac{(1+\lambda)A}{\left(\frac{s}{r}-1\right)\alpha+1} \left(\frac{\alpha}{r}\right)^{\alpha} \left(\frac{1-\alpha}{e}\right)^{1-\alpha} \right]^{1/\gamma} \tag{19}$$

Using the definition of the function that gives the long-run cost-minimizing amount of capital stock needed to transport a given output (cf., equation (2)), the associated amount of capital stock is $K_{ce} = K(Q_{ce})$:

$$K_{ce} = \left(\frac{e\alpha}{r(1-\alpha)} \right)^{1-\alpha} \left[\frac{(1+\lambda)A}{\left(\frac{s}{r} - 1 \right)\alpha + 1} \left(\frac{\alpha}{r} \right)^{\alpha} \left(\frac{1-\alpha}{e} \right)^{1-\alpha} \right]^{\beta/\gamma} \tag{20}$$

Two remarks can be formulated on the cost-efficient capital-output combination (K_{ce}, Q_{ce}). First, by construction, that combination verifies the *ex post* rate-of-return constraint (17). So, if the regulated firm *ex ante* installs an amount of capital stock equal to K_{ce}, the output level Q_{ce} represents a candidate output level that can be considered *ex-post* by the regulated firm. Thus, this is an instance for which a closed-form expression exists for a candidate output level.

Second, it should be noted that the cost-efficient capital stock K_{ce} in (20) is parameterized by the fair rate of return s. One may also recall that, following Lemma 1, it has already been highlighted that K^* the amount of capital stock decided by the regulated firm also varies with s. Hence, one may wish to explore whether that rate s could be adjusted at a level such that the regulated firm rationally decides to install *ex ante* the amount of capital stock K^* that equals K_{ce}.

Obtaining *ex ante* the installation of the cost-efficient amount of capital stock

We now explore the condition for the regulated firm to rationally decide to immobilize the capital stock K^* that equals the cost-efficient level $K_{ce} = K(Q_{ce})$ presented in (20).

Remarking that $K(Q_{ce}) = \left(Q_{ce}/Q^*\right)^{\beta} K(Q^*)$ and introducing the output level Q_M chosen by an unregulated monopoly facing the inverse demand function (4), the condition $K^* = K_{ce}$ is logically equivalent to:

$$\frac{K^*}{K(Q^*)} \times \left(\frac{Q^*}{Q_M} \times \frac{Q_M}{Q_{ce}} \right)^{\beta} = 1, \tag{21}$$

Proceeding in this fashion allows to readily identify the two ratios introduced in Callen et al. (1976) and reviewed in our section 3.2 (cf. Table 1): the capital ratio $K^*/K(Q^*)$ presented in equation (13) and the output ratio Q^*/Q_M detailed in equation (11). The ratio Q_M/Q_{ce} is easy to evaluate using the value of the output level Q_M indicated in (27) (see Appendix A) and that of Q_{ce} detailed in (19).

Substituting these results into equation (21) and simplifying, the condition for a firm facing a rate of return s and a given demand expansion factor λ to install *ex-ante* an amount of capital equal to the *ex-post* cost-efficient amount of capital becomes:

$$\frac{(1-\varepsilon)}{\beta}\left[\left(\frac{s}{r}-1\right)\alpha+1\right]\left(\frac{r}{s}\frac{\eta}{(1-\varepsilon)\alpha}\right)^{\eta/\beta}-1=\lambda\,. \tag{22}$$

The following proposition clarifies the conditions for that equation to hold.

Proposition 2: *For any* λ *such that* $0<\lambda<\bar{\lambda}$ *where* $\bar{\lambda}\equiv\left[\frac{\eta}{(1-\varepsilon)\alpha}\right]^{\frac{\eta}{\beta}}\left[\frac{1-\varepsilon}{\beta}\right]-1$, *there exists a unique fair rate of return* s_λ *in the open interval* (r,s^M) *such that the condition (22) is verified and the amount of capital installed* K^* *equals* $K_{ce}=K(Q_{ce})$ *the cost-minimizing amount needed to transport* Q_{ce} *given by the capital requirement function in* (2). *Moreover, the fair rate of return* s_λ *is monotonically decreasing with the demand parameter* λ.

So, if the *ex post* demand expansion coefficient λ is lower than the threshold value $\bar{\lambda}$, and if the regulator sets the fair rate of return at the level s_λ, the regulated firm's best response to that rate is to *ex ante* install the capital stock $K^*=K(Q_{ce})$. Proposition 1 indicates that, *ex post,* the regulated firm must adjusts its output to verify the *ex post* rate-of-return constraint (16) by either raising it to Q_e^* or lowering it to Q_c^*. By construction, the cost-efficient output level Q_{ce} verifies that *ex post* constraint and thus equals one of the two candidate output levels Q_c^* and Q_e^*. The following corollary should be useful to further characterize whether or not Q_{ce} involves an expansion beyond the *ex ante* output level Q^*.

Corollary 2: *For any* λ *such that* $0<\lambda<\bar{\lambda}$ *where* $\bar{\lambda}\equiv\left[\frac{\eta}{(1-\varepsilon)\alpha}\right]^{\frac{\eta}{\beta}}\left[\frac{1-\varepsilon}{\beta}\right]-1$, *if the regulator imposes the fair rate of return* $s=s_\lambda$, *the output level* Q_{ce} *in* (19) *is monotonically increasing with the demand parameter* λ.

Recall that (cf., the characterization derived from Proposition 1 and Corollary 1) Q_e^* is the unique output level that verifies the *ex post* rate-of-return constraint and is monotonically increasing with the demand parameter λ. This corollary thus confirms that $Q_e^*=Q_{ce}$.

To summarize, we have just shown that, if the *ex post* demand expansion coefficient λ is lower than the threshold value $\bar{\lambda}$, there exists a unique fair rate of return s_λ with $r<s_\lambda<s_M$ such that: (i) the regulated firm *ex ante* rationally installs the capital stock $K^*=K(Q_{ce})$ to supply the output Q^*, and (ii) *ex post*, the regulated firm reacts to the enlarged demand by increasing its output to attain the level Q_{ce} such that the *ex post* capital-output combination (K^*,Q_e^*) is cost efficient, that is $(K^*,Q_e^*)=(K(Q_{ce}),Q_{ce})$.

IV.4 Discussion

IV.4.1 Application to a natural gas pipeline project

One can use the values of the technological parameters for the natural gas pipeline system (i.e., $\alpha=8/11$ and $\beta=9/11$) to evaluate the threshold value $\bar{\lambda}$ for various conceivable values of the demand price elasticity $1/\varepsilon$ listed in Table 2 (column I). These threshold values $\bar{\lambda}$ are tabulated in Table 2 (column II). For each value of the demand price elasticity $1/\varepsilon$ in column I, it is also possible to numerically evaluate the ratio s_λ/r of the fair rate of return to the market price of capital that solves the equation (22) (i.e., that guarantees *ex post* efficiency) for a given value of the *ex post* demand expansion parameter λ that is in the range $0<\lambda<\bar{\lambda}$. The assumed values for the parameter λ are presented in column III and the associated values for the ratio s_λ/r are presented in column IV.

To explore the implications of setting the fair rate of return at the level s_λ, we consider two series of indicators that are evaluated for each pair of parameters $1/\varepsilon$ and λ. The first series is presented in columns V to VIII and focuses on the *ex ante* situation when the regulated firm solely serves the FDI-financed mega projects. It includes the indicators listed in Callen et al. (1976) and reviewed above:[27] the output ratio Q^*/Q_M (column V); the overcapitalization ratio $K^*/K(Q^*)$ (column VI); the cost ratio $C^*/C(Q^*)$ (column VII); and the net social welfare ratio $(W^*-W_M)/(W_a-W_M)$ (column VIII) where W_M (respectively W_a) is the net social welfare obtained if the *ex ante* demand is served by an unregulated private monopoly (respectively by a social planner applying the average-cost-pricing rule). The ratio $(W^*-W_M)/(W_a-W_M)$ compares the gain in net social welfare (W^*-W_M) resulting from the application of rate-of-return regulation on a private monopoly and the gain in net social welfare (W_a-W_M) that would be obtained by a social planner applying the average-cost-pricing rule in a previously monopolistically-controlled industry.

The second series of indicators is tabulated in columns IX to XI and focuses on the *ex post* situation. These indicators document the magnitude of the change observed in the firm's output, in the price and in the net social welfare attained *ex post* once the expanded demand materializes. In column IX, we report the values of the output expansion ratio Q_{ce}/Q^* that compares the firm's *ex post* and *ex ante* output levels. Column X presents the price ratio $P_\lambda(Q_{ce})/P(Q^*)$ that relates the *ex post* price level $P_\lambda(Q_{ce})$ observed when the firm produces the output Q_{ce} to the *ex ante* price level $P(Q^*)$. Lastly, column XI documents the social performance of the regulated sector once the expanded demand materializes by reporting the ratio $(W_{ce}^\lambda-W_M^\lambda)/(W_a^\lambda-W_M^\lambda)$ where W_{ce}^λ is the net social welfare attained *ex post* and W_M^λ (respectively W_a^λ) is the net social welfare obtained if the *ex post* demand is served by an unregulated private monopoly (respectively by a social planner applying the average-cost-pricing rule). For the sake of concision, the technical derivation of a closed-form expression for each of these three ratios is detailed in Appendix C. We simply underline here that these ratios are entirely

[27] See Table 1- Panel 3.

determined by the technological parameters α and β, the ratio s_λ/r, the demand price elasticity $1/\varepsilon$ and the *ex post* demand expansion parameter λ and are invariant with the input prices e and r.

Table 2 Rate of return, output, cost, price and welfare gain ratios for alternative demand elasticities and demand expansion parameters

Column # I	II	III	IV	V	VI	VII	VIII	IX	X	XI
				Ex ante ratios				*Ex post* ratios		
$\frac{1}{\varepsilon}$	$\bar{\lambda}$	λ	$\frac{s_\lambda}{r}$	$\frac{Q^*}{Q_M}$	$\frac{K^*}{K(Q^*)}$	$\frac{C^*}{C(Q^*)}$	$\frac{W^*-W_M}{W_a-W_M}$	$\frac{Q_{ce}}{Q^*}$	$\frac{P_\lambda(Q_{ce})}{P(Q^*)}$	$\frac{W_{ce}^\lambda-W_M^\lambda}{W_a^\lambda-W_M^\lambda}$
1.001	0.371	0.000	1,125.750	1.000	1.000	1.000	0.000	1.000	1.000	0.000
		0.050	7.195	89.736	3.967	2.892	0.604	5.388	0.195	0.824
		0.100	3.660	163.778	4.770	3.474	0.669	6.750	0.163	0.908
		0.150	2.455	233.631	5.319	3.871	0.699	7.711	0.149	0.949
		0.200	1.848	300.908	5.748	4.183	0.716	8.478	0.142	0.971
		0.300	1.236	430.329	6.414	4.666	0.728	9.693	0.134	0.993
		0.371	1.000	519.647	6.795	4.944	0.728	10.403	0.132	1.000
1.150	0.200	0.000	8.250	1.000	1.000	1.000	0.000	1.000	1.000	0.000
		0.050	2.355	3.764	1.408	1.133	0.554	1.519	0.730	0.724
		0.100	1.594	5.687	1.566	1.221	0.673	1.730	0.683	0.885
		0.150	1.225	7.518	1.682	1.292	0.720	1.889	0.662	0.962
		0.200	1.000	9.314	1.778	1.352	0.729	2.021	0.651	1.000
1.300	0.131	0.000	4.500	1.000	1.000	1.000	0.000	1.000	1.000	0.000
		0.050	1.632	3.511	1.319	1.089	0.616	1.402	0.810	0.783
		0.100	1.163	5.338	1.446	1.154	0.732	1.570	0.778	0.951
		0.131	1.000	6.438	1.507	1.187	0.750	1.651	0.769	1.000
1.500	0.082	0.000	3.000	1.000	1.000	1.000	0.000	1.000	1.000	0.000
		0.050	1.235	3.787	1.274	1.070	0.707	1.344	0.862	0.876
		0.082	1.000	5.196	1.349	1.104	0.781	1.442	0.847	1.000
1.700	0.055	0.000	2.357	1.000	1.000	1.000	0.000	1.000	1.000	0.000
		0.050	1.033	4.374	1.252	1.060	0.797	1.316	0.893	0.976
		0.055	1.000	4.638	1.263	1.065	0.811	1.331	0.891	1.000

The *ex ante* ratios in Table 2 indicate that, in case of a large demand expansion coefficient λ that is close to $\bar{\lambda}$, the allowable rate of return s_λ has to be set close to the cost of capital r to obtain a sufficiently large degree of overcapitalization (cf. column VI). Of course, this overcapitalization imposes a cost increase that can be substantial (cf. column VII). Yet, it should be noted that the output of the regulated firm is considerably larger than that of an unregulated monopoly. Thus, despite the Averch-Johnson distortion, the gain in net social welfare obtained by imposing the rate-of-return regulation to an unregulated monopoly is larger than 70% of the somehow theoretical gain obtained by changing the unregulated monopolist into a social planner applying the average-cost-pricing rule.

The most important conclusions to be drawn from the *ex post* ratios listed in Table 2 are:

i. that *ex post*, the occurrence of a demand that is larger than initially expected (i.e., $\lambda > 0$) forces the regulated firm to substantially expand its output (cf., column IX);
ii. that this expansion is large enough to systematically yield to a price decline *ex post* (cf., column X);
iii. and that the elimination of the Averch-Johnson distortion *ex post* substantially improves the social performance of the regulated sector (cf. the *ex post* social welfare ratios presented in column XI compared with the *ex ante* values listed in column VIII).

By construction, the magnitude of these effects is larger if the allowable rate of return s_λ is close to the market price of capital, i.e. if the demand expansion coefficient λ is large (but lower than the threshold value $\bar{\lambda}$). Indeed, if the demand expansion coefficient λ is close to the threshold value $\bar{\lambda}$, the *ex post* behavior of the regulated firm becomes identical to the theoretical benchmark of a social planner applying the average-cost-pricing rule.

This discussion highlights that, in case of a large increase of the *ex post* demand (i.e. a coefficient λ that is close to the threshold value $\bar{\lambda}$), the allowable rate of return s_λ needed to optimally build ahead of demand is close to the market price of capital r. From a regulator's perspective, that situation allows to catch two birds with a single stone since the policy objective to build ahead of demand is perfectly aligned with the usual goal to augment the net social welfare by limiting the exertion of market power. In contrast, in case of a small coefficient λ (i.e., a small increase of the *ex post* demand), the ambition to optimally build ahead of demand recommends the adoption of an allowable rate of return s_λ that is significantly higher than the market price of capital r. Indeed, if λ is tiny and close to zero, there is a limited need to overcapitalize *ex ante* and the goal to serve the *ex post* demand using a cost-efficient combination of input results in an allowable rate of return s_λ that is close to the rate of return obtained by an unregulated private monopoly s_M (because an unregulated monopoly *de facto* uses a cost-efficient combination of inputs). Such a decision would obviously have a detrimental on the net social welfare, particularly on the *ex ante* consumers that are critically needed to justify the construction of the infrastructure.

This remark echoes the policy discussion in Joskow (1999) who pointed that the public policy objectives assigned to the regulator (e.g., maximizing the net social welfare, favoring the use of a cost-efficient combination of inputs) can be conflicting goals. Rather than trying to prioritize them, we now explore the conditions under which these two objectives are aligned.

IV.4.2 Conflicting regulatory objectives?

Our analysis is structured in three successive steps. First, we briefly recall two results proposed in Callen et al. (1976) regarding: (i) the net social welfare, and (ii) the socially desirable rate of return that can be selected by a regulator. Then, we show how their results can be used to identify a range of allowable rates of return such that the net social welfare obtained when considering the *ex ante* demand is close to the social optimum. Lastly, we discuss the conditions needed for these rates to be compatible with the regulator's ambition to optimally build ahead of demand.

Insights from Callen et al. (1976)

Callen et al. (1976) examine the case of a regulated monopolist that considers the inverse demand function (4) and serves Q^* units. They provide a closed-form expression for the net social welfare W^* that is parameterized by s, the allowable rate or return set by the regulator (see Table 1–equation (10)).

For that demand schedule, they also examine how a regulatory agency could set this allowable rate of return so as to maximize the net social welfare given the regulated firm's reaction to that rate. They formally prove that the socially desirable rate can be larger than the market price of capital and is as follows:

$$s^{opt} = \max\left\{ r, \frac{\eta^2}{\alpha\left[\beta-(1-\alpha)(1-\varepsilon)^2\right]} r \right\}. \tag{23}$$

Preserving the net social welfare obtained *ex ante*

From the socially desirable rate in equation (23), we can derive the following proposition:

Proposition 3: *The socially desirable rate of return s^{opt} is bounded and verifies: $s^{opt} \leq \beta r/\alpha$.*

From a practitioner's perspective, this result has important implications. If a social welfare maximizing regulator has zero information on the value of the price elasticity of the demand and thus cannot exactly evaluate s^{opt}, that regulator should not implement a rate of return that is larger than β/α times the market price of capital. The ratio β/α is determined by the technological parameters. In the case of a natural gas pipeline system, we have $\alpha = 8/11$, $\beta = 9/11$ and thus $\beta/\alpha = 1.125$.

From a social welfare perspective, it can be interesting to gauge the welfare losses associated with the arbitrary selection of an allowable rate of return s within the range $r < s \leq \beta r/\alpha$ in spite of s^{opt} the socially desirable one. The following corollary addresses this concern.

Corollary 3: *If the regulator arbitrarily sets the allowable rate of return s in the range $r < s \leq \beta r/\alpha$, the net social welfare obtained ex ante $W^*(s)$ is possibly suboptimal (i.e. $W^*(s) \leq W^*(s^{opt})$) but not smaller than the lower bound $\underline{W^*} = \min\{W^*(r), W^*(\beta r/\alpha)\}$*

.

To document the welfare losses associated with such arbitrary decision, Table 3 compares, for a series of conceivable values of the demand price elasticity $1/\varepsilon$, two versions of the net social welfare gain ratio obtained *ex ante* that has been presented in Section 4.1: (i) the ratio $\left(W^*(s^{opt}) - W_M\right)/\left(W_a - W_M\right)$ obtained when the socially desirable rate of return s^{opt} is implemented and (ii) and the ratio $\left(\underline{W^*} - W_M\right)/\left(W_a - W_M\right)$ that provides a lower bound for the value of the ratio

$\left(W^*(s)-W_M\right)/\left(W_a-W_M\right)$ obtained when implementing an allowable rate of return s in the range $r<s\le \beta r/\alpha$.

Table 3. Welfare gain ratios for alternative demand elasticity parameters

$\frac{1}{\varepsilon}$	$\frac{s^{opt}}{r}$	$\frac{W^*\left(s^{opt}\right)-W_M}{W_a-W_M}$	$\frac{\underline{W^*}-W_M}{W_a-W_M}$
1.001	1.124	0.729	0.728
1.150	1.035	0.729	0.727
1.300	1.000	0.750	0.738
1.500	1.000	0.781	0.748
1.700	1.000	0.811	0.749

The results presented in Table 3 show that arbitrarily setting the allowable rate of return in the range $r<s\le \beta r/\alpha$ has a modest impact on the net social welfare obtained *ex ante* (compared with the use of s^{opt}).

Preserving the net social welfare obtained *ex ante* while building ahead of demand

The analysis in Section 3.4 has examined the problem of a regulator that seeks to set the allowable rate of return at a level s_λ so that the regulated firm *ex ante* installs the capital stock that will be needed to serve the *ex post* demand in a cost-efficient manner. Proposition 2 indicates that: there is a one-to-one mapping between the demand expansion parameter λ in the range $0<\lambda<\bar{\lambda}$ and the rate of return s_λ in the range $r<s_\lambda<s^M$; and that s_λ is smoothly and monotonically decreasing with λ. If the demand expansion parameter λ takes a small value, the allowed rate-of-return s_λ that will guarantee a cost-efficient operation *ex post* is likely to be large and close to the monopolist's rate s^M. In the preceding paragraph, we have just shown that it is preferable to keep the allowable rate low and preferably below the threshold $\beta r/\alpha$ to maintain a high level of net social welfare *ex ante*. Therefore, one can wonder whether there exists a range of values for the demand expansion parameter λ such that the rate of return s_λ is not too harmful for the net social welfare obtained *ex ante* while allowing the desired overcapitalization that will be needed *ex post*. This idea provides the motivation for the following proposition.

Proposition 4: *For any* λ *such that* $\underline{\lambda} \leq \lambda < \overline{\lambda}$ *where* $\underline{\lambda} \equiv \frac{(1-\varepsilon)}{\beta}[\beta-\alpha+1]\left(\frac{\eta}{(1-\varepsilon)\beta}\right)^{\eta/\beta} - 1$, *there exists a unique fair rate of return* s_λ *in the interval* $\left(r, \frac{\beta}{\alpha}r\right]$ *such that the condition (22) is verified.*

That proposition indicates that, if the demand expansion coefficient λ verifies $\underline{\lambda} \leq \lambda < \overline{\lambda}$, the regulator can set the allowable rate of return at a level s_λ that is not greater than the upper bound $\beta r/\alpha$ and thus jointly fulfill the two public policy objectives of preserving a high level of net social welfare *ex ante* and inducing the regulated firm to build ahead of demand by installing the targeted amount of extra-capital stock.

To gain insights on the width of the interval $\left[\underline{\lambda}, \overline{\lambda}\right)$ for the case of a natural gas pipeline, Table 4 reports the values of the upper and lower bounds for a series of conceivable values of the demand price elasticity $1/\varepsilon$. Table 4 also reports the output expansion ratio Q_{ce}/Q^* that compares the firm's *ex post* and *ex ante* output levels obtained under the two cases $\lambda = \underline{\lambda}$ and $\lambda = \overline{\lambda}$.

Table 4. The range of demand expansion (or output expansions) that makes the public policy objectives of building ahead of demand and preserving the *ex ante* net social welfare aligned

$\frac{1}{\varepsilon}$	$\underline{\lambda}$	$\overline{\lambda}$	$\frac{Q_{ce}}{Q^*}\left(\underline{\lambda}\right)$	$\frac{Q_{ce}}{Q^*}\left(\overline{\lambda}\right)$
1.001	0.330	0.371	10.002	10.403
1.150	0.170	0.200	1.943	2.021
1.300	0.106	0.131	1.587	1.651
1.500	0.063	0.082	1.387	1.442
1.700	0.039	0.055	1.280	1.331

From the figures detailed in Table 4, we notice that the interval $\left[\underline{\lambda}, \overline{\lambda}\right)$ is narrow which indicates that the conditions for optimally building ahead of demand with a limited detrimental impact on the *ex ante* net social welfare are restrictive. As it is very unlikely that the condition $\lambda \in \left[\underline{\lambda}, \overline{\lambda}\right)$ will be verified in a gas pipeline project similar to the one envisioned in Mozambique,[28] an important policy recommendation can be derived from this finding: in a LDC, it is important to clearly prioritize the public policy objectives assigned to the agency that regulates the natural gas pipeline sector.

IV.5 Conclusions and Policy Implications

Developing countries trying to develop natural gas resources through pipeline infrastructure face a number of challenges. On the one hand, they need to impose a clear and manageable regulatory framework to the future pipeline operator, such as the long-advocated rate-of-return regulation. But this framework presents its own flaws, as it is suspected of generating over-investment through the Averch-Johnson effect. On the other hand, they must provide an infrastructure at times where initial demand is almost inexistent and prepare for future demand growth. While the irreversibility and increasing returns of pipeline investments, derived from their engineering characteristics, advise them to build ahead of demand, this would likely collide with the more conservative approach of foreign investors.

We show in this paper that economic analysis can help to a certain extent to address these challenges simultaneously. Using classical rate-of-return regulation models, we examine the design choices of a regulated firm based on ex ante conservative demand estimates, and extend the literature by characterizing its operating decisions once it reacts to an ex post demand larger than expected. We then prove that a regulator chose the allowed rate-of-return ex ante so as to induce the firm to build ahead of demand. This is a crucial finding, as it guarantees an efficient ex post operation and a reduction of Averch-Johnson distortions.

This strategy has several limitations though. It can only be applied for a demand growth under an identified threshold and may impact the initial welfare in case of large allowed rates of return. We show using numerical data that the range of demand growth ratios for which ex post welfare can be

[28] Based on development scenarios detailed in ICF (2012), possible combinations of initial projects - two power plants in Pemba and Nacala amounting to 9.5 Bcf/y each and fertilizer and methanol plants in Nacala for 15.6 Bcf/y and 18 Bcf/y respectively – with possible extensions – three power plants in Nacala and further down the pipeline and a 159 Bcf/y GTL plant in Nacala – show growth ratios ranging from 1.18 to 5.35. While for reasonable values of demand elasticity nearing 1.3, some of the more conservative combinations might fall in the desired range, this remains very unlikely as each would require very specific values of elasticity. This is the consequence of the narrowness of this strategic window, and of the lumpiness of mega-projects investments.

improved and initial adverse effects kept limited is so narrow, that regulator will likely have to prioritize one goal over the other in practice.

The analysis developed here provides important novel insights for development planners and regulators involved with pipeline infrastructure projects in developing countries. Given the significant investments and large economic potential at stake in such projects, it can greatly contribute to addressing the contradictory challenges they face, as shown for the case of the Rovuma fields in Mozambique. It also demonstrates that these are infrastructure projects for which it is crucial to clearly prioritize policy goals to achieve desired outcomes.

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IV.7 References

André, J., & Bonnans J.F., 2011. Optimal structure of gas transmission trunklines. *Optimization and Engineering*, 12(1-2), 175–198.

Auty, R.M., 1988a. Oil-exporters' disappointing diversification into resource-based industry: The external causes. *Energy Policy*, 16(3), 230–242.

Auty, R.M., 1988b. State enterprise and resource based industry in oil exporting countries. *Resources Policy*, 14 (4), 275–287.

Auty, R.M., Gelb, A., 1986. Oil windfalls in a small parliamentary democracy: Their impact on Trinidad and Tobago. *World Development,* 14(9), 1161–1175.

Averch, H., Johnson, L., 1962. Behavior of the firm under regulatory constraint. *American Economic Review*, 52(5), 1052–1069.

Callen J.L., Mathewson G.F., Mohring H., 1976. The Benefits and Costs of Rate of Return Regulation. *American Economic Review*, 66(3), 290–297.

Caselli, F., Tesei, A., 2016. Resource Windfalls, Political Regimes, and Political Stability. *The Review of Economics and Statistics*, 98(3), 573–590.

Cesur, R., Tekin, E., Ulker, A. (2017), Air Pollution and Infant Mortality: Evidence from the Expansion of Natural Gas Infrastructure. *The Economic Journal*, 127, 330–362.

Chenery, H.B., 1949. Engineering production functions. *Quarterly Journal of Economics*, 63(4), 507–531.

Chenery, H.B., 1952. Overcapacity and the acceleration. *Econometrica*, 20(1), 1–28.

Demierre, J., Bazilian, M., Carbajal, J., Sherpa, S., Modi, V., 2015. Potential for regional use of East Africa's natural gas. *Applied Energy*, 143: 414–436.

Franza, L., 2013. Gas in East Africa, assessing the potential for various stakeholders. *Clingendael International Energy Programme*, 2013/05. The Hague, The Netherlands

Hirschman A.O., 1958. The Strategy of Economic Development. *Yale University press*.

ICF, 2012. The Future of Natural Gas in Mozambique: Towards a Gas Master Plan. Report submitted to: The World Bank and Government of Mozambique Steering Committee. December 20, 2012.

IEA, 2014. Africa Energy Outlook. *World Energy Outlook Special Report*. International Energy Agency: Paris.

IMF, 2016. Republic of Mozambique, selected issues. *IMF Country Report No. 16/10.* International Monetary Fund, Washington, D.C.

Joskow, P.L., 1999. Regulatory Priorities for Reforming Infrastructure Sectors in Developing Countries. *Annual World Bank Conference on Development Economics 1998*, pp. 191-223. The World Bank, Washington, D.C.

Kabirian, A., Hemmati, M. R., 2007. A strategic planning model for natural gas transmission networks. *Energy Policy*, 35(11): 5656–5670.

Kahn, A.E., 1988. *The Economics of Regulation Pricing and Institutions*. Cambridge, MA: MIT Press.

Klein, B., Crawford, R.G., Alchian, A.A., 1978.Vertical Integration, Appropriable Rents, and the Competitive Contracting Process. *The Journal of Law & Economics*, 21(2), 297–326.

Klevorick, A.K., 1971. The 'Optimal' Fair Rate of Return. *The Bell Journal of Economics and Management Science*, 2(1), 122–53.

Laffont, J.-J., Tirole, J., 1993. *A Theory of Incentives in Procurement and Regulation*. Cambridge, MA: MIT Press.

Mahumane, G., Mulder, P., 2016. Introducing MOZLEAP: An integrated long-run scenario model of the emerging energy sector of Mozambique. *Energy Economics*, 59, 275–289.

Manne, A.S., 1961. Capacity expansion and probabilistic growth. *Econometrica*, 29(4), 632–649.

Massol, O., 2011. A cost function for the natural gas industry: further considerations. *The Engineering Economist*, 56(2), 95–122.

Massol, O., Banal-Estañol, A., 2014. Export diversification through resource-based industrialization: The case of natural gas. *European Journal of Operational Research*, 237 (3), 1067–1082.

Melina, G., Xiong, Y., 2013. Natural Gas, Public Investment and Debt Sustainability in Mozambique. *IMF Working Paper*, WP/13/261.

Ministério da Planificação E Desenvolvimento, 2014. *Estratégia nacional de desenvolvimento (2015-2035)*. República de Moçambique: Maputo.

Parker. D., Kirkpatrick, C., 2005. Privatisation in Developing Countries: A Review of the Evidence and the Policy Lessons. *Journal of Development Studies*, 41(4): 513-541.

Perrotton F., Massol, O. 2017. The technology of a natural gas pipeline: Insights for costs and rate-of-return regulation. *Working Paper.* City University of London.

Sovacool, B.K., 2009. Energy policy and cooperation in Southeast Asia: The history, challenges, and implications of the trans-ASEAN gas pipeline (TAGP) network. *Energy Policy*, 37(6), 2356–2367.

Yépez, R.A., 2008. A cost function for the natural gas industry. *The Engineering Economist*, 53(1), 68–83.

Xu, C., Bell, L., 2013. Worldwide reserves, oil production post modest rise. Oil & Gas Journal, 111 (12), December 2nd, 2013.

Appendix A – The polar cases of a monopoly and of a social planner

In this appendix, we consider the *ex ante* inverse demand function (4) and review the standard outcomes obtained under two polar cases: (i) the profit-maximizing unregulated monopoly that charges a non-discriminatory price (Table A.1 – Column 1); and (ii) a welfare-maximizing social planner that behaves so as to maximize the sum of the producers' and consumers' surpluses (i.e., the net social welfare) while ensuring that the firm obtains zero economic profit (Table A.1 – Column 2).

Table A1. The optimal decisions taken by a profit-maximizing unregulated monopoly and a welfare-maximizing social planner providing zero profit to the firm

	The unregulated monopoly	The welfare-maximizing planner that provides zero-profits to the firm
Optimization program	$\underset{K,Q}{\text{Max}}$ (24) $\Pi_M(Q) = P(Q)Q - rK - eE(Q,K)$	$\underset{K,Q}{\text{Max}}$ (25) $W(Q) = \int_0^Q P(q)\,dq - rK - eE(Q,K)$ s.t. $P(Q)Q - rK - eE(Q,K) = 0$ (26)
Solution:		
Output	$Q_M = \left[\frac{A(1-\varepsilon)}{\beta}\left(\frac{\alpha}{r}\right)^{\alpha}\left(\frac{1-\alpha}{e}\right)^{1-\alpha}\right]^{\frac{1}{\gamma}}$ (27)	$Q_a = \left[A\left(\frac{\alpha}{r}\right)^{\alpha}\left(\frac{1-\alpha}{e}\right)^{1-\alpha}\right]^{\frac{1}{\gamma}}$ (28)
Capital	$K_M = \left(\frac{e\alpha}{r(1-\alpha)}\right)^{1-\alpha}(Q_M)^{\beta}$ (29)	$K_a = \left(\frac{e\alpha}{r(1-\alpha)}\right)^{1-\alpha}(Q_a)^{\beta}$ (30)

Note: The objective function (C.3) is the firm's profit, i.e.: the difference between the total revenue $P(Q)Q$ *and the sum of the capital cost* rK *and the energy cost* $eE(Q,K)$. *The objective function (C.4) is the net social welfare defined as the sum of the consumer surplus* $\int_0^Q P(q)\,dq - P(Q)Q$ *and the producer's surplus* $P(Q)Q - rK - eE(Q,K)$. *The constraint (26) states that the firm is compelled to obtain zero economic profit.*

These outcomes are subscripted with M and a respectively. In both cases, production is cost efficient and uses the cost-minimizing amount of capital stock, that is, $K_M = K(Q_M)$ and $K_a = K(Q_a)$. Note also that, for the social planner, substitution of the optimal decisions Q_a and K_a in the zero profit condition (26) gives $P(Q_a)Q_a - C(Q_a) = 0$ which means that the output is set at a level such that the price equals the long-run average cost.

Following Callen et al. (1976), one can define s_M the monopolist's rate of return on invested capital which is the ratio of: the accounting profit derived from the production of the output Q_M (that is: $P(Q_M)Q_M - eE(Q_M, K_M)$), and K_M the profit-maximizing capital stock:

$$s_M \equiv \frac{\left[\beta/(1-\varepsilon)\right]-(1-\alpha)}{\alpha} r. \tag{31}$$

Appendix B – Mathematical proofs

Proof of Proposition 1: *Ex post, the capital stock is fixed and equals K^*. The regulated firm's profit is given by the single variable function $\Pi : Q \mapsto P(Q)Q - rK^* - eE(Q, K^*)$ that is twice-differentiable, strictly concave and verifies $\Pi(0) = -rK^*$ and $\lim_{Q \to +\infty} \Pi(Q) = -\infty$. We let M denote the unique profit-maximizing output. Here, the profit function Π is monotonically increasing (respectively decreasing) on the left interval $[0, M]$ (respectively the right interval $[M, +\infty)$) and there is a one-to-one correspondence between the left interval $[0, M]$ (respectively the right interval $[M, +\infty)$) and the image $[\Pi(0), \Pi(M)]$ (respectively the interval $(-\infty, \Pi(M)]$).*

Recall that we are looking for an output level Q such that the ex post rate-of-return constraint (16) *is verified. The condition* (16) *is logically equivalent to $\Pi(Q) = (s-r)K^*$.*

We first focus on the right interval $[M, +\infty)$ and are going to prove that the image interval $(-\infty, \Pi(M)]$ contains the value $(s-r)K^*$. Notice that the output level Q^* verifies (C.9) *and that $\Pi(Q^*) = \lambda P(Q^*)Q^* + (s-r)K^*$. As $\lambda > 0$ and $Q^* > 0$, we obtain $\Pi(Q^*) > (s-r)K^*$. Using the definition of a maximum: $\Pi(M) \geq \Pi(Q^*)$. So, $\Pi(M) > (s-r)K^*$ which proves that the open interval*

$(-\infty,\Pi(M))$ *contains* $(s-r)K^*$. *Hence, there exists a unique output level* Q_e^* *in* $(M,+\infty)$ *such that* $\Pi(Q_e^*)=(s-r)K^*$ *and the condition* (16) *holds.*

Then, we examine the left interval $[0,M]$. *As* $s>r$, *we have* $(s-r)K^*>0$ *and thus* $(s-r)K^*>\Pi(0)$. *As we have already shown that* $\Pi(M)>(s-r)K^*$, *we can now affirm that the open interval* $(\Pi(0),\Pi(M))$ *also contains* $(s-r)K^*$. *So, there also exists a unique output level* Q_c^* *in* $(0,M)$ *such that* $\Pi(Q_c^*)=(s-r)K^*$ *and the constraint* (16) *is verified.*

We have just shown that there exist two solutions Q_c^* *and* Q_e^* *that verify* $Q_c^*<M<Q_e^*$. *Now, recall that the pair* Q^* *and* K^* *verifies* (C.11). *As* $s>r$, *the firm's ex post marginal profit evaluated at* Q^* *thus verifies* $\Pi'(Q^*)=\lambda\left[P'(Q^*)Q^*+P(Q^*)\right]$ *which is positive because:* $\lambda>0$; $P'(Q^*)Q^*+P(Q^*)=eE_Q(Q^*,K^*)$ *(cf., equation (C.11)) and* $E_Q(Q,K)>0$. *As* $\Pi'(Q^*)>0$, *the marginal profit function is locally monotonically increasing. Because of the strict concavity of the profit function, it means that* $Q^*<M$ *and thus* $Q^*<Q_e^*$. *Recall that we have shown above that* $\Pi(Q^*)>(s-r)K^*$. *As the profit function is monotonically increasing on the interval* $[0,M]$, *the condition* $Q_c^*<Q^*$ *also holds. So, the two solutions verify* $Q_c^*<Q^*<Q_e^*$.

Q.E.D.

Proof of Corollary 1: *Recalling that* Q_e^* *(respectively* Q_c^**) verifies the ex post regulatory constraint* (16) *which is logically equivalent to* $(1+\lambda)P(Q)Q-eE(Q,K^*)=(s-r)K^*$, *the implicit function theorem can be invoked to assess the sign of* $dQ_e^*/d\lambda$ *(respectively* $dQ_c^*/d\lambda$*). As:* $\partial\left[(1+\lambda)P(Q)Q-eE(Q,K^*)-(s-r)K^*\right]/\partial\lambda$ *evaluated at* $Q=Q_e^*$ *(respectively* $Q=Q_c^*$*) equals* $P(Q_e^*)Q_e^*$ *(respectively* $P(Q_c^*)Q_c^*$*) which is positive, and* $\partial\left[(1+\lambda)P(Q)Q-eE(Q,K^*)-(s-r)K^*\right]/\partial Q$ *evaluated at* $Q=Q_e^*$ *(respectively* $Q=Q_c^*$*) equals the ex post marginal profit* $\Pi'(Q_e^*)$ *(respectively*

$\Pi'\left(Q_c^*\right)$*) introduced in the preceding proof which is negative (respectively positive), the implicit function theorem reveals that* $dQ_e^* / d\lambda > 0$ *(respectively* $dQ_c^* / d\lambda < 0$*).* *Q.E.D.*

Proof of Proposition 2: *Recall that the condition* $K^* = K_{ce}$ *is equivalent to:*

$$\frac{(1-\varepsilon)}{\beta}\left[\left(\frac{s}{r}-1\right)\alpha+1\right]\left(\frac{r}{s}\frac{\eta}{(1-\varepsilon)\alpha}\right)^{\eta/\beta}-1=\lambda . \tag{32}$$

We let $x \in (1, s_M/r)$ *denote the ratio* s/r *and let* $f : x \mapsto [\quad)\alpha+1]x^{-\eta/\beta}$. *We are going to prove that this smooth univariate function is a monotonically decreasing one. We let:* $v : x \mapsto [\quad 1)\alpha+1]x$. *Remarking that* $v(x) > 0$ *and* $f(x) > 0$ *for any* $x > 1$, *it is clear that the sign of* $v(x)\dfrac{f'(x)}{f(x)} = -\eta\left[(x-1)\alpha+1\right]+\beta\alpha x$ *is identical to that of* $f'(x)$ *the gradient of* f *w.r.t.* x *evaluated at* x. *Recalling that* $\eta \equiv \beta-(1-\varepsilon)(1-\alpha)$ *and rearranging, we obtain* $v(x)\dfrac{f'(x)}{f(x)} = (1-\alpha)\left[(1-\varepsilon)\alpha x-\eta\right]$. *As* $(1-\varepsilon)\alpha > 0$ *and* $(1-\alpha) > 0$, *the expression* $(1-\alpha)\left[(1-\varepsilon)\alpha x-\eta\right]$ *which is a linear function of the variable* x *has a positive slope coefficient and is thus a monotonically increasing function. So,* $v(x)\dfrac{f'(x)}{f(x)} < v\left(\dfrac{s_M}{r}\right)\dfrac{f'(s_M/r)}{f(s_M/r)}$ *for any* $x \in (1, s_M/r)$. *As* $s_M/r = \left[\beta/(1-\varepsilon)-(1-\alpha)\right]/\alpha$, *we have* $v\left(\dfrac{s_M}{r}\right)\dfrac{f'(s_M/r)}{f(s_M/r)} = 0$ *which proves that* $f'(x) < 0$ *for any* $x \in (1, s_M/r)$. *We have just shown that the smooth univariate function* f *is monotonically decreasing which indicates that the smooth univariate function* $h : s \mapsto [\quad)/\beta]\left[\eta/(\alpha(1-\varepsilon))\right]^{\eta/\beta} f(s/r)-1$ *is also monotonically decreasing. Hence,* h *is a one-to-*

one mapping from the open interval (r,s_M) *to the image interval* $\left(h(s_M),h(r)\right)$ *where* $h(s_M)=0$ *and* $h(r)=\left[\frac{\eta}{(1-\varepsilon)\alpha}\right]^{\frac{\eta}{\beta}}\left[\frac{1-\varepsilon}{\beta}\right]-1$ *that is* $\bar{\lambda}$.

As the function h *is invertible, we let* $g:\lambda\mapsto$ *denote its inverse. By construction,* g *is also a one-to-one mapping from the open interval* $\left(0,\bar{\lambda}\right)$ *to the interval* (r,s_M) *and the value of its derivative for any* $\lambda\in\left(0,\bar{\lambda}\right)$ *is* $g'(\lambda)=1/h'(s)$ *where* s *is the unique return in* $\left(r,s^M\right)$ *such that* $s=g(\lambda)$ *(cf., the inverse function theorem). As the sign of* $h'(s)$ *equals the one of* $f'(s/r)$ *and it has been shown above that the latter is negative for any* $s\in\left(r,s^M\right)$*, we thus have* $g'(\lambda)<0$ *which indicates that* g *is a monotonically decreasing function of the demand parameter* λ.

Q.E.D.

Proof of Corollary 2: *We assume that* s *is set at the level* s_λ *mentioned in Proposition 2. Inserting first* s_λ *in the closed-form expression of the output level* Q_{ce} *detailed in equation (19) and then remarking that* $A\left(\frac{\alpha}{r}\right)^{\alpha}\left(\frac{1-\alpha}{e}\right)^{1-\alpha}>0$*, that* $\lambda\geq 0$ *and that* $s_\lambda\geq r$ *, the sign of the gradient of* Q_{ce} *with respect to the demand parameter* λ *is:*

$$sign\left(\frac{dQ_{ce}}{d\lambda}\right)=sign\left(\frac{1}{(1+\lambda)\gamma}-\frac{1}{\gamma}.\frac{ds_\lambda}{d\lambda}.\frac{1}{\left(\frac{s_\lambda}{r}-1\right)\alpha+1}\right) \tag{33}$$

Recall that γ *is positive (as by assumption* $\varepsilon>1-\beta$*), that* $s_\lambda\geq r$ *and that Proposition 2 indicates that* $\left(ds_\lambda/d\lambda\right)<0$. *So,* $\left(dQ_{ce}/d\lambda\right)>0$ *and the cost-efficient output level* Q_{ce} *is monotonically increasing with the demand parameter* λ.

Q.E.D.

Proof of Proposition 3: *By assumption, the technology is such that* $0<\alpha<\beta<1$ *and the demand elasticity parameter verifies* $1-\beta<\varepsilon<1$. *As* $\eta \equiv \beta-(1-\varepsilon)(1-\alpha)$, *we have* $\eta>0$. *We now consider the function* $f:\varepsilon \mapsto \left[\quad -\alpha)(1-\varepsilon)\right]^2 \Big/ \left[\alpha\left(\beta-(1-\alpha)(1-\varepsilon)^2\right)\right]$ *which is defined for any* ε *in* $[1-\beta,1]$. *We evaluate* f' *the gradient of* f *w.r.t.* ε *and divide it by* $2(1-\alpha)\eta$ *which is a positive term. After simplification, one can notice that the sign of* $f'(\varepsilon)$ *is identical to that of* $\varepsilon\beta$ *and thus positive. Hence,* f *is a smooth and monotonically increasing function and for any* ε *in* $(1-\beta,1)$, *we have* $f(\varepsilon)<f(1)$ *that is:* $f(\varepsilon)<\beta/\alpha$. *As* $0<\alpha<\beta<1$, *we have* $\beta/\alpha>1$ *and thus* $s^{opt}=r.\max\{1,f(\varepsilon)\}$ *verifies* $s^{opt}<\beta r/\alpha$.

Q.E.D.

Proof of Corollary 3: *If the regulator sets the allowable rate of return at a level* $s>r$, *the net social welfare is given by the smooth univariate function* $W^*:s\mapsto \frac{A}{1-\varepsilon}\left[\,Q^*(s)\right]^{1-\varepsilon}-rK^*(s)-eE\left(Q^*(s),K^*(s)\right)$, *where* $Q^*(s)$ *and* $K^*(s)$ *are the regulated firm's optimal decisions. Inserting (8) into* $E\left(Q^*(s),K^*(s)\right)$, *the total cost of the energy input is* $\frac{e}{\delta}\left[Q^*(s)\right]^{1-\varepsilon}$. *Using the rate-of-return constraint (C.9), the total capital cost* $rK^*(s)$ *is:* $\frac{r}{s}\left[P\left(Q^*(s)\right)Q^*(s)-eE\left(Q^*(s),K^*(s)\right)\right]=\frac{r}{s}\left(A-\frac{e}{\delta}\right)\left[Q^*(s)\right]^{1-\varepsilon}$. *Simplifying, we have the smooth univariate function :* $W^*(s)=\frac{A}{\beta}\left(\frac{\beta-(1-\alpha)(1-\varepsilon)^2}{1-\varepsilon}-\frac{r}{s}\eta\right)\left[Q^*(s)\right]^{1-\varepsilon}$ *where* $Q^*(s)$ *is detailed in* (7) .

The gradient of W^* *w.r.t.* s *is:* $\frac{dW^*}{ds}(s)=\frac{A}{\beta s\gamma}\left[\frac{r}{s}\eta^2-\alpha\left[\beta-(1-\alpha)(1-\varepsilon)^2\right]\right]\left[Q^*(s)\right]^{1-\varepsilon}$ *which is positive iff.* $s<f(\varepsilon)r$, *equal to zero iff* $s=f(\varepsilon)r$ *and negative iff.* $s>f(\varepsilon)r$, *where* $f:\varepsilon\mapsto \quad /\left[\quad\left(\beta-(1-\alpha)(1-\varepsilon)^2\right)\right]$ *is the smooth function defined in the Proof of Proposition 3. We have shown there that* f *is monotonically increasing for any* ε *in* $[1-\beta,1]$. *So,* f *is a one-to-one* *mapping between* $\varepsilon \in [1-\beta, 1]$ *and* $f(\varepsilon) \in [f(1-\beta), f(1)]$. *By assumption,* $0 < \alpha < \beta < 1$, thus: $f(1) = \beta/\alpha > 1$ *and* $f(1-\beta) = \left[1 \Big/ \left(\dfrac{1-\beta}{\alpha\beta} + 1\right)\right] < 1$. *So, there exists a unique* ε^* *in* $(1-\beta, 1)$ *such that* $f(\varepsilon^*) = 1$. *Two cases have to be discussed depending on whether the elasticity parameter* ε *verifies* $1-\beta < \varepsilon \le \varepsilon^*$ *or* $\varepsilon^* < \varepsilon < 1$.

<u>*Case 1:*</u> $\varepsilon \in (1-\beta, \varepsilon^*]$. *As* f *is monotonically increasing, we have* $f(\varepsilon) \le 1$ *and thus, for any s in the interval* $s \in \left(r, \dfrac{\beta r}{\alpha}\right)$, *the condition* $s > f(\varepsilon) r$ *is verified. Thus,* W^* *is monotonically decreasing on the interval* $s \in \left(r, \dfrac{\beta r}{\alpha}\right)$ *and* $W^*(s) > W^*(\beta r/\alpha)$. *So,* $W^*(s) > \mathrm{Min}\{W^*(r), W^*(\beta r/\alpha)\}$ *holds.*

<u>*Case 2:*</u> $\varepsilon \in (\varepsilon^*, 1)$. *As* f *is monotonically increasing, we have* $f(\varepsilon) > 1$. *Moreover, we know that* $f(\varepsilon) < \beta/\alpha$ *(cf. the proof of Proposition 3). Thus, for any* $s \in (r, f(\varepsilon) r)$ *(respectively* $s \in \left(f(\varepsilon) r, \dfrac{\beta}{\alpha} r\right)$*), the function* W^* *is smooth and monotonically increasing (respectively decreasing) and the condition* $W^*(s) > W^*(r)$ *(respectively* $W^*(s) > W^*(\beta r/\alpha)$*) holds. So, the condition* $W^*(s) > \mathrm{Min}\{W^*(r), W^*(\beta r/\alpha)\}$ *holds.* *Q.E.D.*

Proof of Proposition 4: *Here, we simply have to highlight that* $\underline{\lambda}$ *is the value of the demand expansion parameter obtained by substituting* $s_\lambda = \beta r/\alpha$ *into equation (22). As it has been assumed that* $\beta > \alpha > 0$, *the rate* $s_\lambda = \beta r/\alpha$ *is in the interval* (r, s^M). *Thus, one can use the one-to-one mapping highlighted in the Proof of Proposition 2, to claim: (i) that setting* $s_\lambda = \beta r/\alpha$ *is the unique fair rate of return such that the equation (22) is verified when* $\lambda = \underline{\lambda}$, *and (ii) that* $\underline{\lambda}$ *belongs to the interval* $(0, \overline{\lambda})$. *As Proposition 2 also indicates that the rate of return* s_λ *is monotonically decreasing with* λ*, we can conclude that for any* λ *in the interval* $[\underline{\lambda}, \overline{\lambda})$, *there exists a rate of return* s_λ *the* *interval* $\left(r, \frac{\beta}{\alpha} r\right]$ *such that the condition (22) is verified.*

Q.E.D.

Appendix C – Ex post performance ratios

In this appendix, we derive the closed-form expressions of the three ratios used in Section 4 to assess the *ex post* performance of the regulation (i.e., once the demand expansion materializes). These three ratios respectively document the output expansion, the price variation and the impact on the net social welfare. Hereafter, it is assumed that the fair rate of return is set at the level s_λ indicated in Proposition 2.

IV.7.1 Output expansion ratio

We consider the output expansion ratio Q_{ce}/Q^* that compares the regulated firm's *ex post* output level Q_{ce} and the *ex ante* one Q^* to document the magnitude of the change in the firm's production plan. To rapidly obtain a closed-form expression for that ratio, we use the following reformulation where the output level Q_M chosen by an unregulated monopoly facing the *ex ante* inverse demand (4) is introduced:

$$\frac{Q_{ce}}{Q^*} = \frac{Q_{ce}}{Q_M} \times \frac{Q_M}{Q^*} . \tag{34}$$

A closed-form expression of the ratio Q^*/Q_M has been presented above (cf., equation (11)). Using that expression, the value of Q_{ce} in equation (19), those of Q_M presented in Appendix A- equation (27) and simplifying, we can rewrite the output expansion ratio as follows:

$$\frac{Q_{ce}}{Q^*} = \left[\frac{(1+\lambda)\beta}{(1-\varepsilon)\left[\left(\frac{s_\lambda}{r} - 1\right)\alpha + 1\right]} \right]^{1/\gamma} \times \left(\frac{s_\lambda}{r} \frac{(1-\varepsilon)\alpha}{\eta} \right)^{\alpha/\gamma} . \tag{35}$$

IV.7.2 Price ratio

We now examine the price ratio $P_\lambda(Q_{ce})/P(Q^*)$ that provides a rapid comparison between the *ex post* price level $P_\lambda(Q_{ce})$ observed when the regulated firm produces the output Q_{ce} and the *ex ante* price level $P(Q^*)$. Using the definitions of the inverse demand functions in equations (4) and (15), we obtain:

$$\frac{P_\lambda(Q_{ce})}{P(Q^*)} = (1+\lambda)\left(\frac{Q_{ce}}{Q^*}\right)^{-\varepsilon} \tag{36}$$

where Q_{ce}/Q^* is the output expansion ratio in (35).

IV.7.3 Net social welfare

To document the implications on the net social welfare, we consider the ratio $\left(W_{ce}^\lambda - W_M^\lambda\right)/\left(W_a^\lambda - W_M^\lambda\right)$ where W_{ce}^λ is the net social welfare attained *ex post* and W_M^λ (respectively W_a^λ) is the net social welfare that would have been obtained if the *ex post* demand had been served by an unregulated private monopoly (respectively by a social planner applying the average-cost-pricing rule) that could freely decide the optimal output-capital pair needed to serve the *ex post* demand.

To begin with, we evaluate the net social welfares W_M^λ and W_a^λ. Substituting the *ex post* inverse demand function (15) in the optimization program stated in Table A.1 (cf., Appendix A) yields the optimal output level Q_M^λ (respectively Q_a^λ) decided by the unregulated private monopoly (respectively the social planner):

$$Q_M^\lambda = \left[\frac{(1+\lambda)A(1-\varepsilon)}{\beta}\left(\frac{\alpha}{r}\right)^\alpha\left(\frac{1-\alpha}{e}\right)^{1-\alpha}\right]^{\frac{1}{\gamma}} \tag{37}$$

$$Q_a^\lambda = \left[(1+\lambda)A\left(\frac{\alpha}{r}\right)^\alpha\left(\frac{1-\alpha}{e}\right)^{1-\alpha}\right]^{\frac{1}{\gamma}} \tag{38}$$

Moreover, with the constant elasticity demand schedule (15), the net social welfare associated with Q units of output can be written: $W^{\lambda} = \left[(1+\lambda)A/(1-\varepsilon)\right]Q^{1-\varepsilon} - C(Q)$ where $C(Q)$ is the total cost indicated in equation (3).

As Q_a^{λ} is the output such that price equals the average cost: $(1+\lambda)A\left(Q_a^{\lambda}\right)^{1-\varepsilon} = C\left(Q_a^{\lambda}\right)$, the net social welfare obtained under a social planner applying the average-cost-pricing rule is:

$$W_a^{\lambda} = P\left(Q_a^{\lambda}\right)Q_a^{\lambda}\left[\varepsilon/(1-\varepsilon)\right]. \tag{39}$$

Remarking that $Q_M^{\lambda} = \sqrt[\gamma]{((1-\varepsilon)/\beta)}Q_a^{\lambda}$ and using the relation $(1+\lambda)A\left(Q_a^{\lambda}\right)^{1-\varepsilon} = C\left(Q_a^{\lambda}\right)$, the net social welfare obtained in the case of a monopoly is:

$$W_M^{\lambda} = P\left(Q_a^{\lambda}\right)Q_a^{\lambda}.\left(\frac{1}{1-\varepsilon}\left(\frac{1-\varepsilon}{\beta}\right)^{\frac{1-\varepsilon}{\gamma}} - \left(\frac{1-\varepsilon}{\beta}\right)^{\frac{\beta}{\gamma}}\right). \tag{40}$$

Similarly, one can observe that the *ex post* cost efficient output of the regulated firm is directly proportional to the output chosen by the social planner applying the average-cost-pricing rule as:

$$Q_{ce} = \left[\left(\frac{s_{\lambda}}{r} - 1\right)\alpha + 1\right]^{-1/\gamma} Q_a^{\lambda}. \tag{41}$$

Hence, one can also use the relation $(1+\lambda)A\left(Q_a^{\lambda}\right)^{1-\varepsilon} = C\left(Q_a^{\lambda}\right)$ to write the net social welfare obtained by the regulated monopoly that increases its output to cope with the augmented demand:

$$W_{ce}^{\lambda} = P\left(Q_a^{\lambda}\right)Q_a^{\lambda}.\left(\frac{1}{1-\varepsilon}\left[\left(\frac{s_{\lambda}}{r} - 1\right)\alpha + 1\right]^{\frac{\varepsilon-1}{\gamma}} - \left[\left(\frac{s_{\lambda}}{r} - 1\right)\alpha + 1\right]^{\frac{-\beta}{\gamma}}\right). \tag{42}$$

Using (42), (40) and (39) and simplifying, one can readily obtain a simple expression for the ratio $\left(W_{ce}^{\lambda} - W_M^{\lambda}\right)/\left(W_a^{\lambda} - W_M^{\lambda}\right)$ that solely depends on the technological parameters (i.e., α and β), the demand price elasticity and the ratio s_{λ}/r of the fair rate of return to the market price of capital.

Chapter V. Identifying inefficiencies in an Entry-Exit gas system

V.1 Introduction

The Entry-Exit market design has been chosen for the European gas market as it reduces the number gas trading locations to a limited set of virtual trading points (VTPs) in the whole European market, among other reasons. Suppliers are largely relieved of spatial considerations, which appear only in the entry and exit tariffs they pay according to their nominations and in the capacity limits which determine how much gas they can ship. Thus, contractual gas trading between suppliers is almost fully separated from actual physical gas flows, dealt with by the Transmission System Operator (TSO). While this makes gas trading easier for suppliers and increases the liquidity of the market, it undermines coordination in gas network operation, which may in turn generate losses. The overall efficiency of the Entry-Exit system, often gauged through market liquidity, depends on the balance between those two effects. Thus, it appears critical to compare the magnitude of the losses inherent to this market organization with the gains in market performance that it allows. In this paper, we provide a comprehensive framework to analyze and quantify the losses in network usage resulting from this market organization, compared to an integrated management.

The framework provided here also clarifies the incentives created by transmission network tariffs and capacity limits in an Entry-Exit system. This is essential to understand how future regulatory changes could impact the efficiency of network allocation. It is particularly important in a context where the European gas market faces new challenges, as gas demand declines or stagnates, but also tends fluctuate more and more. This is likely to trigger the revision of many aspects of market organization, such as the size of the different Entry-Exit zones, the daily timelines and network tariffs, to cite only a few. The launch of the "Quo vadis market regulatory framework" (European Commission 2016) study by the European Commission and the regular update of Gas Target Model (ACER 2015, CEER 2011) go in that direction, and would benefit from a better understanding of the Entry-Exit arrangements on network allocation.

In order to provide a coherent and comprehensive picture of this complex market organization, that we briefly describe in section V.2, we introduce a combined modeling of the Entry-Exit gas market and of its underlying network operation. We first detail the model in section V.3. Building on this formulation, we analytically demonstrate the existence of inefficiencies in the Entry-Exit system in section V.4 and numerically illustrate them. Since the complexity of the model only allows us to quantify the inefficiencies for simple market and network cases, section V.5 gives valuable insights on which factors influence those inefficiencies and makes the case for working toward the application of this analysis to real markets. We conclude in section V.6.

V.2 Background

In this section, we start by describing the main aspects of the Entry-Exit system that lead to a decoupling of economic and technical decisions and identify the inefficiencies that this generates. We then show that the literature has mainly focused on trading issues and that a quantification of network allocation inefficiencies is sorely needed.

V.2.1 Advantages and drawbacks of the Entry-Exit system

The decoupling of economic and technical decisions improves liquidity

One of the pillars of the European Union is the creation of a Single European Market. Following this goal, the gas industry, like others, has been progressively liberalized in order to create a single, competitive European gas market. Three legislative packages have defined and implemented the market design now in place in Europe, known as the Entry-Exit system (European Union 1998, 2003, 2009a, b). The historical structure of the former national, vertically integrated, state-owned gas companies in Europe weighted heavily on the choice of this market design. While the transmission assets have been unbundled from generation, supply and production, the gas market remains divided into zones, still largely defined by national borders.

In each zone, a transmission system operator (TSO) is in charge of operating the network on behalf of network users, which we will call shippers thereafter (comprising national gas suppliers, gas trading operators, gas importers, and gas producers). Transport is offered by the TSO on a non-discriminatory and transparent basis through the sale of capacity products, which allow the entry or the exit of a certain amount of gas in the zone for a given period of time. Before the corresponding time of use, capacity holders must notify the TSO of how much of that capacity they intend to use in a process called nomination. The TSO will then do reasonable efforts to satisfy the nominations of all network users. This has been described as a “common access” paradigm, compared to other existing gas market designs (Glachant et al. 2014).

Regarding commercial transactions, any entity can trade gas with another inside a zone, as long as a corresponding entry or exit capacity is purchased at some point to secure the right to inject or withdraw gas at nomination time. No other technical or geographical considerations are necessary to trade in the European market. This reduced number of market places and the simple conditions required to access the network were intentionally designed to foster gas trading and increase overall market liquidity. However, as a result, contractual gas trading between suppliers is almost fully separated from actual physical gas flows, dealt with by the TSO.

This relative decoupling may not be a concern, as long as the economic signals received by network users reflect the consequence of their choices on network costs. In gas networks, the location of a gas injection or a withdrawal may change the overall network cost. This is partly due to compressors' operating costs, which depend on the amount of gas to be moved, but also to congestion that injections or withdrawals may create, which can motivate changes in other injection and withdrawal patterns. Thus, those signals should include spatial considerations, i.e. be locational signals, which is only partially true in the Entry-Exit system.

Impaired locational signals are likely to generate inefficient use of the network

In the Entry-Exit market design, the only signals received by shippers regarding network costs are the price at which entry and exit capacity is sold and the amount of capacity available for sale at each entry and exit point. The sale of entry and exit capacity is organized by the TSO through auctions and is intended at recovering operating and capital costs. For each entry and exit point, the capacity price is at least equal to a fixed administrative reference price, which is set by the TSO and approved by the national energy regulator. In the rest of this paper, we will refer to this reference price as the "capacity tariff". The amount of capacity to be sold at each entry and exit point is capped by a capacity limit, which is also set by the TSO in line with technical and safety goals. Given that shippers are free to use the capacity they purchase in total, partly, or not at all, capacity limits are chosen so that most injections and withdrawals combinations within those bounds are technically feasible. Capacity prices resulting from the auction only rise above capacity tariffs when all the capacity available for a given entry or exit point is sold.

In theory, those two economic tools could be used by the TSO to convey locational signals to shippers. Capacity limits always provide some, since they are linked to technical considerations and differ for each entry and exit points. On the contrary, the European regulation lets TSOs choose whether capacity tariffs should include locational signals or not (European Commission 2017)[29], i.e. if they should be different among entry or exit points and if those differences should account for location-related changes in network cost. However, regulation also states that those tariffs should be set for periods of at least one year called "tariff periods", which prevents those locational signals, even when they are enforced, to be adapted to each flow situations.

It has been argued in the literature that inability of those signals to adapt to each flow situation lead to inefficiencies in network use (Hallack and Vazquez 2013, Hunt 2008, Rious and Hallack 2009, Vazquez et al. 2012). The most obvious ones are short-term and related to the increase in compressor expenditures given the distortion of costs signals received by shippers. These inefficiencies also change the overall cost that must be recovered through those tariffs, which scrambles incentives even

[29] Two methodologies to set capacity tariffs are recommended by the network code on tariffs (European Commission 2017). In the postage stamp methodology, all entry or exit tariffs are set equal, which does not provide any locational signals. Conversely, the capacity-weighted distance methodology leads to differentiated tariffs among entry and exit points, based on booking scenarios.

more. On the long-term, this is also likely to undermine efficient network reinforcement, since investment decisions will be based on a non-efficient usage state network.

V.2.2 The overall benefits of the Entry-Exit system need to be quantified

The overall efficiency of the Entry-Exit system depends on the balance between the gains obtained through increased market liquidity and the losses resulting from the decoupling of economic and physical gas flows.

Literature is generally focused on liquidity and trading issues

So far, debates pertaining to the Entry-Exit system have often focused on trading arrangements and on the ways to improve market functioning, tackling liquidity, market power exercise, contracts and security of supply issues.

Following earlier gas market integration studies such as (Siliverstovs et al. 2005), a first stream of literature focuses on the effect of European regulatory measures on market prices. (Growitsch et al. 2015) measure price convergence between the Dutch and German markets and (Hulshof et al. 2016, Kuper and Mulder 2016) analyze the efficiency of the Dutch day-ahead gas market. They rely spot prices data, and infrastructure use for the latter. The liquidity and the development of trading hubs is also analyzed (Miriello and Polo 2015), as well as their impact on market participants, such as gas storage operators (Felix et al. 2013). Other authors examine market power exercise in the European market through complementarity models (Egging et al. 2008, Gabriel and Smeers 2006), and the dynamics of long-term contracts (Abada et al. 2013, 2014, Creti and Villeneuve 2004). Security of supply in the European context has also been extensively studied (Abada and Massol 2011, Austvik 2016, Chaton et al. 2009).

Quantitative analyses of network use efficiency are lacking

By comparison, the literature on the consequences of the Entry-Exit market design on network use is scarce. As already mentioned, a small number of qualitative analyses have examined the decoupling of gas trading and physical flows peculiar to the Entry-Exit system and raised questions about its overall efficiency (Hallack and Vazquez 2013, Hunt 2008, Rious and Hallack 2009, Vazquez et al. 2012). Only a few authors take a quantitative approach. Some examine tariff-setting methodologies based on booking scenarios (Vos et al. 2013), while others describe their implementations in specific countries (Alonso et al. 2010, Bermúdez et al. 2013). They all rely on more or less detailed network representations to set the tariffs, but the impact of those tariffs on shippers' decisions is not investigated. And when it is (Brandão et al. 2014), the consequences on network operation are not measured.

While models that represent simultaneously gas markets and network operation have been recently introduced, they do not examine the tariff and capacity arrangements of the Entry-Exit market design.

In (Midthun et al. 2009), the importance of adding technical representations of the gas network in market models is highlighted for the first time, and the last chapter of the corresponding PhD thesis (Midthun 2007) introduces a seminal model of the impact of network decisions on capacity booking in the specific case of the Norwegian market, close but somewhat different to the European Entry-Exit system. Following the same path and taking into account the peculiarities of the Entry-Exit system, (Keyaerts et al. 2011, 2014, Keyaerts and D'haeseleer 2012) introduce gas market models with detailed network representation, including linepack storage, and imbalance regulation but focus on intraday flexibility and imbalance management. (Fodstad et al. 2015) analyze the opportunity of offering interruptible capacity booking, but again in a setting inspired by the Norwegian case. To our knowledge, no model representing the overall economic and technical impact of network capacity allocation in an Entry-Exit system has been proposed and implemented to date. Given the value at stake in compressors' fuel consumption, gas procurement costs and in gas infrastructure investment, we believe that there is a strong need for more research in that direction.

In this paper, we examine whether the Entry-Exit network access rules based on tariffs and capacity limits, can provide sufficient locational signals to guarantee an efficient use of the transmission network. We then try to quantify the potential related inefficiencies. To do so, we propose a model representing the three main decision stages occurring in a single-zone, Entry-Exit market. In a first stage, a TSO sets tariffs and capacity limits, anticipating the second stage actions of shippers on the gas market, as well as the technical network-operation decisions of the third stage. Regarding tariffs and capacity limits, we set free of any of the previously mentioned methodologies and assume that they are derived simultaneously, in a way that maximizes the overall welfare, and perfectly anticipating market risks. The gas market is assumed to be competitive and the network operator is expected to minimize operating costs. Although these assumptions might seem restrictive and unrealistic, they let us observe the ideal optimal outcome of the Entry-Exit system. Any evidence of inefficiencies in this model would guarantee the existence of inefficiencies in practice.

V.3 Modeling the Entry-Exit system

In this section, we introduce notations and two mathematical models that will be used in sections V.4 and V.5 to derive analytical and numerical results. The first one describes the European Entry-Exit system, while the second is a benchmark, the ideal integrated utility operating the gas network efficiently.

V.3.1 The Entry-Exit system model

We consider an Entry-Exit system composed of a single market zone. It features an existing gas network with legacy investment costs to recover. We focus on a single tariff period, that can be described as a succession of three decision stages. A network operator first sets for the whole tariff period (typically a year) the maximum amount of capacity to be sold at each entry and exit points, as well as the tariffs at which capacity will be sold. Gas shippers then trade gas into and out of the zone for each time step (typically a day each), taking into account these tariffs, as well as supply costs and demand. Finally, they notify their injection and withdrawal decisions to the network operator that dispatches gas so as to satisfy those decisions while minimizing transport costs.

The gas transmission network is represented by a set of nodes $\mathcal{N}$ containing $N_{\mathcal{N}}$ nodes and a set of arcs $\mathcal{A} \subset \mathcal{N} \times \mathcal{N}$ which accounts for the $N_{\mathcal{A}}$ pipes between these nodes. We assume that this network links existing demand and supply locations and that each pipe a_{ij} running from node i to node j has a given length $L_{a_{ij}}$. The subset $\mathcal{N}^{En} \subset \mathcal{N}$ denotes the $N_{\mathcal{N}^{En}}$ supply nodes of the network, i.e. the *entry* nodes of the zone, while the subset $\mathcal{N}^{Ex} \subset \mathcal{N}$ denotes the $N_{\mathcal{N}^{Ex}}$ demand nodes, i.e. the *exit* nodes. Flows in and out of the network are to be dispatched for the $N_{\mathcal{T}}$ time steps of set $\mathcal{T}$, the tariff period considered. To account for uncertainty, each possible outcome of a variable at a time step t is also indexed by the scenario s among the $N_{\mathcal{S}}$ scenarios of set $\mathcal{S}$. Variables are written as lowercase letters, while parameters of the model are referred to using capital letters.

Shippers' decisions

We first focus on the decisions made by the shippers. At each demand node i, and for each time step t, shippers can decide to withdraw a certain amount of gas $q_{t,i,s}^{with}$ given the uncertain inverse demand function $P_{t,i,s}(q_{t,i,s}^{with})$ at this node. Similarly, at each supply node i, a quantity $q_{t,i,s}^{inj}$ can be injected, knowing the associated uncertain marginal supply cost $C_{t,i,s}(q_{t,i,s}^{inj})$. Those variables are positive:

$$0 \leq q_{t,i,s}^{with} \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^{Ex}, \forall s \in \mathcal{S} \tag{1}$$

$$0 \leq q_{t,i,s}^{inj} \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^{En}, \forall s \in \mathcal{S} \tag{2}$$

We assume that the marginal supply cost at the entry points and the level of demand at the exit points are independent from the transactions happening in this zone[30].

To be allowed to flow gas into the zone at a given entry node for a time step t, shippers must buy a volume of entry capacity $k_{t,i,s}^{inj}$ at least equal to the volume of gas to be injected. Similarly, an appropriate amount of exit capacity $k_{t,i,s}^{with}$ must be purchased:

[30] This is of course a restrictive assumption, as inter-zonal transactions would influence gas prices in all the zones involved. However, since a model of this kind is new to our knowledge, restricting its scope to a single zone already provides valuable results.

$$q_{t,i,s}^{with} \leq k_{t,i,s}^{with} \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^{Ex}, \forall s \in \mathcal{S} \qquad (3)$$

$$q_{t,i,s}^{inj} \leq k_{t,i,s}^{inj} \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^{En}, \forall s \in \mathcal{S} \qquad (4)$$

However, only a limited amount of capacity $k_i^{M,with}$ (resp. $k_i^{M,inj}$), fixed by the network operator, can be purchased at each exit (resp. entry):

$$k_{t,i,s}^{with} \leq k_i^{M,with} \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^{Ex}, \forall s \in \mathcal{S} \qquad (5)$$

$$k_{t,i,s}^{inj} \leq k_i^{M,inj} \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^{En}, \forall s \in \mathcal{S} \qquad (6)$$

Capacity is auctioned by the network operator before each time step, after uncertainty has been revealed[31]. A positive reference price t_i^{with} (resp. t_i^{inj}), known as capacity tariff and fixed by the network operator, is enforced for each exit (resp. entry) capacity auction.

In the following, we assume that shippers behave competitively. Hence, for each time step, we represent them as a single entity maximizing shippers' welfare $w_{t,s}^{S}$, which is the expected surplus gained from trading gas minus the expected cost of purchasing entry and exit capacity in a given scenario:

$$w_{t,s}^{S} = \sum_{i \in \mathcal{N}^{Ex}} \int_0^{q_{t,i,s}^{with}} P_{t,i,s}(q_{t,i,s}^{with}) - \sum_{i \in \mathcal{N}^{En}} \int_0^{q_{t,i,s}^{inj}} C_{t,i,s}(\mathrm{q}_{t,i,s}^{inj}) - \sum_{i \in \mathcal{N}^{Ex}} t_i^{with} \cdot k_{t,i,s}^{with} - \sum_{i \in \mathcal{N}^{En}} t_i^{inj} \cdot k_{t,i,s}^{inj} \qquad (7)$$

In our competitive setting, the cost of purchasing capacities is written as the cost of purchasing them at a price equal to the tariff, since the existence of an auction premium at a node would only serve to discriminate between shippers, but would not change overall flow decisions[32].

Apart from the details of the auction procedures that we just described, the only constraint that we impose to the market is to feature balanced transactions:

$$\sum_{i \in \mathcal{N}^{En}} q_{t,i,s}^{inj} - \sum_{i \in \mathcal{N}^{Ex}} q_{t,i,s}^{with} = 0 \qquad \forall t \in \mathcal{T}, \forall s \in \mathcal{S} \qquad (8)$$

This is actually not guaranteed in the current implementation of European Entry-Exit system, as injection and withdrawal requests can be made independently (European Commission 2014). However, shippers are incentivized to balance their daily portfolio by ex-post imbalance settlements, which should in practice lead to an equivalent situation.

Thus, for each time step and in each scenario, we can state the model of shippers' decisions as the maximization of the objective function defined in equation (7) corresponding to this time step and this scenario, under the associated instances of constraints (1), (2), (3), (4), (5), (6) and (8). Since the

[31] We disregard inefficiencies in the allocation of capacity between market participants, i.e. we assume that a secondary market for capacity exists and is well functioning. This may not be the case in certain European markets yet.

[32] In the European system, excess revenues collected when capacity prices exceed reference prices can be used by the TSO to reduce future tariffs or physical congestion (European Commission 2017). In a way, this amounts to redistributing them to shippers.

maximum capacity limits and tariff variables are set independently and in advance by the TSO, they are considered as fixed from the shippers' point of view.

Remarking that constraints (3) and (4) are always binding, the capacity variables $k_{t,i,s}^{with}$ (resp. $k_{t,i,s}^{inj}$) can be directly replaced by the withdrawal (resp. injection) variables $q_{t,i,s}^{with}$ (resp. $q_{t,i,s}^{inj}$). Moreover, all time steps and scenario realizations being strictly independent, those one-time-step, one-scenario programs can be replaced by a single program maximizing the expected[33] total shippers' welfare w^S:

$$w^S = \mathbb{E}_s\left[\sum_{t\in\mathcal{T}}\left[\sum_{i\in\mathcal{N}^{Ex}}\int_0^{q_{t,i,s}^{with}} P_{t,i,s}\left(q_{t,i,s}^{with}\right) - \sum_{i\in\mathcal{N}^{En}}\int_0^{q_{t,i,s}^{inj}} C_{t,i,s}\left(q_{t,i,s}^{inj}\right) - \sum_{i\in\mathcal{N}^{Ex}} t_i^{with}\cdot k_{t,i,s}^{with} - \sum_{i\in\mathcal{N}^{En}} t_i^{inj}\cdot k_{t,i,s}^{inj}\right]\right] \tag{9}$$

The reformulated shippers' program is denoted S and dual variables (written using the Greek alphabet) are associated to each constraint:

Shippers' problem S:

$$\max_{q_{t,i,s}^{with},q_{t,i,s}^{inj}} \mathbb{E}_s\left[\sum_{t\in\mathcal{T}}\left[\sum_{i\in\mathcal{N}^{Ex}}\int_0^{q_{t,i,s}^{with}} P_{t,i,s}\left(q_{t,i,s}^{with}\right) - \sum_{i\in\mathcal{N}^{En}}\int_0^{q_{t,i,s}^{inj}} C_{t,i,s}\left(q_{t,i,s}^{inj}\right) - \sum_{i\in\mathcal{N}^{Ex}} t_i^{with}\cdot q_{t,i,s}^{with} - \sum_{i\in\mathcal{N}^{En}} t_i^{inj}\cdot q_{t,i,s}^{inj}\right]\right] \tag{S-1}$$

s.t. $$\sum_{i\in\mathcal{N}^{En}} q_{t,i,s}^{inj} - \sum_{i\in\mathcal{N}^{Ex}} q_{t,i,s}^{with} = 0 \qquad \alpha_{t,s}^S \qquad \forall t\in\mathcal{T},\forall s\in\mathcal{S} \tag{S-2}$$

$$q_{t,i,s}^{with} \le k_i^{M,with} \qquad \beta_{t,i,s}^{S,with} \qquad \forall t\in\mathcal{T},\forall i\in\mathcal{N}^{Ex},\forall s\in\mathcal{S} \tag{S-3}$$

$$q_{t,i,s}^{inj} \le k_i^{M,inj} \qquad \beta_{t,i,s}^{S,inj} \qquad \forall t\in\mathcal{T},\forall i\in\mathcal{N}^{En},\forall s\in\mathcal{S} \tag{S-4}$$

$$0 \le q_{t,i,s}^{with} \qquad \theta_{t,i,s}^{S,with} \qquad \forall t\in\mathcal{T},\forall i\in\mathcal{N}^{Ex},\forall s\in\mathcal{S} \tag{S-5}$$

$$0 \le q_{t,i,s}^{inj} \qquad \theta_{t,i,s}^{S,inj} \qquad \forall t\in\mathcal{T},\forall i\in\mathcal{N}^{En},\forall s\in\mathcal{S} \tag{S-6}$$

If the demand and supply functions are assumed to be linear, the shippers' problem becomes a linear optimization program.

Technical network operation

When the day-ahead market for a given day ends, shippers communicate their injection and withdrawal requests to the network operator in a process called nomination. The network operator then

[33] We choose the expectation for its simplicity in analytical and numerical derivations. Moreover, we believe that other risk measures, while producing different equations, would lead to similar results.

has to find a cost-efficient way to operate compressors and other active network elements[34]. For each time step and in each scenario, it must satisfy the injection and withdrawal decisions of the shippers while minimizing the cost of operating the network $c_{t,s}^{ope}$. This cost is the product of the energy consumed by each compressor $e_{t,a_{ij},s}$ and the cost of the energy used to run the compressors V:

$$c_{t,s}^{ope} = \sum_{a_{ij} \in \mathcal{A}} V \cdot e_{t,a_{ij},s} \qquad \forall t \in \mathcal{T}, \forall s \in \mathcal{S} \tag{10}$$

The network operator is subject to physical network constraints. They state how much gas can technically be shipped in the network, and at which cost. We assume that the network is operated in a steady state for each of the time step of our time horizon[35], which means that the flow of gas at the entry and at the exit of a pipe are identical and constant for each given time step. Hence, the flow of gas through a pipe a_{ij} is unique and is denoted $q_{t,a_{ij},s}$. It is positive when gas is flowing from node i to node j. We highlight that in a steady-state regime each time step and scenario is independent, as no gas can be stored in the network from a time step to another. Moreover, to avoid introducing integer numbers in our already complex models, we assume that network flows are always directed the same way[36] :

$$0 \leq q_{t,a_{ij},s} \qquad \forall t \in \mathcal{T}, \forall a_{ij} \in \mathcal{A}, \forall s \in \mathcal{S} \tag{11}$$

An essential network constraint is the nodal balance constraint, which ensures that the same amount of gas flows in and out of a node:

$$q_{t,i,s}^{inj} + \sum_{j \in \mathcal{N} | a_{ij} \in \mathcal{A}} q_{t,a_{ij},s} = q_{t,i,s}^{with} + \sum_{j \in \mathcal{N} | a_{ij} \in \mathcal{A}} q_{t,a_{ij},s} \qquad \forall t \in \mathcal{T},\ \forall i \in \mathcal{N}, \forall s \in \mathcal{S} \tag{12}$$

Once a feasible flow pattern has been determined, the cost of shipping gas through the network must be calculated. The non-convexities introduced by classical gas network representation, most notably due to compressors' physics, would prevent us from deriving analytical results. Therefore, we use an approximate network representation introduced in (Perrotton and Massol). Starting from a detailed, steady-state network representation, they assume that the compression levels remain low, as in most real transmission pipelines, and that a compressor station is present at the inlet of each pipe. Under those assumptions, they derive a unique equation of a Cobb-Douglas form that links the gas flow in a pipe to the amount of capital used to build it, $K_{a_{ij}}$, and to the energy consumed by the compressors:

$$q_{t,a_{ij},s}{}^{H} = B_{a_{ij}} \cdot K_{a_{ij}}{}^{G} \cdot e_{t,a_{ij},s}{}^{1-G} \qquad \forall t \in \mathcal{T}, \forall a_{ij} \in \mathcal{A}, \forall s \in \mathcal{S} \tag{13}$$

where $G = 8/11$ and $H = 9/11$ and $B_{a_{ij}}$ is a constant described in Appendix C.

[34] Although none of them has been included in this model, valves, pressure reducers for instance could be included (Schmidt et al. 2012), but it would significantly increase the complexity of the model.

[35] This amounts to neglecting the transient effects of gas flow. Those are important in practice but they would make our models much less tractable while not changing fundamentally our numerical results based on daily flows.

[36] This is the case for most real pipelines, as changing flow direction requires specific infrastructure.

Hence, for each time step and in each scenario, the network operator solves a program that maximizes the opposite of the objective function presented in the instance of equation (10) that corresponds to this time step and this scenario, under the matching instances of constraints (11), (12) and (13).

We remind the reader that all time steps are independent regarding network operation. Therefore, we can replace this multitude of single-time-step optimization programs by a unique technical network operation program, denoted T, which minimizes the expectation of the operating cost over the whole time horizon, taking the injection and withdrawal decisions of the shippers' problem S as inputs:

Technical network operation problem T:

$$\max_{\substack{q_{t,a_{ij},s} \\ e_{t,a_{ij},s}}} -\mathbb{E}_s\left[\sum_{t\in\mathcal{T}}\sum_{a_{ij}\in\mathcal{A}} V\cdot e_{t,a_{ij},s}\right] \tag{T-1}$$

$$\text{s.t. } 0 \leq q_{t,a_{ij},s} \qquad \forall t\in\mathcal{T}, \forall a_{ij}\in\mathcal{A}, \forall s\in\mathcal{S} \tag{T-2}$$

$$q_{t,i,s}^{inj} + \sum_{j\in\mathcal{N}|a_{ji}\in\mathcal{A}} q_{t,a_{ji},s} = q_{t,i,s}^{with} + \sum_{j\in\mathcal{N}|a_{ij}\in\mathcal{A}} q_{t,a_{ij},s} \qquad \forall t\in\mathcal{T},\ \forall i\in\mathcal{N}, \forall s\in\mathcal{S} \tag{T-3}$$

$${q_{t,a_{ij},s}}^{H} = B\cdot {K_{a_{ij}}}^{G}\cdot {e_{t,a_{ij},s}}^{1-G} \qquad \forall t\in\mathcal{T}, \forall a_{ij}\in\mathcal{A}, \forall s\in\mathcal{S} \tag{T-4}$$

Given the values of parameters G and H and by eliminating the energy variables in the objective function using equation (T-4), this problem becomes a non-linear, convex optimization program.

Network operator's tariffs and capacity limits determination

Before day-ahead markets take place and gas is actually dispatched, the network operator must set the positive tariffs and maximum capacity limits for all the time steps of the tariff period:

$$0 \leq t_i^{with} \qquad \forall i\in\mathcal{N}^{Ex} \tag{14}$$

$$0 \leq t_i^{inj} \qquad \forall i\in\mathcal{N}^{En} \tag{15}$$

$$0 \leq k_i^{M,with} \qquad \forall i\in\mathcal{N}^{Ex} \tag{16}$$

$$0 \leq k_i^{M,inj} \qquad \forall i\in\mathcal{N}^{En} \tag{17}$$

This happens at the beginning of the time horizon, before uncertainty is revealed. The network operator anticipates the decisions of the shippers and the subsequent technical network operation and tries to maximize the total expected social welfare w^T. This welfare is the difference between the surplus of offer and demand and the network costs, for all the time steps of the tariff period. For each pipe, network costs are the energy consumed by the compressors as expressed before. Hence, the total expected social welfare is:

$$w^T = \mathbb{E}_s\left[\sum_{t\in\mathcal{T}}\left[\sum_{i\in\mathcal{N}^{Ex}}\int_0^{q_{t,i,s}^{with}} P_{t,i,s}\left(q_{t,i,s}^{with}\right) - \sum_{i\in\mathcal{N}^{En}}\int_0^{q_{t,i,s}^{inj}} C_{t,i,s}\left(q_{t,i,s}^{inj}\right) - \sum_{a_{ij}\in\mathcal{A}} V\cdot e_{t,a_{ij},s}\right]\right] \quad (18)$$

To represent the Entry-Exit system as it is implemented in the EU, a cost-reflectivity constraint must be enforced. Regulation states that the network operator must try to recover through capacity sale just as much as it spends for capital remuneration and network operation. The return on invested capital due to network owners is defined as $R\cdot K_{a_{ij}}$, where R is the rate of return on capital for the time horizon considered. Given the uncertain nature of future operation expenses at the time when tariffs are set, we assume that this constraint is enforced in a probabilistic way, using the expectation of network costs and capacity revenues:

$$\mathbb{E}_s\left[\sum_{t\in\mathcal{T}}\sum_{a_{ij}\in\mathcal{A}} V\cdot e_{t,a_{ij},s}\right] + \sum_{a_{ij}\in\mathcal{A}} R\cdot K_{a_{ij}} = \mathbb{E}_s\left[\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}^{Ex}} t_i^{with}\cdot k_{t,i,s}^{with} + \sum_{i\in\mathcal{N}^{En}} t_i^{inj}\cdot k_{t,i,s}^{inj}\right] \quad (19)$$

We note here that instead of this strict cost-reflectivity constraint in which expected expenses and revenues must be equal, we rather use a weaker cost-*recovery* constraint in numerical applications. This makes it easier to compute numerical solutions, while any inefficiency arising with a cost-recovery constraint would be identical to that arising with a cost-reflectivity constraint. Moreover, to be coherent with our reformulated shippers' problem S, capacity purchases can here also be replaced by the injection and withdrawal decisions of the shippers. Thus, the reformulated cost-reflectivity and cost-recovery constraints are:

$$\mathbb{E}_s\left[\sum_{t\in\mathcal{T}}\sum_{a_{ij}\in\mathcal{A}} V\cdot e_{t,a_{ij},s}\right] + \sum_{a_{ij}\in\mathcal{A}} R\cdot k_{a_{ij}} = \mathbb{E}_s\left[\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}^{Ex}} t_i^{with}\cdot q_{t,i,s}^{with} + \sum_{i\in\mathcal{N}^{En}} t_i^{inj}\cdot q_{t,i,s}^{inj}\right] \quad (20)$$

$$\mathbb{E}_s\left[\sum_{t\in\mathcal{T}}\sum_{a_{ij}\in\mathcal{A}} V\cdot e_{t,a_{ij},s}\right] + \sum_{a_{ij}\in\mathcal{A}} R\cdot k_{a_{ij}} \leq \mathbb{E}_s\left[\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}^{Ex}} t_i^{with}\cdot q_{t,i,s}^{with} + \sum_{i\in\mathcal{N}^{En}} t_i^{inj}\cdot q_{t,i,s}^{inj}\right] \quad (21)$$

To sum-up, the model representing the Entry-Exit system is a three-stage model. The first stage, which represents the TSO setting tariffs and capacity limits, is the maximization of the objective function presented in (18) subject to the cost-reflectivity constraint (20) and to the positivity constraints (14), (15), (16) and (17). The two following stages are sequential, and occur in that order:

- (a) S, the problem of the shippers, taking the tariffs and capacity limits set by the network operator in the first stage as parameters and deciding for withdrawal and injection flows;

- (b) T, the technical network operation problem, taking the injection and withdrawal decisions set by the shippers in problem S as parameters and yielding the minimum energy consumption required to operate the network.

The second and third stage decision variables appear in the objective function and constraints of the first stage problem since the TSO anticipates market outcomes and network operation. We note that there is no such hierarchical relationship but only a succession relationship between the second and third stage, i.e. the decision variables of the third stage have no impact on the second stage problem. This three-stage model, denoted E_3, will be used for our analytical developments.

To compute numerical solutions, our model can be reformulated as a bilevel program. One can note that the objective function of the technical network operation problem T also appears in the objective function of the first stage problem, and that the parameters of the former (shippers' flow decisions) are anticipated variables in the first stage. Similarly to (Grimm et al. 2016), these two programs can be collapsed in a single stage, by simply including the constraints of the technical network operation problem in the first stage problem. This reformulated Entry-Exit problem, denoted E_2, is the maximization of the objective function presented in (18) subject to the cost-recovery constraint (21), to the technical network operation constraints (11), (12), (13), to the positivity constraints (14), (15), (16) and (17), and to the shippers' problem S. If the demand and supply functions are assumed to be linear, this program is a non-linear bilevel program, with a linear lower-level problem. Thus, this Mathematical problem with Equilibrium Constraints (MPEC) can be solved by adding the Karush-Kuhn-Tucker (K.K.T.) conditions of the lower-level problem as complementarity constraints to the upper-stage problem.

V.3.2 Ideal integrated utility benchmark

We assess the performance of the Entry-Exit system by comparing it to an ideal integrated welfare-maximizing utility. This utility decides how gas will be dispatched for all the time steps of the tariff period in order to maximize the total expected welfare w^T.

This lets us define the problem of the integrated, welfare-maximizing utility, denoted I (the dual variables associated to each constraint are written using the Greek alphabet):

Integrated utility's problem I:

$$\max_{\substack{q_{t,i,s}^{with}, q_{t,i,s}^{inj} \\ q_{t,a_{ij},s}, e_{t,a_{ij},s}}} \mathbb{E}_s \left[\sum_{t \in \mathcal{T}} \left[\sum_{i \in \mathcal{N}^{Ex}} \int_0^{q_{t,i,s}^{with}} P_{t,i,s}(q_{t,i,s}^{with}) - \sum_{i \in \mathcal{N}^{En}} \int_0^{q_{t,i,s}^{inj}} C_{t,i,s}(q_{t,i,s}^{inj}) - \sum_{a_{ij} \in \mathcal{A}} V \cdot e_{t,a_{ij},s} \right] \right] \quad \text{(I-1)}$$

$$\text{s.t. } q_{t,i,s}^{inj} + \sum_{j \in \mathcal{N} | a_{ji} \in \mathcal{A}} q_{t,a_{ji},s} = q_{t,i,s}^{with} + \sum_{j \in \mathcal{N} | a_{ij} \in \mathcal{A}} q_{t,a_{ij},s} \qquad \alpha_{t,i,s}^{I} \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}, \forall s \in \mathcal{S} \quad \text{(I-2)}$$

$$q_{t,a_{ij},s}{}^{H} = B \cdot K_{a_{ij}}{}^{G} \cdot e_{t,a_{ij},s}{}^{1-G} \qquad \beta_{t,a_{ij},s}^{I} \quad \forall t \in \mathcal{T}, \forall a_{ij} \in \mathcal{A}, \forall s \in \mathcal{S} \quad \text{(I-3)}$$

$$0 \le q_{t,a_{ij},s} \qquad \theta_{t,a_{ij},s}^{I} \quad \forall t \in \mathcal{T}, \forall a_{ij} \in \mathcal{A}, \forall s \in \mathcal{S} \quad \text{(I-4)}$$

$$0 \le q_{t,i,s}^{with} \qquad \theta_{t,i,s}^{I,with} \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^{Ex}, \forall s \in \mathcal{S} \quad \text{(I-5)}$$

$$0 \le q_{t,i,s}^{inj} \qquad \theta_{t,i,s}^{I,inj} \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^{En}, \forall s \in \mathcal{S} \quad \text{(I-6)}$$

It is a non-linear, non-convex optimization program. However, one can notice that if linear demand and supply functions are used, it becomes a non-linear convex optimization program.

In the next section, we use these models to identify the inefficiency sources of the Entry-Exit system when it is applied to the gas transmission network.

V.4 Identifying the inefficiencies of the Entry-Exit system

We now use model E_3 to assess the efficiency of market-driven network use in the Entry-Exit system, compared to an ideal integrated-utility benchmark. In a first sub-section, we analytically prove that the Entry-Exit system becomes inefficient when the tariffs and capacity limits are set in advance for more than two different demand instances, which is always the case in practice. We then highlight that the enforcement of a cost-reflectivity constraint makes the conditions of efficiency even more stringent. Finally, we show that setting tariffs and capacity limits in two separate processes also impacts efficiency. In this section, we assume that the demand and offer functions are linear. For the sake of concision, all technical developments and proofs are presented in Appendix A.

V.4.1 Efficiency of the Entry-Exit market design

We first examine the efficiency of the Entry-Exit system when it is not subject to a cost-reflectivity constraint.

We can write the optimality conditions of our integrated-utility benchmark I that fully characterize the efficient dispatch. Similarly, for a given set of tariffs and capacity limits, the optimality conditions of the shippers' problem S, which characterize the trading decisions of the shippers, can be derived. By comparing those two sets of optimality conditions, one can examine in which conditions it is possible for a network operator subject to the Entry-Exit system to find a set of tariffs and capacity limits that induce shippers to favor a dispatch identical to that of the benchmark. Conclusions are detailed in the following proposition.

Proposition 1: *It is not possible to find a set of Entry-Exit tariffs and capacity limits* $\left(t_i^{with}, k_i^{M,with}\right)_{i=(1,\dots,N_{\mathcal{N}Ex})}$ *and* $\left(t_i^{inj}, k_i^{M,inj}\right)_{i=(1,\dots,N_{\mathcal{N}En})}$ *that induces an efficient dispatch if* $N_{\mathcal{T}} \cdot N_{\mathcal{S}} > 2$.

To put it differently, the Entry-Exit system leads to an inefficient allocation of network resources if shippers have a risk to face more than two different demand and supply instances during the period for which tariffs and capacity limits are fixed. An instance is the realization of the demand and supply functions for a given time step in a given scenario. Considering daily time steps and scenarios, this occurs if the tariff period is longer than two days, or if it is one-day long but three or more different demand and supply scenarios are to be anticipated. Since tariffs and capacity limits are in practice set for long periods of time (typically months) and well in advance (typically once a year), the Entry-Exit system is bound to be less efficient than the ideal integrated-utility benchmark.

V.4.2 Effect of the enforcement of a cost-reflectivity condition

We now examine how the cost-reflectivity condition that is usually imposed to network operators under an Entry-Exit system changes the previous conclusion. The effect of adding a cost-reflectivity condition is detailed in the following proposition.

Proposition 2: *When a cost-reflectivity condition is enforced, it is not possible to find a set of Entry-Exit tariffs and capacity limits* $\left(t_i^{with}, k_i^{M,with}\right)_{i=(1,\dots,N_{\mathcal{N}Ex})}$ *and* $\left(t_i^{inj}, k_i^{M,inj}\right)_{i=(1,\dots,N_{\mathcal{N}En})}$ *that induces an efficient dispatch if* $N_{\mathcal{T}} \cdot N_{\mathcal{S}} > 1$.

Thus, an Entry-Exit system in which a cost-reflectivity constraint is implemented may induce an efficient allocation of network resources only if tariffs and capacity limits are set individually for each demand and supply instance. Once again, since tariffs and capacity limits are in practice set for long periods of time (typically months) and well in advance (typically once a year), an Entry-Exit system in which a cost-reflectivity constraint is enforced is less efficient than the ideal integrated-utility benchmark. This confirms that the main inefficiency source of the Entry-Exit system is the impossibility to adapt network economic signals in real-time. However, even assuming that this would be possible, this system offers no way for the TSO to reveal the willingness to pay of shippers in real-time, which makes tariffs determination only rely on the anticipation capability of the TSO. Hence, adapting tariffs to flow configurations on the short term would not be feasible nor advisable in practice.

V.4.3 Impact of the open access paradigm on the Entry-Exit system efficiency

In the ideal Entry-Exit system that we outlined in models E_2 and E_3, the network operator has the opportunity to use both tariffs and capacity limits as tools to send locational signals to shippers and achieve gas market efficiency. However, the anticipation and modelling effort that this would require becomes tremendously complex when the size of the network at stake or the length of the tariff period increase, and when interconnections to other Entry-Exit zones exist. In practice, due to the intractability of such a task, capacity limits are set independently through a separate technical analysis. Since the open access paradigm of the Entry-Exit system (Glachant et al. 2014) links the right to inject or withdraw gas only to the ownership of capacity, the network operator sets capacity limits so as to ensure that most of the dispatches involving injections and withdrawals within those limits are feasible[37]. Tariffs are determined separately, capacity limits being considered as fixed. The consequence of setting capacity limits independently from tariffs is detailed in the following proposition.

> ***Proposition 3:*** *When capacity limits* $\left(k_i^{M,with}\right)_{i=(1,\ldots,N_{\mathcal{N}Ex})}$ *and* $\left(k_i^{M,inj}\right)_{i=(1,\ldots,N_{\mathcal{N}En})}$ *are considered as fixed, it is not possible to find a set of Entry-Exit tariffs and capacity limits* $\left(t_i^{with}, k_i^{M,with}\right)_{i=(1,\ldots,N_{\mathcal{N}Ex})}$ *and* $\left(t_i^{inj}, k_i^{M,inj}\right)_{i=(1,\ldots,N_{\mathcal{N}En})}$ *that induces an efficient dispatch if* $N_{\mathcal{T}} \cdot N_{\mathcal{S}} > 1$.

Hence, an Entry-Exit system in which capacity and tariffs are determined separately can induce an efficient allocation of network resources only if tariffs are set individually for each demand and supply instance. Here also, since tariffs are in practice set for long periods of time (typically months) and well in advance (typically once a year), an Entry-Exit system in which capacity limits are set independently from tariffs is less efficient than the ideal integrated-utility benchmark.

V.4.4 Numerical case study

We have shown analytically the existence of network allocation inefficiencies in an Entry-Exit system. We now try to quantify the magnitude of these inefficiencies using model E_2. Given its complexity, we were unable to apply it on large, realistic network and market cases. Hence, we illustrate the previous propositions on a simple network case, composed of a single pipe, with injection taking place at first node i_1 and withdrawal at end node i_2. We use model E_2 with or without a cost-recovery constraint, and with endogenous or exogenous capacity limits to analyze the performance of the Entry-Exit system on such a network. We also rely on model I to compute the decisions of the

[37] To avoid being too restrictive, a probabilistic analysis may be used to exclude situations that would be very unlikely to occur but that would have a high impact on capacity limits (Koch et al. 2015).

ideal integrated-utility benchmark. We assume that the demand and offer functions are linear and that only demand is uncertain[38]. Models are solved in GAMS using the EMP framework (Ferris et al. 2009, GAMS Development Corporation 2016), and at a 10^{-12} solving precision. All numerical results are presented using the following units: US dollars for costs and surpluses, million standard cubic feet per day (MSCFD) for flows and capacity limits, dollars per million standard cubic feet per day ($/MSCFD) for tariffs. The details of those case study can be found in Appendix B.

We first illustrate Proposition 1 by computing the performance of the Entry-Exit system for a tariff period lasting one day, when two or three one-day demand scenarios are anticipated by the network operator. The results, detailed in Table 1, are coherent with Proposition 1: the Entry-Exit system is efficient when two scenarios are anticipated, while it performs worse than the integrated utility when three scenarios are anticipated. Moreover, the magnitude of the inefficiency is very small (0,0010%). Regarding the very low level of these inefficiencies, it must be underlined that these results were obtained for a very simple network, for a single day set-up and for an idealistic Entry-Exit system.

Table 1: Illustration of Proposition 1 on a single-pipe network

		Flow			Tariff		Capacity		Welfare	
		$q^{with}_{t_1,i_2,s_1}$	$q^{with}_{t_1,i_2,s_2}$	$q^{with}_{t_1,i_2,s_3}$	$t^{inj}_{i_1}$	$t^{with}_{i_2}$	$k^{M,inj}_{i_1}$	$k^{M,with}_{i_2}$	w^T	$\frac{w^T_I - w^T_E}{w^T_I}$
$N_S = 2$	Integrated utility (I)	13.69	6.90	–	–	–	–	–	418.645	
	Entry-Exit (E) no cost-recovery	13.69	6.90	–	0.029	0.419	14.78	13.69	418.645	**0.0000%**
$N_S = 3$	Integrated utility (I)	13.67	10.30	6.90	–	–	–	–	404.525	
	Entry-Exit (E) no cost recovery	13.67	10.34	6.86	0.394	0.386	13.67	13.73	404.521	**0.0010%**

Note: this table compares the withdrawn flows, injection and withdrawal tariffs, injection and withdrawal capacity limits as well as total expected welfare for a single pipe under an Entry-Exit system when no cost-recovery constraint is enforced, to that of the same network under the management of an integrated utility. The last column presents the relative difference between their total expected welfares.

We now illustrate Proposition 2 and 3 by comparing the performance of three variants of the Entry-Exit system. The first one, when no cost-recovery constraint is enforced and capacity limits are set endogenously is the less constrained one. The second one differs by introducing a cost-recovery constraint. Finally, the third still has no cost-recovery constraint enforced, but capacity limits are arbitrarily set exogenously at a value 50% higher than their optimum in the first variant. In all cases, two one-day demand scenarios are anticipated by the network operator. The results, detailed in Table 2, confirm that when a cost-recovery constraint is enforced or when the choices of tariffs and capacity limits are decoupled, the allocation of network resources is inefficient compared to the benchmark of the integrated utility.

[38] Uncertainty can easily be added on the supply side as well and would produce similar results.

Table 2: Illustration of Propositions 2 and 3 on a single-pipe network

$N_S = 2$	Flow		Tariff		Capacity				Welfare	
	$q^{with}_{t_1,i_2,s_1}$	$q^{with}_{t_1,i_2,s_2}$	$t^{inj}_{i_1}$	$t^{with}_{i_2}$	$k^{M,inj}_{i_1}$	$k^{M,with}_{i_2}$	Tariff revenue	Network costs	w^T	$\frac{w_I^T - w_E^T}{w_I^T}$
Entry-Exit (E)	13.69	6.90	0.029	0.419	14.78	13.69	4.607	16.599	418.645	**0.0000%**
Entry-Exit (E) with cost-recovery	13.69	6.74	0.811	0.811	13.69	14.84	16.564	16.564	418.596	**0.0117%**
Entry-Exit (E) with exogenous capacities limits	13.78	6.81	0.555	0.555	20.53	20.53	11.416	16.659	418.614	**0.0073%**

Notes: this table compares the withdrawn flows, injection and withdrawal tariffs, injection and withdrawal capacity limits as well as total expected tariff revenue, total expected network costs and total expected welfare for a single pipe for three Entry-Exit system settings. The last column presents the relative difference between the total expected welfares of those Entry-Exit systems and that of the integrated utility benchmark for the same demand scenarios which was already listed in Table 1, line 2.

The magnitude of the inefficiencies is slightly higher, but remains quite small. This is not surprising given the low level of operation expenses compared to capital remuneration, the small size of the network, the shortness of the tariff period considered and the ideal Entry-Exit system considered. How these inefficiencies scale on large network, with a diverse and numerous user base and for longer tariff periods is unknown though. Computing similar results for a realistic network case, if feasible, would require the use or the development of much more sophisticated solving procedures. We try to explore on simple cases whether this would be worth the effort, i.e. if inefficiencies could potentially be larger in real Entry-Exit markets[39].

V.5 Discussion

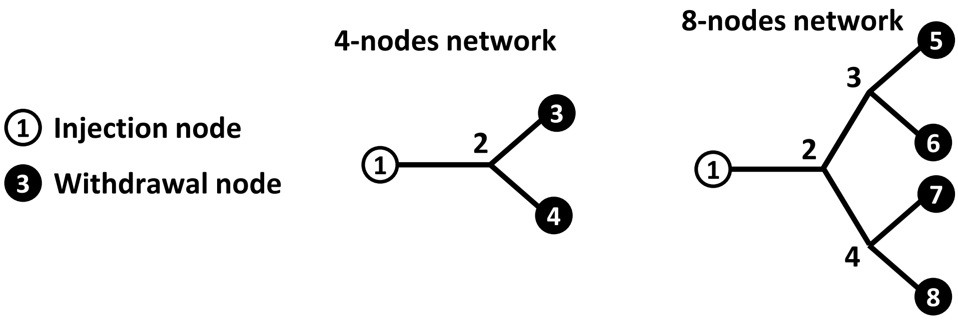
In this section, we numerically analyze the effect of various parameters on the magnitude of the inefficiencies uncovered analytically and numerically in section V.4. We first show that these inefficiencies may increase with network size and demand heterogeneity. Then, we try to rank inefficiency sources for under-sized and over-sized networks. We conclude on the need for further research efforts leading to the analysis of real case studies.

V.5.1 Influence of network size

We start by showing that inefficiencies may be higher in larger networks. We numerically compare the dispatch of a four-nodes network under an Entry-Exit system to that of an eight-nodes network, as shown in Figure 1.

[39] Other results, presented in Appendix D and Appendix E, also rule out that our analytical results, based on the assumption of exogenous network investment and simplified network representation, would change with endogenous investment or detailed network representation.

Figure 1: 4-node and 8-node network cases



In both cases, three demand scenarios are anticipated for a one-day tariff period. Those are evenly spaced around a central mean demand value and identical for each demand nodes. No cost-recovery constraint is enforced. The results, presented in Table, show that the inefficiencies more than double with this increase in network size. This indicates that the inefficiencies in real networks could be higher than those observed on simple networks. Moreover, while the two networks used here feature an increase in size, they retain the same relatively simple tree structure. Although we were unable to test it, we suspect that the often more complex, meshed structure of European gas networks could also induce an increase in inefficiencies. Finally, this has important policy implications regarding future changes in the geographical definition the European Entry-Exit zones. Any merger of existing zones, by creating a larger zone, increases the potential for network allocation inefficiencies. This confirms the risk of a trade-off between liquidity improvements and network use efficiency and calls for a thorough cost-benefit analysis of merger projects.

Table 3: Influence of network size on dispatch inefficiencies

$N_S = 3$		Flow			Tariff		Capacity		Welfare	
		$q^{with}_{t_1,i_3,s_1}$	$q^{with}_{t_1,i_3,s_2}$	$q^{with}_{t_1,i_3,s_3}$	$t^{inj}_{i_1}$	$t^{with}_{i_3}$	$k^{M,inj}_{i_1}$	$k^{M,with}_{i_3}$	w^T	$\frac{w^T_I - w^T_E}{w^T_I}$
$N_{\mathcal{N}} = 4$	Integrated utility (I)	13.44	10.17	6.84	–	–	–	–	780.647	
	Entry-Exit (E)	13.44	10.24	6.76	0.46	1.01	26.95	13.44	780.618	**0.0038%**
		$q^{with}_{t_1,i_5,s_1}$	$q^{with}_{t_1,i_5,s_2}$	$q^{with}_{t_1,i_5,s_3}$	$t^{inj}_{i_1}$	$t^{with}_{i_5}$	$k^{M,inj}_{i_1}$	$k^{M,with}_{i_5}$	w^T	$\frac{w^T_I - w^T_E}{w^T_I}$
$N_{\mathcal{N}} = 8$	Integrated utility (I)	9.24	7.02	4.74	–	–	–	–	1050.372	
	Entry-Exit (E)	9.24	7.10	4.66	0.45	1.79	36.94	9.24	1050.276	**0.0092%**

V.5.2 Influence of the structure of uncertainties

We now look at the impact of the structure of demand uncertainty on inefficiencies. Table 4 compares the magnitude of the inefficiencies in a single-pipe Entry-Exit system, where no cost-recovery constraint is enforced, for two different sets of one-day demand scenarios. The first set of demand scenarios is characterized by four demand scenarios evenly spread around a mean demand value (given that linear demands are used, we vary the intercept of the inverse demand function, while its slope remains unchanged among scenarios). In the second set of demand scenarios, the mean demand value remains the same, but the scenarios "diverge", with the two first scenarios being close, high demand scenarios, while the two lasts are close, low demand scenarios (See Figure B-1 in Appendix B). We consider the same single-pipe network as used in section V.4.

Table 4: Influence of the structure of demand scenarios on dispatch inefficiencies

$N_S = 4$		Flow				Tariff		Capacity		Welfare	
		$q^{with}_{t_1,i_2,s_1}$	$q^{with}_{t_1,i_2,s_2}$	$q^{with}_{t_1,i_2,s_3}$	$q^{with}_{t_1,i_2,s_4}$	$t^{inj}_{i_1}$	$t^{with}_{i_2}$	$k^{M,inj}_{i_1}$	$k^{M,with}_{i_2}$	w^T	$\frac{w^T_I - w^T_E}{w^T_I}$
Even scenarios	Integrated utility (I)	13.67	11.42	9.17	6.90	–	–	–	–	399.824	
	Entry-Exit (E)	13.67	11.48	9.16	6.84	0.45	0.45	13.67	13.67	399.818	**0.0016%**
Divergent scenarios	Integrated utility (I)	15.12	13.70	7.99	6.55	–	–	–	–	442.627	
	Entry-Exit (E)	15.12	13.81	7.95	6.48	0.41	0.41	15.12	15.12	442.611	**0.0036%**

Results show that the magnitude of the inefficiencies is more than doubled when the demand scenarios diverge instead of being evenly spread around a mean demand. This means that a single set of tariff and capacity limit better accommodates similar demands rather than demands that differ wildly. Those conclusions would symmetrically apply to supply uncertainty. In real Entry-Exit markets, this means that inefficiencies may increase when entry and exit points encompass shippers' activity with disparate injection or withdrawal patterns. Similarly, if the duration of the tariff period for which tariffs and capacity limits are defined comprises heterogeneous demand and injection patterns, inefficiencies could increase. Since gas demand fluctuations are expected to increase in the future due to electricity-generation uses, it is important to better assess the efficiency of the Entry-Exit system to generate efficient network allocation with a variable demand.

V.5.3 Influence of infrastructure use level

Investing in gas transmission networks is costly and irreversible. Hence, it often lags behind usage trends. In this subsection, we analyze the effect of over- and under-investment on the inefficiencies of the Entry-Exit system. We use the same single-pipe test cases used at the end of section V.4, but present two variants where the exogenous capital is set 30% above or under the optimal capital determined by the integrated utility. Results appear in Table 5.

We see that the inefficiency generated in a cost-recovery regime is much higher when the network is over-sized. This is due to the fact that the cost of excess investment is born by a smaller user base, generating higher distortions. It is also interesting to note that this cost-recovery constraint has no effect on dispatch in an under-sized network. Conversely, exogenously setting large capacity limits introduces a much higher distortion when the network is under-sized than when it is oversized. This may come from the fact that the network operator must make a stringent use of tariffs to bring back injection and withdrawal bids to acceptable levels, since they are not capped by wisely chosen capacity limits. Thus, the inadequacy of real networks to current use patterns may create higher inefficiencies than those revealed in our test cases. As the European gas demand currently declines or stagnates, the distortions in network allocation created by the costs of legacy infrastructure built for a larger demand should be carefully assessed.

Table 5: Assessment of Propositions 1, 2 and 3 in case of under- and over-investment

$N_S = 3$		Flow			Tariff		Capacity		Welfare	
		$q^{with}_{t_1,i_2,s_1}$	$q^{with}_{t_1,i_2,s_2}$	$q^{with}_{t_1,i_2,s_3}$	$t^{inj}_{i_1}$	$t^{with}_{i_2}$	$k^{M,inj}_{i_1}$	$k^{M,with}_{i_2}$	w^T	$\frac{w^T_I - w^T_E}{w^T_I}$
Optimal capacity: $k_{a_{12}}$=143	Integrated utility (I)	13.67	10.30	6.90	–	–	–	–	404.525	
Over-capacity: $k_{a_{12}}$=186	Integrated utility (I)	13.80	10.37	6.93	–	–	–	–	403.241	
	Entry-Exit (E)	13.80	10.39	6.91	0.12	0.27	16.23	13.80	403.240	0.0003%
	Entry-Exit (E) cost-recovery	13.69	10.21	6.73	0.86	0.86	1.14E+05	1.79E+11	403.144	**0.0242%**
	Entry-Exit (E) exogenous capacity limits	13.85	10.37	6.89	0.29	0.29	20.70	20.70	403.236	0.0015%
Under-capacity: $k_{a_{12}}$=100	Integrated utility (I)	13.29	10.08	6.80	–	–	–	–	401.300	
	Entry-Exit (E)	13.29	10.18	6.70	0.03	1.90	13.29	13.29	401.275	0.0063%
	Entry-Exit (E) cost-recovery	13.29	10.18	6.70	1.75	0.17	13.29	15.26	401.275	0.0063%
	Entry-Exit (E) exogenous capacity limits	13.54	10.05	6.57	1.41	1.42	19.94	19.94	401.156	**0.0358%**

The inefficiencies in network resource allocation of the Entry-Exit system that were quantified in section V.4 were very small, but also derived from simple market and network cases. The results obtained in this section all point out that the inefficiencies to be found in real Entry-Exit markets may be much larger. This stresses the need for further research aiming at quantifying those inefficiencies on real case studies.

V.6 Conclusion

The Entry-Exit system applied in Europe is argued to foster liquidity in the gas market. However, by decoupling physical flows from economic trades, it also reduces the network-cost incentives that shippers receive. At a time of renewed debates on the European gas market design, its efficiency should be assessed as the balance of those two effects. However, the inefficiencies arising from the current network allocation have received much less attention, not even mentioning quantification efforts, compared to trading issues. For the first time, we propose and implement a modelling framework to analyze them, which includes tariffs-setting and capacity-allocation mechanisms on a single market zone.

We analytically conclude that network allocation is in practice inefficient in an Entry-Exit system compared to the ideal integrated-utility benchmark. This stems from the fact that the network operator sets tariffs in advance and freezes them for long periods of time. Moreover, as the Entry-Exit system features no real-time mechanism to reveal the willingness to pay of network users, the minimization of this inefficiency solely relies on the anticipation capabilities of the network operator. These are the main sources of inefficiency in network allocation of the Entry-Exit system.

We then use our modeling framework to quantify these network allocation inefficiencies. Given the complexity of the model, only simple network and demand cases have been studied. Results show small inefficiencies on those simplistic cases. However, they also show that inefficiencies might be much higher in real markets. Therefore, we underline the need for regulatory bodies and TSOs to pursue this effort in quantifying the effects of the Entry-Exit market design on network allocation for real market cases. More specifically, we unveil a possible increase of inefficiencies with network size, which confirms that any project aimed at merging zones, or reforming tariffs should carefully balance benefits and costs. We also observe an increase in inefficiencies with demand heterogeneity. Given the shared belief that gas-demand variability will rise in the coming years, this calls for a debate on the adequacy of the Entry-Exit setup to the future of the European gas market. Finally, we confirm that if the current stagnation or decline of gas demand in Europe leads existing transmission infrastructure to be considered oversized, inefficiencies in network allocation may increase.

Keeping in mind that the ideal integrated-utility benchmark is by essence, idealistic, only the balance of liquidity gains and network allocation inefficiencies can tell whether the Entry-Exit system is a good compromise in practice when compared to other gas market design options. Given the economic value at stake, this should be carefully measured to justify consolidating, adapting or moving away from the Entry-Exit system. This work is a strong motivation and a first step toward highly challenging realistic case studies, as the only way to provide more insights on the efficiency of existing Entry-Exit gas markets.

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V.8 References

Abada I, Ehrenmann A, Smeers Y (2014) Modeling Gas Markets with Endogenous Long-Term Contracts. *CORE Discussion Paper*.

Abada I, Gabriel S, Briat V, Massol O (2013) A Generalized Nash–Cournot Model for the Northwestern European Natural Gas Markets with a Fuel Substitution Demand Function: The GaMMES Model. *Networks and Spatial Economics* 13(1):1–42.

Abada I, Massol O (2011) Security of supply and retail competition in the European gas market. *Energy Policy* 39(7):4077–4088.

ACER (2015) *European Gas Target Model - Review and Update* (ACER).

Alonso A, Olmos L, Serrano M (2010) Application of an entry–exit tariff model to the gas transport system in Spain. *Energy Policy* 38(9):5133–5140.

Austvik OG (2016) The Energy Union and security-of-gas supply. *Energy Policy* 96:372–382.

Babonneau F, Nesterov Y, Vial JP (2012) Design and Operations of Gas Transmission Networks. *Operations Research* 60(1):34–47.

Bermúdez A, González-Díaz J, González-Diéguez FJ, González-Rueda ÁM (2013) Gas transport networks: Entry–exit tariffs via least squares methodology. *Energy Policy* 63:252–260.

Brandão A, Soares I, Sarmento P, Resende J, Pinho J (2014) Regulating international gas transport: Welfare effects of postage stamp and entry–exit systems. *Energy* 69:86–95.

CEER (2011) *Draft Vision for a European Gas Target Model* (CEER).

Chaton C, Creti A, Villeneuve B (2009) Storage and security of supply in the medium run. *Resource and Energy Economics* 31(1):24–38.

Coelho PM, Pinho C (2007) Considerations about equations for steady state flow in natural gas pipelines. *Journal of the Brazilian Society of Mechanical Sciences and Engineering* 29(3):262–273.

Creti A, Villeneuve B (2004) Longterm Contracts and Take-or-Pay Clauses in Natural Gas Markets. *Energy Studies Review* 13(1).

De Wolf D, Smeers Y (1996) Optimal Dimensioning of Pipe Networks with Application to Gas Transmission Networks. *Operations Research* 44(4):596–608.

De Wolf D, Smeers Y (2000) The gas transmission problem solved by an extension of the simplex algorithm. *Management Science* 46(11):1454–1465.

Egging R, Gabriel SA, Holz F, Zhuang J (2008) A complementarity model for the European natural gas market. *Energy Policy* 36(7):2385–2414.

European Commission (2014) *Commission Regulation (EU) No 312/2014 of 26 March 2014 establishing a Network Code on Gas Balancing of Transmission Networks* (European Commission).

European Commission (2016) Quo vadis gas market regulatory framework. *EU Commission Energy Studies*. Retrieved (October 15, 2017), https://ec.europa.eu/energy/en/studies?field_associated_topic_tid=63.

European Commission (2017) *Commission Regulation (EU) 2017/460 of 16 March 2017 establishing a network code on harmonised transmission tariff structure for gas* (European Commission).

European Union (1998) *Directive 98/30/EC of the European Parliament and of the Council of 22 June 1998 concerning common rules for the internal market in natural gas* (European Union).

European Union (2003) *Directive 2003/55/EC of the European Parliament and of the Council of 26 June 2003 concerning common rules for the internal market in natural gas* (European Union).

European Union (2009a) *Directive 2009/73/EC of the European Parliament and of the Council of 13 July 2009 concerning common rules for the internal market in natural gas and repealing Directive 2003/55/EC* (European Union).

European Union (2009b) *Regulation (EC) No 715/2009 of the European Parliament and of the Council of 13 July 2009 on Conditions for Access to the Natural Gas Transmission Networks and Repealing Regulation (EC) No 1775/2005* (European Union).

Felix B, Woll O, Weber C (2013) Gas storage valuation under limited market liquidity: an application in Germany. *The European Journal of Finance* 19(7–8):715–733.

Ferris MC, Dirkse SP, Jagla JH, Meeraus A (2009) An extended mathematical programming framework. *Computers & Chemical Engineering* 33(12):1973–1982.

Fodstad M, Midthun KT, Tomasgard A (2015) Adding flexibility in a natural gas transportation network using interruptible transportation services. *European Journal of Operational Research* 243(2):647–657.

Gabriel S, Smeers Y (2006) *Complementarity problems in restructured natural gas markets* (Springer).

GAMS Development Corporation (2016) *General Algebraic Modeling System (GAMS) Release 24.7.1* (Washington, DC, USA).

Glachant JM, Hallack M, Vazquez M (2014) *Gas network and market diversity in the US, the EU and Australia: a story of network access rights* (European University Institute).

Grimm V, Martin A, Schmidt M, Weibelzahl M, Zöttl G (2016) Transmission and generation investment in electricity markets: The effects of market splitting and network fee regimes. *European Journal of Operational Research* 254(2):493–509.

Growitsch C, Stronzik M, Nepal R (2015) Price Convergence and Information Efficiency in German Natural Gas Markets. *German Economic Review* 16(1):87–103.

Hallack M, Vazquez M (2013) European Union regulation of gas transmission services: Challenges in the allocation of network resources through entry/exit schemes. *Utilities Policy* 25:23–32.

Hulshof D, van der Maat JP, Mulder M (2016) Market fundamentals, competition and natural-gas prices. *Energy Policy* 94:480–491.

Hunt P (2008) *Entry–Exit Transmission Pricing with Notional Hubs: Can it Deliver a Pan-European Wholesale Market in Gas?* (Oxford Institute for Energy Studies).

Keyaerts N, Delarue E, Rombauts Y, D'haeseleer W (2014) Impact of unpredictable renewables on gas-balancing design in Europe. *Applied Energy* 119:266–277.

Keyaerts N, D'haeseleer W (2012) Increasing efficiency through market-based cross-border procurement of gas-balancing services in Europe. *Energy* 47(1):564–576.

Keyaerts N, Hallack M, Glachant JM, D'haeseleer W (2011) Gas market distorting effects of imbalanced gas balancing rules: Inefficient regulation of pipeline flexibility. *Energy Policy* 39(2):865–876.

Koch T, Hiller B, Pfetsch ME, Schewe L eds. (2015) *Evaluating Gas Network Capacities* (Society for Industrial and Applied Mathematics, Philadelphia, PA).

Kuper GH, Mulder M (2016) Cross-border constraints, institutional changes and integration of the Dutch–German gas market. *Energy Economics* 53:182–192.

Menon ES (2005) *Gas Pipeline Hydraulics* (CRC Press).

Midthun KT (2007) *Optimization models for liberalized natural gas markets*. dissertation. (Norwegian University of Science and Technology).

Midthun KT, Bjorndal M, Tomasgard A (2009) Modeling optimal economic dispatch and system effects in natural gas networks. *The Energy Journal* 30(4):155–180.

Miriello C, Polo M (2015) The development of gas hubs in Europe. *Energy Policy* 84:177–190.

Möller M (2004) *Mixed integer models for the optimisation of gas networks in the stationary case*. (TU Darmstadt).

O'Neill RP, Williard M, Wilkins B, Pike R (1979) A Mathematical Programming Model for Allocation of Natural Gas. *Operations Research* 27(5):857–873.

Perrotton F, Massol O The technology of a natural gas pipeline: Insights for costs and rate-of-return regulation. *Submitted to Utilities Policy*.

Rious V, Hallack M (2009) *Tariffs for European Gas Transmission Networks* (Florence School of Regulation).

Rossi B, Casoetto B, Le Maitre A, Jacquiau A, Moumini N (2012) Computing And Optimizing the Available Linepack In a Gas Network to Address the Issue of Flexibility. *PSIG Annual Meeting*.

Schmidt M, Steinbach MC, Willert BM (2012) High detail stationary optimization models for gas networks—Part 1: Model components. *IfAM Preprint* 94.

Siliverstovs B, L'Hégaret G, Neumann A, von Hirschhausen C (2005) International market integration for natural gas? A cointegration analysis of prices in Europe, North America and Japan. *Energy Economics* 27(4):603–615.

The World Bank Commodity Markets Review. *Monthly Prices*. Retrieved (March 8, 2017), http://www.worldbank.org/en/research/commodity-markets.

Vazquez M, Hallack M, Glachant JM (2012) Designing the European Gas Market: More Liquid & Less Natural? *Economics of Energy & Environmental Policy* 1(3):25–38.

Vos M, Kiewiet B, Petrov K (2013) *Gas Transmission Pricing Models For Entry-Exit System and Implementation Options* (DNV KEMA, Groningen, The Netherlands).

Yépez RA (2008) A Cost Function for the Natural Gas Transmission Industry. *The Engineering Economist* 53(1):68–83.

Appendix A – Mathematical proofs

Proof of Proposition 1:

We assume that the demand and supply functions $P_{t,i,s}$ and $C_{t,i,s}$ are linear.

Ideal dispatch characterization

The integrated utility problem I is a convex non-linear optimization problem. Hence, its K.K.T. conditions can be derived. Out of those, the two equations and the two complementarity conditions that involve injection and withdrawal decisions are:

$$P_{t,i,s}\left(q_{t,i,s}^{with}\right) + \alpha_{t,i,s}^{I} + \theta_{t,i,s}^{I,with} = 0 \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^{Ex}, \forall s \in \mathcal{S} \qquad \text{(A-1)}$$

$$-C_{t,i,s}\left(q_{t,i,s}^{inj}\right) - \alpha_{t,i,s}^{I} + \theta_{t,i,s}^{I,inj} = 0 \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^{En}, \forall s \in \mathcal{S} \qquad \text{(A-2)}$$

$$0 \leq \theta_{t,i,s}^{I,with} \perp -q_{t,i,s}^{with} \leq 0 \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^{Ex}, \forall s \in \mathcal{S} \qquad \text{(A-3)}$$

$$0 \leq \theta_{t,i,s}^{I,inj} \perp -q_{t,i,s}^{inj} \leq 0 \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^{En}, \forall s \in \mathcal{S} \qquad \text{(A-4)}$$

The solution to this problem is unique, and only depends on the chosen set of demand and supply functions $U = \left(P_{t,i,s}, C_{t,i,s}\right)$. Thus, the value taken by variables at solution, denoted with a star exponent, can be written as functions of those parameters: $q_{t,i,s}^{with,*}(U)$, $q_{t,i,s}^{inj,*}(U)$, $\alpha_{t,i,s}^{I,*}(U)$, $\theta_{t,i,s}^{I,with,*}(U)$, $\theta_{t,i,s}^{I,inj,*}(U)$. As already mentioned, all time steps and scenarios are independent (this is straightforward from the K.K.T., as each equation involves only a unique time-step and scenario combination). Consequently, the value at solution of a variable associated to a given time-step and scenario pair only depends on the set of demand and supply functions corresponding to this time-step and scenario pair $U_{t,s} = \left(P_{t,i,s}, C_{t,i,s}\right)$. Hence the values at solution can be rewritten as functions of these specific parameters: $q_{t,i,s}^{with,*}\left(U_{t,s}\right)$, $q_{t,i,s}^{inj,*}\left(U_{t,s}\right)$, $\alpha_{t,i,s}^{I,*}\left(U_{t,s}\right)$, $\theta_{t,i,s}^{I,with,*}\left(U_{t,s}\right)$, $\theta_{t,i,s}^{I,inj,*}\left(U_{t,s}\right)$.

Shippers' trading decisions

Assuming a given set of tariffs and capacity limits $v = \left(t_i^{with}, t_i^{inj}, k_i^{M,with}, k_i^{M,\mathrm{inj}}\right)$, the problem of the shippers, S, is a linear optimization problem. Hence, its K.K.T. conditions can be written as follows:

$$P_{t,i,s}\left(q_{t,i,s}^{with}\right) - t_i^{with} + \alpha_{t,s}^{S} - \beta_{t,i,s}^{S,with} + \theta_{t,i,s}^{S,with} = 0 \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^{Ex}, \forall s \in \mathcal{S} \qquad \text{(A-5)}$$

$$-C_{t,i,s}\left(q_{t,i,s}^{inj}\right) - t_i^{inj} - \alpha_{t,s}^{S} - \beta_{t,i,s}^{S,inj} + \theta_{t,i,s}^{S,inj} = 0 \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^{En}, \forall s \in \mathcal{S} \qquad \text{(A-6)}$$

$$0 \leq \beta_{t,i,s}^{S,with} \perp q_{t,i,s}^{with} - k_i^{M,with} \leq 0 \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^{Ex}, \forall s \in \mathcal{S} \qquad \text{(A-7)}$$

$$0 \leq \beta_{t,i,s}^{S,\mathrm{inj}} \perp q_{t,i,s}^{inj} - k_i^{M,inj} \leq 0 \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^{En}, \forall s \in \mathcal{S} \qquad \text{(A-8)}$$

$$0 \leq \theta_{t,i,s}^{S,with} \perp -q_{t,i,s}^{with} \leq 0 \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^{Ex}, \forall s \in \mathcal{S} \qquad \text{(A-9)}$$

$$0 \leq \theta_{t,i,s}^{S,inj} \perp -q_{t,i,s}^{inj} \leq 0 \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^{En}, \forall s \in \mathcal{S} \qquad \text{(A-10)}$$

The solution to this set of equations is unique and depends on the tariffs and capacity limits parameters v, as well as the demand and supply parameters in U. Once again, it is easy to see that the previous equations always involve a single time-step and scenario combination, which means that the value taken by the variables of a given time-step and scenario pair at solution only depends on the demand and supply parameters of the corresponding pair $U_{t,s}$. Hence, the value of the variables at solution, denoted with a star exponent, can be written as a function of those parameters: $\alpha_{t,s}^{S,*}(v, U_{t,s})$, $\beta_{t,i,s}^{S,with,*}(v, U_{t,s})$, $\theta_{t,i,s}^{S,with,*}(v, U_{t,s})$, $q_{t,i,s}^{with,*}(v, U_{t,s})$, $q_{t,i,s}^{inj,*}(v, U_{t,s})$.

Conditions for efficiency of the Entry-Exit system

By comparing equations (A-1) and (A-2) to equations (A-4) and (A-5), one can state that the withdrawal and injection decisions of the shippers $q_{t,i,s}^{with,*}(v, U_{t,s})$ and $q_{t,i,s}^{inj,*}(v, U_{t,s})$ will be identical to those of the integrated utility problem $q_{t,i,s}^{with,*}(U)$ and $q_{t,i,s}^{inj,*}(U)$ if the two following conditions hold:

$$\begin{aligned} -t_i^{with} + \alpha_{t,s}^{S,*}(v, U_{t,s}) - \beta_{t,i,s}^{S,with,*}(v, U_{t,s}) + \theta_{t,i,s}^{S,with,*}(v, U_{t,s}) \\ = \alpha_{t,i,s}^{I,*}(U_{t,s}) + \theta_{t,i,s}^{I,with,*}(U_{t,s}) \end{aligned} \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^{Ex}, \forall s \in \mathcal{S} \qquad \text{(A-11)}$$

$$\begin{aligned} -t_i^{inj} - \alpha_{t,s}^{S,*}(v, U_{t,s}) - \beta_{t,i,s}^{S,inj,*}(v, U_{t,s}) + \theta_{t,i,s}^{S,inj,*}(v, U_{t,s}) \\ = -\alpha_{t,i,s}^{I,*}(U_{t,s}) + \theta_{t,i,s}^{I,inj,*}(U_{t,s}) \end{aligned} \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^{En}, \forall s \in \mathcal{S} \qquad \text{(A-12)}$$

Whether those equations are verified or not depends on the values of the tariffs and capacity limits in set v. While there are $N_{\mathcal{T}} \cdot (N_{\mathcal{N}^{Ex}} + N_{\mathcal{N}^{En}}) \cdot N_{\mathcal{S}}$ equations, set v contains $2 \cdot (N_{\mathcal{N}^{Ex}} + N_{\mathcal{N}^{En}})$ variables. Therefore, assuming all equations are independent, a set of tariffs and capacity limits verifying those equations can only be found if the number of equations is lower or equal to the number of variables, i.e. if $N_{\mathcal{T}} \cdot N_{\mathcal{S}} \leq 2$ (if some equations are redundant, solutions can be found if $N_{\mathcal{T}} \cdot N_{\mathcal{S}} \leq 2 + l$, where l is the number of redundant equations). Given the form of equations (A-11) and (A-12), redundancy can only occur between instances of different time-step and scenario pairs: the instance of equation (A-11) (resp. (A-12)) associated to time step t and scenario s is redundant to another instance of that equation associated to different time step t' and scenario s' if the demand and supply parameters $U_{t,s}$ is identical to that of the other time-step and scenario pair $U_{t',s'}$. In other words, redundancy only occurs when the demand and supply parameters are identical for two different time-step and scenario pairs. Therefore, one can conclude that the Entry-Exit system cannot lead to an efficient dispatch if tariffs and capacity limits are set identically for more than two *different* demand and supply instances.

Q.E.D.

Proof of Proposition 2:

As in Proposition 1, we assume that the demand and supply functions $P_{t,i,s}$ and $C_{t,i,s}$ are linear. While the choice of tariffs and capacity limits was completely free in the case developed in the proof of Proposition 1, tariffs and capacity limits must now be set so that the cost-reflectivity constraint (20) holds. We also assume that set $\mathcal{T} \times \mathcal{S}$ does not contain more than two elements (i.e. the time horizon for which tariffs and capacity limits are set does not exceed two time-step and scenario pairs). This means that an efficient set of tariffs and capacity limits can be found when no cost-reflectivity or cost-recovery condition is enforced, and that any inefficiency is in this case directly attributable to the addition of such a constraint. Following the same arguments and notations as in the proof of Proposition 1 above, the withdrawal and injection decisions of the shippers $q_{t,i,s}^{with,*}(v, U_{t,s})$ and $q_{t,i,s}^{inj,*}(v, U_{t,s})$ will now be identical to those of the integrated utility $q_{t,i,s}^{with,*}(U)$ and $q_{t,i,s}^{inj,*}(U)$ if equations (A-11) and (A-12) hold, as well as the following equation:

$$\mathbb{E}_S\left[\sum_{t\in\mathcal{T}}\sum_{a_{ij}\in\mathcal{A}} E \cdot e_{t,a_{ij},s}^{*}(v, U_{t,s})\right] + \sum_{a_{ij}\in\mathcal{A}} R \cdot K_{a_{ij}} = \mathbb{E}_S\left[\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}^{Ex}} t_i^{with} \cdot q_{t,i,s}^{with,*}(v, U_{t,s}) + \sum_{i\in\mathcal{N}^{En}} t_i^{inj} \cdot q_{t,i,s}^{inj,*}(v, U_{t,s})\right] \tag{A-13}$$

Following the procedure described in the Entry-Exit problem E_3, the optimal energy use $e_{t,a_{ij},s}^{*}(v, U_{t,s})$ is the unique solution to the convex, non-linear network operation problem T, taking the withdrawal and injection decisions of the shippers as parameters. As already mentioned, the operation of the network is independent for each time-step and scenario pair. Hence, the value at solution of a variable of problem T associated to a given time-step and scenario pair only depends on the demand and supply parameters corresponding to this time step $U_{t,s}$. Since the withdrawal and injection decisions of the shippers $q_{t,i,s}^{with,*}(v, U_{t,s})$ and $q_{t,i,s}^{inj,*}(v, U_{t,s})$ are fixed inputs to the network operation problem T, its solution also implicitly depends on the tariffs and capacity limits in set v. This justifies the notation $e_{t,a_{ij},s}^{*}(v, U_{t,s})$. Like in Proposition 1, whether those equations are verified or not depends on the values of the tariffs and capacity limits parameters in set v. This time however, there are $N_{\mathcal{T}} \cdot (N_{\mathcal{N}^{Ex}} + N_{\mathcal{N}^{En}}) \cdot N_{\mathcal{S}} + 1$ equations, while set v still contains $2 \cdot (N_{\mathcal{N}^{Ex}} + N_{\mathcal{N}^{En}})$ variables. By assumption, $N_{\mathcal{T}} \cdot N_{\mathcal{S}} \leq 2$, so two cases must be considered. If $N_{\mathcal{T}} \cdot N_{\mathcal{S}} = 1$, i.e. if tariffs and capacity limits are set individually for each time-step and scenario pair, it could be possible to find a set of tariffs and capacity limits such that those equations hold. However, if $N_{\mathcal{T}} \cdot N_{\mathcal{S}} = 2$, no set of tariffs and capacity limits v verifying those equations can be found (unless two equations are redundant, which means that the demand and supply parameters for the two time-step and scenario pairs considered are identical, which brings us back to examining the previous case $N_{\mathcal{T}} \cdot N_{\mathcal{S}} = 1$).

Recalling the implications of Proposition 1 for an unlimited number of time-step and scenario pairs, one can conclude that an Entry-Exit system in which a cost-reflectivity constraint is imposed cannot lead to an efficient dispatch if tariffs and capacity limits are set identically for two or more different demand and supply instances.

Q.E.D.

Proof of Proposition 3:

As in Proposition 1 and Proposition 2, we assume that the demand and supply functions $P_{t,i,s}$ and $C_{t,i,s}$ are linear. We assume that capacity limits are fixed at an arbitrary level. We also assume that set $\mathcal{T} \times \mathcal{S}$ does not contain more than two elements (i.e. the time horizon for which tariffs and capacity limits are set does not exceed two time-step and scenario pairs). This means that an efficient set of tariffs and capacity limits can be found when the choice of both tariffs and capacity limits is free, and that any inefficiency is in this case directly attributable to the exogenous choice of capacity limits. Following the same arguments and notations as in the proof of Proposition 1 above, the withdrawal and injection decisions of the shippers $q_{t,i,s}^{with,*}(v, U_{t,s})$ and $q_{t,i,s}^{inj,*}(v, U_{t,s})$ will be identical to those of the integrated utility $q_{t,i,s}^{with,*}(U)$ and $q_{t,i,s}^{inj,*}(U)$ if equations (A-11) and (A-12) hold.

Like in Proposition 1, whether those equations are verified or not depends on the values of the tariffs and capacity limits parameters in set v. This time however, while there are still $N_{\mathcal{T}} \cdot (N_{\mathcal{N}^{Ex}} + N_{\mathcal{N}^{En}}) \cdot N_{\mathcal{S}}$ equations, set v contains only $(N_{\mathcal{N}^{Ex}} + N_{\mathcal{N}^{En}})$ variables since capacity limits are already fixed. By assumption, $N_{\mathcal{T}} \cdot N_{\mathcal{S}} \leq 2$, so two cases must be considered. If $N_{\mathcal{T}} \cdot N_{\mathcal{S}} = 1$, i.e. if tariffs and capacity limits are set individually for each time-step and scenario pair, it could be possible to find a set of tariffs and capacity limits such that those equations hold. However, if $N_{\mathcal{T}} \cdot N_{\mathcal{S}} = 2$, no set of tariffs and capacity limits v verifying those equations can be found (unless two equations are redundant, which means that the demand and supply parameters for the two time-step and scenario pairs considered are identical, which brings us back to examining the previous case $N_{\mathcal{T}} \cdot N_{\mathcal{S}} = 1$).

Recalling the implications of Proposition 1 for an unlimited number of time-step and scenario pairs, one can conclude that an Entry-Exit system in which capacity limits are considered as fixed cannot lead to an efficient dispatch if tariffs are set identically for two or more different demand and supply instances.

Q.E.D.

Appendix B – Numerical case studies

No matter the number of scenarios they are always assumed to have equal probabilities and in all cases, individual pipes are 46,6 miles long (75km). We assume an annual cost of capital 3%, which amounts for our one-day test cases to a daily cost of capital $R = 0{,}008219\%$.

The intercept of the inverse offer function is set close to the European price for the gas commodity (6,27 $/MMBtu February 2017 (The World Bank)) at 6 K$/MMcfd. The intercepts of the inverse demand functions are identical for all nodes in a given network. They vary only among test cases as detailed in Table B-1. The intercepts used in the specific cases presented in Table 4 are also graphically represented in Figure B-1. In all cases, the intercept is set only slightly above the value the intercept of the offer demand function, to reflect the absence of distribution and commercial costs or taxes in our models.

Table B-1: Parameters of the inverse demand and offer functions

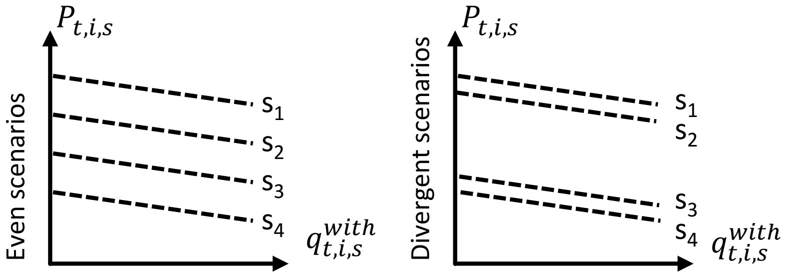
		Demand					Offer	
		Intercept				Slope	Intercept	Slope
		s_1	s_2	s_3	s_4			
Tables 1, 2, 5, D-1, E-1	$N_S = 2$	6.10	6.05	–	–	-0.00359	6.00	0.00359
	$N_S = 3$	6.10	6.08	6.05	–	-0.00359	6.00	0.00359
Table 3	$N_{\mathcal{N}} = 4$	6.10	6.08	6.05	–	-0.00239	6.00	0.00239
	$N_{\mathcal{N}} = 8$	6.10	6.08	6.05	–	-0.00205	6.00	0.00205
Table 4	Even scenarios	6.10	6.09	6.09	6.08	-0.00359	6.00	0.00359
	Divergent scenarios	6.11	6.11	6.10	6.10	-0.00359	6.00	0.00359

Demand is then scaled for each network configuration so as to obtain levels of infrastructure use comparable with that of the French gas transport network. This ensures that the balance between investment costs to be recovered and operation costs is realistic in our test cases. Given that the French gas transport network is 20000 miles long and served 640 TWh of gas in 2011 (Rossi et al. 2012), we set the slopes of the inverse demand and offer function so as to obtain consumption levels in the order of 0,3 Million standard cubic feet per day (MSCFD) per mile of pipe. The absolute values of the slopes of the inverse demand and offer functions are arbitrarily set identical.

The cost of energy is calculated from the base offer gas price (6 K$/MMcfd), leading to a value $R = 5{,}576\text{E-}06$ K$/BJ.

For the test cases based on an existing network (all but those presented in Appendix D), the installed capital (which is related to the diameter of the pipe) is set equal to the optimal capital that would have been chosen by an integrated utility for the same demand scenarios. This capital is computed using model I^K, presented in Appendix D.

Figure B-1 : Illustration of even and divergent demand scenarios



Appendix C – Numerical parameters of the Cobb-Douglas representation of gas pipeline transport

We follow (Perrotton and Massol) to obtain our Cobb-Douglas-like production function of the gas pipeline technology. We here recall the main steps of the reasoning and introduce the numerical values we chose for its parameters.

The Cobb-Douglas production function is based on approximation of a steady-state flow formula that expresses the flow in each pipe $q_{a_{ij}}$ as a function of the pipe's pressure decrease, and physical and technical parameters. Although many versions of this relationship are available (Coelho and Pinho 2007, Menon 2005), we choose the classical Weymouth flow equation:

$$q_{a_{ij}} = C_2 \cdot \left[(P_0 + \Delta P)^2 - {P_1}^2\right]^{1/2} {d_{a_{ij}}}^{5/2} \tag{C-1}$$

where

- P_0 is the pressure at the inlet of the pipe (before the compressor),
- ΔP is the pressure increase created by the compressor placed at the inlet of the pipe,
- P_1 is the pressure at the outlet of the pipe (before the compressor).

C_2 is a physical constant that can be expressed as follows (for USCS units, (Menon 2005)):

$$C_2 = 38.77 \cdot 10^{-6} \frac{T_b}{P_b} \frac{1}{\sqrt{f_D}} \left(\frac{1}{GTZL_{a_{ij}}}\right)^{1/2} \tag{C-2}$$

where $L_{a_{ij}}$ is the length of the pipe and the other parameters take the following values:

- $T_b = 520$ °R (°F +460) is the base temperature,
- $P_b = 14.73$ psia is the base pressure,
- $G = 0.62$ is the specific gravity of gas,
- $T = 520$ °R (°F +460) is the assumed mean flow temperature,

- $Z = 0.8835$ (dimensionless) is the gas compressibility factor at the mean flow temperature.

f_D is the equivalent Darcy friction factor of the Weymouth formula:

$$\frac{1}{\sqrt{f_D}} = 11.18\, d_{a_{ij}} \quad . \tag{C-3}$$

This flow formula can be combined for each pipe with the relationship describing the horsepower $h_{a_{ij}}$ that the compressor located at its inlet must develop to move gas through the pipe (we use the formulation presented in (Yépez 2008) and similar to that of (Menon 2005)):

$$h_{a_{ij}} = C_1 \left[\left(\frac{P_0 + \Delta P}{P_0}\right)^{(\gamma-1)/\gamma} - 1\right] q_{a_{ij}} \tag{C-4}$$

where C_1 is a physical constant that can be expressed as follows (for USCS units):

$$C_1 = 3.0325 \frac{P_b T Z}{T_b} \frac{\gamma}{(\gamma - 1)} \tag{C-5}$$

and $\gamma = 1.2649$ is the adiabatic gas constant (ratio of specific heats).

Assuming that the inlet and outlet pressure are equal ($P_0 = P_1$) and identical for all pipes, and that the compressors' pressure increase remain limited, (Perrotton and Massol) show that these equations can be combined into an approximated production function for gas pipeline transport:

$$q_{a_{ij}} = \sqrt[3]{\frac{2(C_2 \cdot P_0)^2}{C_1 \cdot b \cdot L_{a_{ij}}}}\, d_{a_{ij}}{}^{16/9} h_{a_{ij}}{}^{1/3} \quad . \tag{C-6}$$

It is then possible to transform this equation into a Cobb-Douglas production function of the capital and energy requirement of the pipe. The capital of the pipe $k_{a_{ij}}$ can be roughly approximated by its mass of steel (calculated as the mass of a cylinder) multiplied by the price of steel, as follows:

$$k_{a_{ij}} = P_s \cdot \pi \left(\left(\frac{d_{a_{ij}}}{2} + \tau_{a_{ij}} \right)^2 - \frac{d_{a_{ij}}{}^2}{2} \right) C_4 \cdot L_{a_{ij}} \cdot W_s \tag{C-7}$$

where:

- $P_s = 0.0003$ K\$/kg is the price of steel,
- $W_s = 0.1286$ kg/in^3 is the weight of steel,
- $C_4 = 63360$ in/miles is a conversion factor from miles to inches.

The thickness of the pipe $\tau_{a_{ij}}$ that appears in this equation is linked to its diameter and design parameters related to safety considerations by the following relationship:

$$\tau_{a_{ij}} = C_3 \cdot d_{a_{ij}} \quad . \tag{C-8}$$

The constant C_3 can be calculated using the AGA formula (Menon 2005):

$$C_3 = \frac{DP}{2 \cdot F \cdot SMYS} \tag{C-9}$$

where:

- $DP = 960.1498$ psi is the design pressure,
- $F = 0.6$ is a safety design parameter,
- $SMYS = 52000$ psi is the specified minimum yield strength.

The horsepower requirement can then be translated into an energy $e_{a_{ij}}$ for the given period considered (in our case a day, since we express the flow in MSCFD):

$$e_{a_{ij}} = C_4 \cdot h_{a_{ij}} \tag{C-10}$$

where $C_5 = 0.06442761$ BJ/hp is the conversion factor from mechanical horsepower (hp) to billion Joule for a day of operation.

This lets us define the Cobb-Douglas production function used in this paper:

$${q_{t,a_{ij},s}}^{H} = B_{a_{ij}} \cdot {k_{a_{ij}}}^{G} \cdot {e_{t,a_{ij},s}}^{1-G} \tag{C-11}$$

where $G = 8/11$ and $H = 9/11$ and the constant $B_{a_{ij}}$ can be expressed as follows:

$$B_{a_{ij}} = \left(\frac{2 \cdot (C_2 \cdot P_0)^2}{C_1 \cdot \beta \cdot C_5}\right)^{1/3} \frac{1}{\left(P_s \cdot \pi \cdot C_4\left(C_3 + {C_3}^2\right)W_s\right)^{\frac{8}{9}} {L_{a_{ij}}}^{\frac{11}{9}}} \tag{C-12}$$

with

- $C_1 = 188.464643$
- $C_2 = 9.067528$
- $C_3 = 0.01538701$
- $C_4 = 63360$ in/miles
- $C_5 = 0.06442761$ BJ/hp

The value for the inlet (before compressors) and outlet pressures of each pipe is assumed to be $P_0 = 145.038$ psi.

Appendix D – Endogenous investment

With only a few alterations to models E₂, E₃ and I, investment can be made endogenous. We assume that the network is to be built before at the beginning of the time horizon. This network links the existing demand and supply locations and that its topography is known. For each pipe a_{ij}, its length $L_{a_{ij}}$ is known, only its diameter has to be decided initially.

Entry-Exit model with endogenous investment

Most alterations to previous equations appear in the first stage problem of the network operator. Since the amount of capital $k_{a_{ij}}$ is not a parameter anymore but a positive variable, it must be associated to a positivity constraint:

$$0 \leq k_{a_{ij}} \qquad \forall a_{ij} \in \mathcal{A} \tag{D-1}$$

Then, capital remuneration now adds to compressors' energy cost as a network cost component in the objective function of the network operator. Hence, the total expected social welfare becomes:

$$w^T = \mathbb{E}_s \left[\sum_{t \in \mathcal{T}} \left[\sum_{i \in \mathcal{N}^{Ex}} \int_0^{q_{t,i,s}^{with}} P_{t,i,s}\left(q_{t,i,s}^{with}\right) - \sum_{i \in \mathcal{N}^{En}} \int_0^{q_{t,i,s}^{inj}} C_{t,i,s}\left(q_{t,i,s}^{inj}\right) - \sum_{a_{ij} \in \mathcal{A}} V \cdot e_{t,a_{ij},s} \right] \right] - \sum_{a_{ij} \in \mathcal{A}} R \cdot k_{a_{ij}} \tag{D-2}$$

The cost-reflectivity or cost-recovery constraint does not change, but for the fact that the capital is now a variable. Similarly, in the technical network operation problem T, the flow equation does not change but capital is now a variable.

To sum-up, the Entry-Exit problem E_3 with endogenous investment, denoted E_3^K, is the a three-stage model made of:

- for the first stage, the maximization of the objective function presented in (D-2) subject to the cost-reflectivity constraint (20) and to the positivity constraints (D-1), (14), (15), (16) and (17).

- for the second stage, S, the problem of the shippers, taking the tariffs and capacity limits set by the network operator in the first stage as parameters and deciding for withdrawal and injection flows;

- for the third stage, T, the technical network operation problem, taking as parameters the injection and withdrawal decisions set by the shippers in problem S as well as the capital chosen by the network operator in first stage, and yielding the minimum energy consumption required to operate the network.

As model E_3, it can be reformulated for numerical applications as a bilevel problem with endogenous investment, denoted E_2^K. This time however, its lower level is a non-convex problem.

Ideal integrated-utility model with endogenous investment

Similar alterations to the integrated utility problem I enable to define I^K the problem of the welfare-maximizing integrated utility with endogenous investment (the dual variables associated to each constraint are written using the Greek alphabet):

Integrated utility's problem with endogenous investment I^K:

$$\max_{\substack{q_{t,i,s}^{with}, q_{t,i,s}^{inj} \\ q_{t,a_{ij},s} \\ k_{a_{ij}}, e_{t,a_{ij},s}}} \mathbb{E}_s \left[\sum_{t \in \mathcal{T}} \left[\sum_{i \in \mathcal{N}^{Ex}} \int_0^{q_{t,i,s}^{with}} P_{t,i,s}(q_{t,i,s}^{with}) - \sum_{i \in \mathcal{N}^{En}} \int_0^{q_{t,i,s}^{inj}} C_{t,i,s}(q_{t,i,s}^{inj}) - \sum_{a_{ij} \in \mathcal{A}} V \cdot e_{t,a_{ij},s} \right] \right] - \sum_{a_{ij} \in \mathcal{A}} R \cdot k_{a_{ij}} \quad \text{(D-3)}$$

s.t. $$q_{t,i,s}^{inj} + \sum_{j \in \mathcal{N} | a_{ji} \in \mathcal{A}} q_{t,a_{ji},s} = q_{t,i,s}^{with} + \sum_{j \in \mathcal{N} | a_{ij} \in \mathcal{A}} q_{t,a_{ij},s} \qquad \alpha_{t,i,s}^I \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}, \forall s \in \mathcal{S} \quad \text{(D-4)}$$

$$q_{t,a_{ij},s}{}^H = B \cdot k_{a_{ij}}{}^G \cdot e_{t,a_{ij},s}{}^{1-G} \qquad \beta_{t,a_{ij},s}^I \quad \forall t \in \mathcal{T}, \forall a_{ij} \in \mathcal{A}, \forall s \in \mathcal{S} \quad \text{(D-5)}$$

$$0 \leq k_{a_{ij}} \qquad \omega_{a_{ij}}^I \quad \forall a_{ij} \in \mathcal{A} \quad \text{(D-6)}$$

$$0 \leq q_{t,a_{ij},s} \qquad \theta_{t,a_{ij},s}^I \quad \forall t \in \mathcal{T}, \forall a_{ij} \in \mathcal{A}, \forall s \in \mathcal{S} \quad \text{(D-7)}$$

$$0 \leq q_{t,i,s}^{with} \qquad \theta_{t,i,s}^{I,with} \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^{Ex}, \forall s \in \mathcal{S} \quad \text{(D-8)}$$

$$0 \leq q_{t,i,s}^{inj} \qquad \theta_{t,i,s}^{I,inj} \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}^{En}, \forall s \in \mathcal{S} \quad \text{(D-9)}$$

It is a non-linear, non-convex optimization program.

Numerical results

We previously assumed exogenous network investment, which better represents legacy European gas networks. For numerical implementations, we used the network corresponding to the ideal investment that would have been made by the integrated-utility benchmark facing the same supply and demand scenarios, computed using program I^K. One could argue that current European networks, which were built in a non-liberalized era, are not ideal for the Entry-Exit system. Hence, an adapted network could mitigate the inefficiencies previously highlighted. While our analytical results do not hold with endogenous investment[40], we can numerically assess the magnitude of inefficiencies when the network operator is able to choose network investment simultaneously to tariffs and capacity limits. Solutions to models I^K and E_2^K are computed for the same single-pipe case studies used for the illustration of Propositions 1, 2 and 3 at the end of section V.4. The results listed in Table D-1show that those propositions still seem to hold when investment is endogenous. This would mean that the inefficiencies of the Entry-Exit system cannot be relieved by better network planning.

[40] Due to the network constraints, model I^K is not convex anymore when capital is a decision variable.

Table D-1: Assessment of Propositions 1, 2 and 3 with endogenous investment

		Flow			Tariff		Capacity		Welfare	
$N_{\mathcal{S}} = 2$		$q^{with}_{t_1,i_3,s_1}$	$q^{with}_{t_1,i_3,s_2}$		$t^{inj}_{i_1}$	$t^{with}_{i_3}$	$k^{M,inj}_{i_1}$	$k^{M,with}_{i_3}$	w^T	$\frac{w_I^T - w_E^T}{w_I^T}$
	Integrated utility (I)	13,69	6,90		–	–	–	–	418,645	
	Entry-Exit (E)	13,69	6,90		0,20	0,25	13,69	16,68	418,645	**0,0000%**
	Entry-Exit (E) cost-recovery	13,68	6,74		0,81	0,81	13,68	17,79	418,596	**0,0117%**
	Entry-Exit (E) exogenous capacity limits	13,78	6,81		0,75	0,35	20,53	20,53	418,614	**0,0073%**
$N_{\mathcal{S}} = 3$		$q^{with}_{t_1,i_5,s_1}$	$q^{with}_{t_1,i_5,s_2}$	$q^{with}_{t_1,i_5,s_3}$	$t^{inj}_{i_1}$	$t^{with}_{i_5}$	$k^{M,inj}_{i_1}$	$k^{M,with}_{i_5}$	w^T	$\frac{w_I^T - w_E^T}{w_I^T}$
	Integrated utility (I)	13,67	10,30	6,90	–	–	–	–	404,525	
	Entry-Exit (E)	13,67	10,34	6,86	0,40	0,37	13,67	13,67	404,521	**0,0010%**

Appendix E – Engineering network representation

Entry-Exit model with engineering network representation

The network representation used in the problem of the network operator T can be replaced by other gas network models documented in the literature, featuring various levels of detail and applications (Babonneau et al. 2012, De Wolf and Smeers 1996, 2000, Möller 2004, O'Neill et al. 1979). We propose here a corresponding adaptation of the classical steady-state network representation.

Each node is associated to a pressure level $p_{t,i,s}$. Compressors stations are located at nodes in the pre-defined[41] subset $\mathcal{C} \subset \mathcal{N}$. They increase the pressure at a node by a factor equal to the compression ratio $cr_{t,i,s}$, which is always at least equal to 1:

$$1 \leq cr_{t,i,s} \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{C}, \forall s \in \mathcal{S} \qquad \text{(E-1)}$$

To put it differently, at nodes where a compressors station is present, the compression ratio multiplied by the pressure at this node gives the outlet pressure of the station. At nodes where no compressor exists, this ratio is always equal to 1:

$$cr_{t,i,s} = 1 \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N} \setminus \mathcal{C}, \forall s \in \mathcal{S} \qquad \text{(E-2)}$$

As detailed in Appendix C, depending on those inlet and outlet pressures, the steady-state flow constraints determine how much gas flows through a pipe:

[41] Locating compressors is a technical choice linked to the topology of the network. Taking them as given helps to avoiding integer numbers in our models, and should not influence much our results based on economic operating decisions.

$$q_{t,a_{ij},s}{}^2 \leq 433.44 \cdot 10^{-6} \frac{T_b}{P_b} \left(\frac{1}{GTZL_{a_{ij}}} \right)^{1/2} \cdot \left(\left(cr_{t,i,s} \cdot p_{t,i,s} \right)^2 - p_{t,j,s}{}^2 \right) \cdot d_{a_{ij}}{}^{8/3} \qquad \forall t \in \mathcal{T}, \forall a_{ij} \in \mathcal{A}, \forall s \in \mathcal{S} \qquad \text{(E-3)}$$

where $d_{a_{ij}}$ is the pipe diameter and neglecting pipe elevation. This equation is usually expressed as an equality, but we use its inequality form to make it a convex (flows are positive) and ease the task of finding numerical solutions. This remains coherent as long as there is no incentive to dissipate pressure in the network (which is the case in our compressor-driven, cost-minimizing formulation).

Thus, the maximum flow through a pipe is limited by the upper pressure limits $P_{t,i}^{max}$, stemming from security concerns, and by the lower pressure limits $P_{t,i}^{min}$, which are often part of contractual delivery conditions:

$$P_{t,i}^{min} \leq p_{t,i,s} \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}, \forall s \in \mathcal{S} \qquad \text{(E-4)}$$

$$cr_{t,i,s} \cdot p_{t,i,s} \leq P_{t,i}^{max} \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}, \forall s \in \mathcal{S} \qquad \text{(E-5)}$$

The maximum pressure constraint must be enforced for the maximum pressure at a node, which is the outlet pressure of the compressors station if there is one.

In order to include this network model in our economic model, those operating decisions must be translated in terms of costs. As the energy consumed by compressors is one of the cost component of our objective function, we calculate the energy $e_{t,i,s}$ required to increase pressure at each compressors station (Menon 2005, Yépez 2008):

$$e_{t,i,s} = C_4 \cdot C_1 \cdot \left(cr_{t,i,s}{}^{\beta} - 1 \right) \cdot \sum_{j \in \mathcal{N} | a_{ij} \in \mathcal{A}} q_{t,a_{ij},s} \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{C}, \forall s \in \mathcal{S} \qquad \text{(E-6)}$$

To avoid handling different objective functions depending on the network representation, we assume that compressors are only located at the inlet of pipes, and not at the outlet, and we re-index this constraint by pipes:

$$e_{t,a_{ij},s} = C_4 \cdot C_1 \cdot \left(cr_{t,i,s}{}^{\beta} - 1 \right) \cdot q_{t,a_{ij},s} \qquad \forall t \in \mathcal{T}, \forall a_{ij} \in \mathcal{A}, \forall s \in \mathcal{S} \qquad \text{(E-7)}$$

The other cost component related to the network itself that appears in the objective function accounts for the building of the pipes. As detailed in Appendix C, we calculate this cost in an approximate but simple way as the cost of the steel used to make the pipe[42], which depends only on the diameter variable (pipes' length being fixed):

[42] Other costs, such as preparing land, securing right of way are neglected. However, they do not modify the way capital costs influence the decisions of a pipeline operator. Consequently, this simplifies our analysis, while other costs could easily be included in a more realistic study.

$$k_{a_{ij}} = P_s \cdot \pi\left(C_3 + {C_3}^2\right) \cdot d_{a_{ij}} \cdot C_4 \cdot L_{a_{ij}} \cdot W_s \qquad \forall a_{ij} \in \mathcal{A} \qquad \text{(E-8)}$$

This detailed network representation can be used in models E_2, E_2^K, I and I^K by replacing equation (13), (I-3) or (D-5), by equations (E-1), (E-2), (E-3), (E-4), (E-5), (E-7) and (E-8), and adding the network variables $cr_{t,i,s}$, $p_{t,i,s}$ and $d_{a_{ij}}$ as decision variables to the technical network operation problem T or to problems I and I^K.

One can note that the resulting models are non-linear and non-convex, due to the compressors' and flow constraints.

Numerical results

While we used a simplified network representation to derive analytical results, as well as previous numerical results, the realistic, steady-state network formulation presented above can be used in numerical models. Once again, we use the same single-pipe network case as in section V.4. The results, shown in Table E-1, seem to imply that those propositions still hold with more detailed flow conditions.

Table E-1: Assessment of Propositions 1, 2 and 3 with realistic network modelling

		Flow			Tariff		Capacity		Welfare	
$N_S = 2$		q_{t_1,i_3,s_1}^{with}	q_{t_1,i_3,s_2}^{with}		$t_{i_1}^{inj}$	$t_{i_3}^{with}$	$k_{i_1}^{M,inj}$	$k_{i_3}^{M,with}$	w^T	$\frac{w_I^T - w_E^T}{w_I^T}$
	Integrated utility (I)	13.6922	6.9606		–	–	–	–	419.503846	
	Entry-Exit (E)	13.6922	6.9606		0.02	0.02	13.6922	1.32E+06	419.503846	**0.0000000%**
	Entry-Exit (E) cost-recovery	13.6922	6.7511		1.11	0.43	13.6922	6090.7059	419.425063	**0.0187799%**
	Entry-Exit (E) exogenous capacity limits	13.6922	6.7265		0.86	0.86	20.5383	20.5383	419.405431	**0.0234598%**
$N_S = 3$		q_{t_1,i_5,s_1}^{with}	q_{t_1,i_5,s_2}^{with}	q_{t_1,i_5,s_3}^{with}	$t_{i_1}^{inj}$	$t_{i_5}^{with}$	$k_{i_1}^{M,inj}$	$k_{i_5}^{M,with}$	w^T	$\frac{w_I^T - w_E^T}{w_I^T}$
	Integrated utility (I)	13.5756	10.4421	6.9606	–	–	–	–	405.045015	
	Entry-Exit (E)	13.5756	10.4428	6.9599	0.02	0.02	2.39E+11	13.5756	405.045014	**0.0000003%**

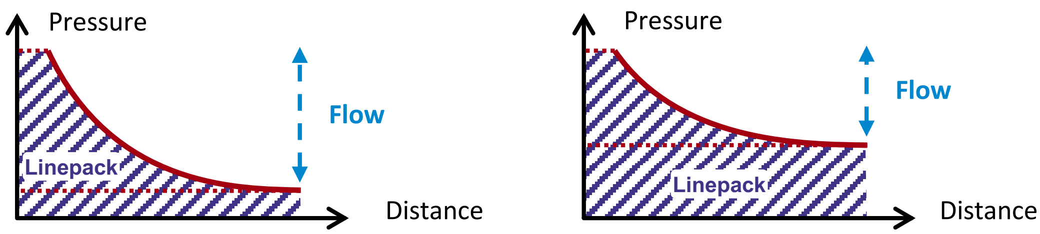
Chapter VI. What short-term market design for efficient flexibility management in gas systems?

VI.1 Introduction

With the increase of intermittent renewables, the electricity system relies more and more on gas-fired power plants to balance the network, as they offer both the flexibility required and lower carbon emissions than other fossil fuels. Hence, uncertainty is transferred to the gas transport network, which hopefully can handle such short-term fluctuations using linepack storage, i.e. by storing gas in pipes (Carter and Rachford 2003, Pietsch et al. 2001). In a path to a successful energy transition, it is critical to make the best out of this flexibility, but it should simultaneously be managed efficiently to remain fair with every network users.

Linepack storage can be managed over the day by adjusting the average pressure level in the gas network. At the same time, the higher the pressure decrease between the inlet and the outlet of a pipe at a given time, the larger the gas flow. Hence, when the inlet pressure hits safety maximum limit, or when the outlet pressure attains the minimum commercial delivery pressure, a trade-off can appear between transporting more gas through the pipe, or storing some for future use (See Figure 1). The latter can be useful to reduce overall procurement costs by buying gas at a time in the day when it is cheap and delivering it later (Arvesen et al. 2013). But the same physical phenomenon that enables storing gas in a pipe also introduces inertia in gas transport (contrarily to the instantaneous flow of electricity). Therefore, packing or depleting gas in advance can also be necessary to allow last-minute changes in delivery planning, i.e. flexibility. This property is often referred to as "linepack flexibility".

Figure 1: Trade-off between storing gas and transporting it for a single pipe



All the market designs currently implemented for gas transport networks let the network operator alone balance the amount of the flexibility and transport services offered to the market. Balancing rules and specific arrangements reserved to users with highly fluctuating demand by some transmission system operators (TSO) in European Entry-Exit system, or flexible delivery contracts offered by pipeline operators in the point-to-point North-American system all rely on implicit, subjective decisions from the network operators, based on an ex-ante assessment of users' needs. Some authors have argued that such an allocation might be inefficient by striking a wrong balance

(Hallack and Vazquez 2013, Keyaerts et al. 2011) and proposed to explicitly offer pipeline flexibility simultaneously to transport services and to the market (Read et al. 2012, Vazquez and Hallack 2013). However, all remained fairly general and disregarded the consequences of practical implementation arrangements. Moreover, none provided quantitative analysis of the potential benefits or losses. Our main contributions are to define in detail, model and quantitatively assess the explicit allocation of flexible services for the gas transport network. Doing so, we try to map the related opportunities and restrictions, including those arising along implementation considerations.

As explained in more details in Section VI.2, we choose to base our analysis on a hypothetical gas transport system operating in a liberalized energy market organized through locational marginal pricing (LMP)[43] auctions. By providing the independent system operator (ISO) with a clear allocation mechanism for transport services, this market design conceptually facilitates the introduction of an explicit allocation of flexibility services and eases the comparison with the base case. In this context, we suggest how the linepack storage capabilities of gas networks can be translated into flexibility products to be offered to the market. We also argue that when such products are offered to the market, the LMP auctions should be designed to take place a single time before each day, contrarily to the usual multiplication of auction rounds needed to deal with demand and offer uncertainty. Finally, we describe the important technical compliance process that must be conducted by the ISO to clear such auctions.

We introduce in Section VI.3 two equilibrium models of LMP auctions for the gas system. The first one describes the usual multiple-round auctions, while the second one introduces the single-auction LMP framework with flexibility services. Several options regarding the structure of flexibility products are proposed for the latter. The resulting models are composed of the economic programs of the gas suppliers and consumers, of the technical and economic program of the ISO, and of the market clearing conditions. In order to deal with linepack storage properly, the ISO's program includes a detailed network representation featuring a simplified transient flow formulation.

Given their computational complexity, these models are then applied to simple network and market cases, which are described in Section VI.4. The results, detailed in Section VI.5 show that the design and the technical implementation of such explicit allocation mechanisms is highly challenging. To be fully efficient, the required diversity of flexibility products is almost as high as the diversity of usage profiles of network users. Real implementations would therefore need to be a compromise between the variety of products and the readability of the auction. Our computational experiments also raise questions regarding the current feasibility of the ISO's daunting task of modeling multiple flow outcomes to clear the auction, due to the non-linearity and non-convexity of gas network physics. In

[43] Such a pricing system has been widely adopted to organize the electricity markets in North America (such as in PJM, NYISO, CAISO, MISO and ERCOT), and has been implemented for gas in the Australian region of Victoria (Pepper et al. 2012).

addition, we highlight that the gains to be obtained by introducing an explicit flexibility allocation highly depend on the structure of the network and on use patterns. Those are mostly relevant for congested networks, and the extent of benefits also largely depends on the level of demand uncertainty. In some cases, practical implementation details even limit the performance of explicit allocation so much that implicit allocation is preferable. The last section concludes the paper and offers policy recommendations.

VI.2 Background and models structure

In this section, we briefly recall the logic behind locational marginal prices and show how it can be applied to gas networks. We then introduce the rationale for gas LMP auctions with flexibility products.

VI.2.1 Applying locational marginal pricing (LMP) to gas markets

Locational marginal pricing, also known as nodal pricing or optimal spot pricing, was first developed as an internal dispatch mechanism for electric utilities (Boiteux and Stasi 1952, Schweppe et al. 1988). Based on its assessment of production costs, network constraints and consumers' demand, the utility would associate a price to electricity sold or bought at each network node. Production units would be ramped up as long as their marginal production cost remained lower than the electricity price at their location, and consumers, informed of the electricity price at their node in real time, would react and adjust their consumption accordingly. If congestion was to appear on a line linking two nodes, the prices at each end of the line would differ, to reflect this congestion. Conditionally to wisely chosen prices, this would result in theory in a network use pattern identical to the welfare-maximizing dispatch.

This scheme has been applied to liberalized gas and electricity markets, by transferring the process of price formation from the single utility to a market, where independent producers, network operators and consumers trade. While local implementations present specificities, they rely on auctions to organize those markets. An independent system operator organizes and clears the auctions while ensuring that the resulting energy flows comply with network constraints. Such auctions are often repeated a few times before delivery time, in order to let network users adjust their positions as uncertainty lifts. Compared to other market organizations applied to electricity or gas markets, LMP lets network users know precisely the impact of their decisions on network congestion, thanks to time- and space-differentiated prices. This property and its resulting theoretically ideal dispatch make it an adapted framework to study the introduction of explicit allocation of network flexibility services in gas markets. Thus, we choose LMP for gas as our base case model, where linepack flexibility is managed solely by the ISO, and introduce the offer of flexibility services in this context.

VI.2.2 Models description

Basic LMP auctions design for gas markets

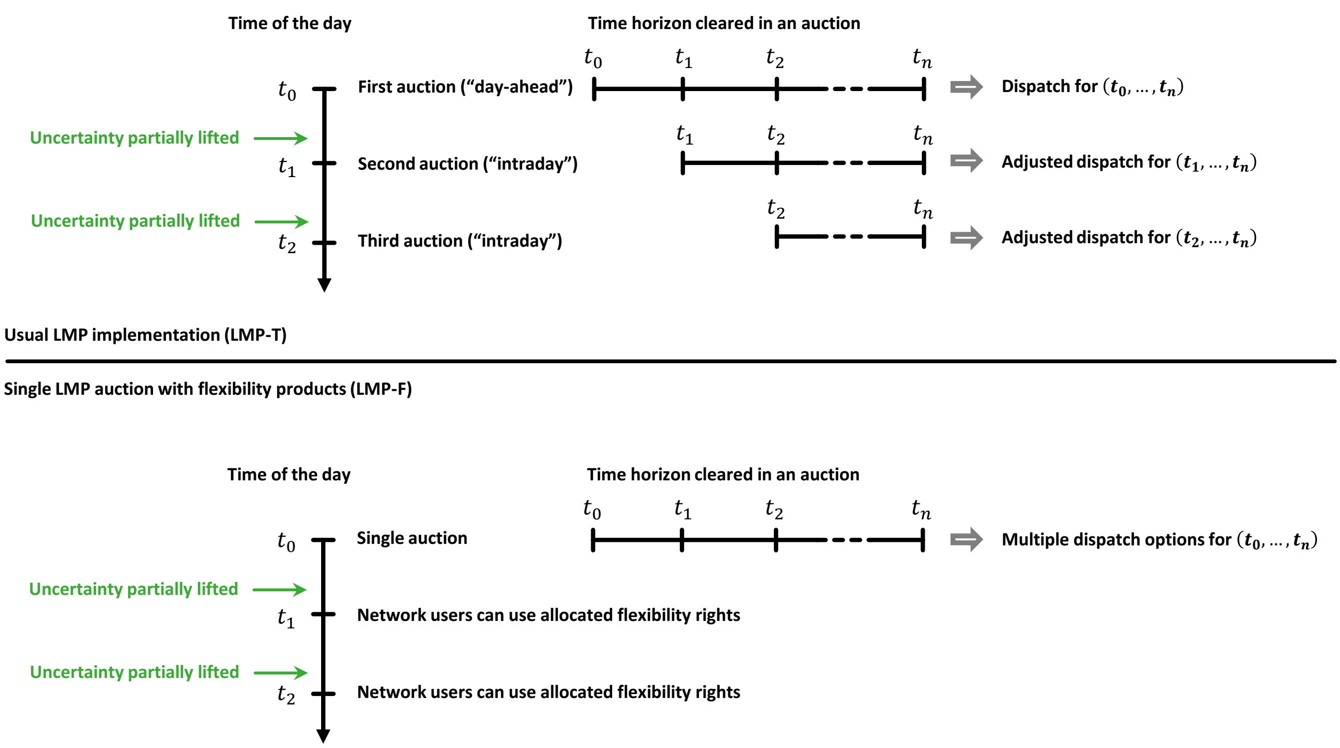
In this subsection, we describe the main elements of the LMP auctions applied to gas natural networks, as well as their potential advantages and weaknesses. This possibility was theoretically explored in (Lochner, Stefan 2009), but the network was only represented through fixed transport capacity, which neglected the impact of gas network physics on this organization. In contrast with electricity networks, the storage properties and inertia of the transport pipes themselves introduce intertemporal links between dispatch occurrences. Dispatch decisions at a certain time impact future dispatch possibilities. Therefore, these decisions must be taken simultaneously to reach an optimal dispatch. The design that was proposed for applying LMP to gas networks by (Read et al. 2012) and that we adopt as the foundation to our LMP model, reflects this (see their extensive study for the details of price formation and hedging possibilities, as well as (Pepper et al. 2012) for practical implementation). The gas day is divided in multiple delivery periods (typically a few hours long) and before its beginning, network users make injection and withdrawal bids for each period. The auctioneer then clears the market for the whole day, ensuring that the intertemporal network constraints are respected. To handle demand or production uncertainty, the ISO can run the auction again, once or more, later in the day for the remaining delivery periods. This allows users to change their bids and readjust their injected or withdrawn quantities.

In this set-up, the ISO clears the auction so as to maximize the value of accepted bids. Given that only firm injection and withdrawal services are offered, this amounts to managing linepack in a way that maximizes the transport capacity offered to the market (Ruff 2012). However, this can restrict the possibility to deviate from the resulting dispatch in subsequent auction rounds. The structure of this model featuring a transport-oriented linepack management and denoted LMP-T is presented in the upper panel of Figure 2.

In each auction, the ISO verifies only the feasibility of the dispatch resulting from the injection and withdrawal linked to the firm products sold in this auction. However, at the time of the auction, some network users may be aware that they will need to update their position in subsequent auctions, once the uncertainty on their willingness to pay for consuming or sell gas will be lifted. Those users have no guaranty that the network state inherited from the execution of the initial dispatch for the first hours of the day will not make those changes technically impossible. Therefore, this market design may be less favorable for users facing high levels of uncertainty.

Moreover, the timing of these auctions can also be awkward for some gas consumers. Those that sell flexibility services in other markets, as CCGTs do through electricity ancillary services, may be unable to adjust their consumption on the short term if the last auction round for this delivery period has already been cleared. Specific network access conditions may have to be established for those players, but as stated before, this could be to the detriment of efficient and fair network management.

Figure 2: Structure of the LMP models, with and without an offer of flexibility services



LMP auctions with flexibility products

Another way to let network users handle uncertainty would be to let the ISO also offer flexible injection or withdrawal products in the auction, in addition to firm products. While the concept of such auctions has been proposed by a few authors, they were not thoroughly defined. We provide a more comprehensive analysis of the idea of flexibility products and explore its consequences on auction design.

Amid a precise description of LMP auctions for gas, (Read et al. 2012) briefly hinted at the possibility to let network users bid for packing gas at a given location in an LMP auction. However, offering the ownership of packed gas at a given location is at best ambiguous, as it does not tell anything about the possibility for the owner to actually use this gas, i.e. injecting or withdrawing at a given location at a given time. This is due to the physics of gas flow: packed gas can simultaneously be an advantage to enable higher withdrawal rates at a given location when the network is not congested, or a liability preventing the increase of withdrawals when the network reaches pressures limits. Since congestion depends on the use patterns of all the other network users simultaneously, offering raw linepack storage would mean offering a product of unknown value in terms of associated future injection or withdrawal rights. This is quite different from ad-hoc electricity storage facilities which can be controlled and allocated independently (Brijs, Geth, et al. 2016, Brijs, Huppmann, et al. 2016).

Instead, (Vazquez and Hallack 2013) suggest to translate linepack management options into clear economic withdrawal and injection rights. They state that a flexibility product should give the owner the right to withdraw (or inject) the corresponding amount of gas over two consecutive periods. However, they limit their analysis to the allocation mechanism of such products through auctions and do not model the actual use of those products at dispatch time. We argue that such products are just a specific kind of flexibility products. The ISO would have to offer a variety of flexibility products that address the particular structure of the uncertainty faced by each network user to efficiently allocate flexibility services. Moreover, it may have to consider purchasing flexibility products itself from some network users, so as to be able to offer more flexibility to others. The next subsection gives an overview of the potential diversity of those flexibility products.

Once again, since gas dispatch decisions for a certain period impact the dispatch possibilities of future periods, the allocation of transport and flexibility services will only be efficient if it is realized simultaneously for all periods. For that same reason, recurring auctions dispatching gas and allocating flexibility products for a single period at a time, as proposed in (Vazquez and Hallack 2013), would be suboptimal as they prevent the ISO to compare the value of users' bids over multiple periods. Therefore, our proposed LMP design with explicit allocation of flexibility services, denoted LMP-F, relies on an auction where network users bid simultaneously for firm injection and withdrawal products as well as flexibility products for every period of the gas day. This auction is run once before the beginning of the gas day, and is not repeated over the day, as shown in the lower panel of Figure 2.

From the ISO's point of view, clearing an LMP-F auction is similar to clearing LMP-T auctions as it still has to verify the technical feasibility of the firm rights to inject and withdraw gas allocated to the participants. However, it must also check the feasibility of the flexible rights to be allocated. And this must be done taking into account that once a flexible product is allocated to a network user, the latter can use it in every way permitted by the terms of use of this product. Hence, the ISO must perform a robust check of the feasibility of network flows, i.e. compute the network states associated to each possible combination of use of flexible products and verify their technical feasibility. In our model, the ISO computes these hypothetical dispatches that result from the possible usage combinations of the flexible products. Given the non-linear and non-convex nature of flow problems for gas networks, this single task is likely to be a first major obstacle to the implementation of such auctions.

In this market design, firm and flexible products are offered simultaneously and competitively. Hence, the allocation of transport capacity and the management of linepack storage are based only their relative value for network users, as expressed by them. Such an explicit allocation of flexibility services relieves the ISO of the subjective anticipation and valuation efforts regarding the flexibility needs of some users. Thus, the allocation of network services is more efficient and fair to everyone. The network users that face uncertainty are also assured that they will be able to adapt their decisions during the day, according to the flexibility products that they purchased. Finally, the gas consumers that sell flexibility services in other markets are guaranteed to be able to adjust their use of the network up to the delivery period, dissipating any concern related to the auction timing.

However, it becomes clear that these flexibility contracts would only have an appeal to network users if the guarantees they provide correspond to the way in which they wish to be able to adjust their schedule. Therefore, flexibility products would have to be tailored to the uncertainty structure perceived by each player.

VI.2.3 Uncertainty management in LMP auctions with flexibility services

Before examining the need for diversified flexibility products, we need to clarify what kind of uncertainty these products are intended to manage.

Scope of the uncertainty targeted by flexibility services

We believe that such products could be useful to mitigate demand or offer fluctuations risks that can be accurately identified at the time of the LMP auction. This includes for instance, the share of variability of domestic demand due to temperature changes that can be forecasted. CCGT which offer a certain share of their capacity on the electricity adjustment market (i.e. which offer ancillary services) would also be concerned. On the opposite, uncontrollable demand fluctuations due to unpredictable short-term events should not be mitigated through flexibility products offered in the LMP auction. Such variability can be observed for instance for domestic consumers with fixed tariff plans that adjust their consumption in an unpredictable way just before consumption time. Or for industrial facilities, acting as demand or offer in the gas market, that may face outages or surges that cannot be forecasted nor controlled. These users or their representative should not be forced to purchase flexibility products as a preventive measure for an uncertainty potential that cannot be reasonably assessed at the time of the auction. If they were, the ISO would have to ensure that a much higher number and much more diverse hypothetical dispatches are feasible before clearing the auction. Hence, using flexibility products to manage these contingencies would dramatically reduce the offer of capacity to the detriment of the overall market. Ancillary services or adjustment mechanisms would still be required to manage such eventualities, in the same way as LMP auctions do.

Need for diversified flexibility products

Now that we have identified the type of uncertainty that is to be addressed by flexibility products, we explore the basic types of products that might be required in LMP-F auctions. These products will be included in our model and their relevance demonstrated through numerical analyses. To keep the analysis as clear as possible, we assume thereafter that suppliers face deterministic supply costs and do not need within-day flexibility. Only consumers are supposed to face an uncertain demand function and are offered products aimed at giving them some degree of flexibility in their withdrawal schedules. Our reasoning could be easily adapted to introduce flexibility products aimed at the management of suppliers' uncertainty though.

We first model the offer of the 2-periods time-flexibility products proposed in (Vazquez and Hallack 2013), which address the need for consumers to be able to decide at the last moment whether to withdraw gas at a given period or during the following one. However, we show that such a product might not be enough, since the uncertainty faced by some users may encompass any type of time intervals. To do so, we also model the provision of 3-periods[44] time-flexibility products that tackle the need users may have to be able to decide when to withdraw gas between two periods separated by a one-period interval. We then explore the possibility that some users may need to choose at the last moment whether to use a quantity of gas that they purchased or to let it in the pipe. To do so, we model the offer of volume-flexibility products that give users the possibility to withdraw or not a quantity of gas for a given period. Were it not for limiting the length of this paper, many other products or combination of products could also be considered.

Need for adjustment products

While these products address the need of some network users to adjust their decisions during the day, they do not incite the others to respond appropriately, or at all, to these changes. This is quite different from classical successive LMP auctions where price changes are observed by all participants and trigger reactions over the day. In a single LMP auction with flexibility products, a supplier would for instance have no incentive during the day to decide to delay injection in response to the late withdrawal of gas from a consumer using a flexibility product that it purchased. Therefore, the ISO might not be able to offer as much network capacity as it would if other players reacted wisely to the final decisions of flexibility products owners. A way to improve this design would be introduce "adjustment" products that let network users surrender some control over a part of their injection or withdrawal schedules to the ISO. This would allow the ISO to coordinate these injection or withdrawal quantities with the activation of flexibility products. At clearing time, the ISO would assess how the control it gains through adjustment products can be used to relieve network constraints in the hypothetical dispatches. We show in Section VI.5 that this mechanism can allow the ISO to offer more capacity to the market.

Adjustment products would however increase the complexity of the clearing process and its related network feasibility assessment. As flexibility products, they would have to be differentiated to account for various time or volume adjustment needs[45]. Suppliers selling (respectively consumers

[44] This is an arbitrary choice, as any type of time-flexibility product could be considered, depending on market needs.

[45] The type of adjustment products would likely be linked to the type of flexibility products offered.

buying) adjustment products would also have to assess the value of injection (respectively withdrawal) scenarios that they do not fully control anymore. Concerning time-adjustment products, the total quantity of gas injected or withdrawn at the end of the day is known. Hence, network users would have to check how their gas procurement costs or use benefits could change depending on when the ISO may trigger the injection or withdrawal of the quantities linked to these products. As for volume-adjustment products, the total quantity actually injected or withdrawn is not guaranteed. Suppliers would have to prepare for the worst case when the ISO triggers the injection of the total quantity of gas linked to the adjustment product. Thus, they would assume that they would face procurement costs for the total quantity linked to a volume-adjustment product and price it accordingly. For consumers though, volume-adjustment products would have no value at all, since the value of a gas quantity that one has no guarantee to be able to use is null[46].

This multiplicity of products can be identified as another drawback of this market design, even when only flexibility products are sold. On the one hand, offering too many different products will negatively impact the readability of the auction mechanism and users may face difficulty to choose how to bid or complain about the lack of transparency of auction results. On the other hand, reducing the variety of flexibility products available in a given market may result in a poorly adapted offer and lead to an inefficient allocation of transport and flexibility services. In Section VI.5, we show how such inefficiencies may arise when the offer of flexibility products is not adapted. Finally, this reinforces the first drawback identified for this market design, as increases even more the complexity of verifying the feasibility of the hypothetical dispatches before when clearing the auction.

To sum-up, the spatial and intertemporal links existing in natural gas networks make transport and flexibility allocation in a market context a challenging task. The explicit allocation of flexibility services through LMP auctions could be a way to handle these technical constraints in a market environment in an efficient and fair way. However, the complexity of implementing it can only be justified if it yields significant gains compared to traditional LMP auctions. In this paper, we document the complexity of this market design and assess whether the intertemporal constraints of natural gas networks can motivate its implementation. To do so, we extend the literature by providing a model of this explicit allocation scheme that also accounts for its subsequent dispatch and by applying it to numerical test cases.

VI.2.4 Notations

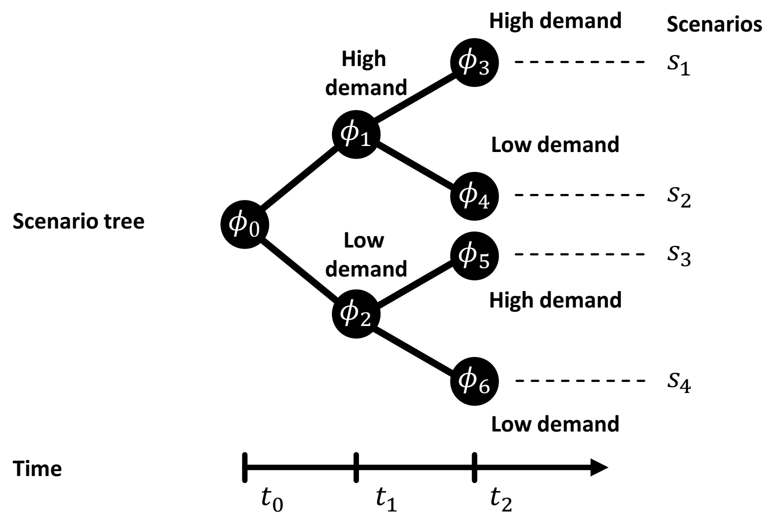
To differentiate the variants of the LMP-F model depending on the type of products available, we add to its former denomination the indexes 2T or 3T when 2-periods or 3-periods time-flexibility products are offered, or the index V for volume-flexibility products. In all variants, the ISO offers

[46] Letting network consumers buy adjustment products can be seen as similar to the demand response contracts currently offered to electricity consumers. Time-adjustment is the counterpart of demand shifting and volume-flexibility the counterpart of load-shedding.

flexibility products to consumers. We may also add the exponent AdI when suppliers can sell injection-adjustment products to the ISO, AdW when consumers can buy withdrawal-adjustment products from the ISO, and AdIW when both are available.

The gas transmission network is represented by a set of nodes $\mathcal{N}$ (indexed by i or j) and a set of arcs $\mathcal{A} \subset \mathcal{N} \times \mathcal{N}$ which accounts for the pipes a_{ij} between those nodes. We assume that this network links a set of existing consumers $c \in \mathcal{C}$ and suppliers $d \in \mathcal{D}$, located at given nodes i_c and i_d. To model uncertainty, we use a tree representation of the possible values of uncertain parameters. An example is shown in Figure 3 for the demand uncertainty of a given user in a three-period case. The root node is always indexed by 0 (and denoted ϕ_0), while the other nodes[47] are numbered using the standard breadth-first method, i.e. by numbering all nodes associated to a time t before moving on to time t+1 ($T(\phi)$ denotes the time associated to node ϕ). Each node ϕ is characterized by a probability θ_ϕ, known by all network users and by the ISO (we assume perfect information). We use notation $\phi \leq \phi'$ to state that node ϕ precedes node ϕ' in the tree or is node ϕ' itself. We also denote the father of node ϕ, i.e. its direct ancestor, ϕ_f. Finally, we associate a scenario s to each leaf of the tree, which refers to the chronicle of the leaf node and all its preceding nodes up to the root node. Scenarios are numbered starting from s_1 and following the increasing leaf-node order, as shown in Figure 3. A separate scenario tree must be built in a similar way for the ISO, to let it compute hypothetical dispatches and clear the auction. The nodes of this second scenario tree are denoted ψ.

Figure 3: Example of the uncertainty structure for a consumer



The notations of the three models, listed below, share the following rules. Small letters denote variables, while capital letters denote parameters. q refers to gas quantities (in millions standard cubic meter, which is analogue to a mass) traded in the market or flowing through the network, p to gas pressure, m to linepack, i.e. gas quantity inside pipes, and π to prices. Lower indices may be added to

[47] In order to avoid confusion with physical network nodes, we may also refer to them as scenario nodes.

refer to time (as t), to a node in the scenario trees (using either ϕ for the overall uncertainty tree or ψ for the ISO's anticipation tree), to a network node (as i or j) or to a pipe (as a_{ij}) to which the variable is associated. Upper level indices can denote market participants (c for consumers, d for suppliers, and ISO) or the type of market product (f for firm injection or withdrawal contracts and lp for flexible contracts relying on linepack management). In the LMP-T model, scenario nodes are also used as upper indices to specify the auction round to which variables belongs. Finally, $lpt+$, $lpt-$ and lpv upper indices only appear in the variables of the LMP-F models that are related to the management of flexibility and adjustment products. All variables but prices are positive, unless otherwise specified.

Sets

Φ	set of scenario nodes ϕ.
Ψ	set of ISO scenario nodes ψ.
X^c	set of scenario nodes χ of consumer $c \in \mathcal{C} \setminus \mathcal{C}_D$.
Ω^s	set of scenario nodes ω of supplier s.
$\mathcal{T}$	time set $(t_0, \dots, t, \dots, t_n)$.
$\mathcal{S}$	scenario set $(s_1, \dots, s, \dots, s_n)$.
$\mathcal{C}$	consumers' set, index c.
$\mathcal{C}_D \subset \mathcal{C}$	subset of consumers with a deterministic demand function.
$\mathcal{D}$	suppliers' set, index d.
$\mathcal{N}$	network nodes, index i or j.
$\mathcal{A}$	pipes, index a_{ij}.

Parameters

$T(\cdot)$	period corresponding to a scenario node.
θ_ϕ	probability of node ϕ.
$C^d(\cdot)$	total cost of supply of supplier d.
$B^c_\phi(\cdot), B^c(\cdot)$	benefit to consume gas for consumer $c \in \mathcal{C} \setminus \mathcal{C}_D$ at node ϕ, resp. for consumer $c \in \mathcal{C}_D$. It is identified as the integral of the inverse demand function.
$R^{lpt-}_{\psi,i}, R^{lpt+}_{\psi,i}$	fraction of the quantity of gas from the time-flexibility products of period $T(\psi)$ purchased at node i that the ISO assumes consumers will actually use at node ψ, the earliest time authorized in the product's terms (resp. at subsequent nodes corresponding to the latest period authorized in the product's terms).
$R^{lpv}_{\psi,i}$	fraction of the quantity of gas from the volume-flexibility products of period $T(\psi)$ purchased at node i that the ISO assumes will actually be used by consumers at node ψ.

$R_{\omega,i}^{adt-,d}$, $R_{\omega,i}^{adt+,d}$ fraction of the quantity of gas from the injection time-adjustment products of period $T(\omega)$ sold at node i that supplier d assumes the ISO will activate at node ω, the earliest time authorized in the product's terms (resp. at subsequent nodes corresponding to the latest period authorized in the product's terms).

$R_{\omega,i}^{adv,d}$ fraction of the quantity of gas from the injection volume-adjustment products of period $T(\omega)$ sold at node i that supplier d assumes the ISO will activate at node ω.

$R_{\chi,i}^{adt-,c}$, $R_{\chi,i}^{adt+,c}$ fraction of the quantity of gas from the withdrawal time-adjustment products of period $T(\chi)$ purchased at node i that consumer $c \in \mathcal{C}_D$ assumes the ISO will activate at node χ, the earliest time authorized in the product's terms (resp. at subsequent nodes corresponding to the latest period authorized in the product's terms).

P_i^{min}, P_i^{max} minimum and maximum pressure at node i.

$M_{a_{ij}}^{ini}$ initial linepack in pipe a_{ij} at the beginning of the gas day.

$M_{a_{ij}}^{ini,\phi}$ initial linepack in pipe a_{ij} at the beginning of node ϕ's auction.

$K_{a_{ij}}^{lp}$ parameter of the linepack equation.

$K_{a_{ij}}^{flow}$ parameter of the flow equation.

Variables (LMP-T model)

$\pi_{t,i}^{f,\phi}$ price of firm gas at node i and period t in node ϕ's auction.

$q_t^{f,c,\phi}$ firm gas purchased (when positive) or sold (when negative) by consumer c for period t in node ϕ's auction.

$q_t^{f,d,\phi}$ firm gas sold (when positive) or purchased (when negative) by supplier d for period t in node ϕ's auction.

$q_t^{c,\phi}$, $q_t^{d,\phi}$ total gas purchased (resp. sold) by consumer c (resp. supplier d) for period t in all auctions up to node ϕ's auction.

$q_{t,i}^{f,ISO,\phi}$ firm gas purchased (if negative) or sold (if positive) by the ISO for period t at node i in node ϕ's auction.

$p_{t,i}^{\phi}$ pressure at network node i and time t associated to the dispatch resulting from the allocation of node ϕ's auction.

$q_{t,i,a_{ij}}^{\phi}$, $q_{t,j,a_{ij}}^{\phi}$ flow in pipe a_{ij} at entry node i (resp. exit node j) at time t resulting from the allocation of node ϕ's auction.

$m_{t,a_{ij}}^{\phi}$ linepack (quantity of gas) in pipe a_{ij} at time t resulting from the allocation of node ϕ's auction.

$m_{a_{ij}}^{fin,\phi}$ final linepack in pipe a_{ij} at the end of the last period t_n resulting from the allocation of node ϕ's auction.

Variables (LMP-F model)

$\pi_{t,i}^{f}$	price of firm gas at node i and period t.
$\pi_{t,i}^{lp}$	price of flexible gas (gas bundled with linepack flexibility service) at node i for period t.
$q_t^{f,c}$, $q_t^{f,d}$	firm gas purchased (resp. sold) by consumer c (resp. supplier d) for period t.
$q_t^{lp,c}$	flexible gas (gas bundled with linepack flexibility service) purchased by consumer $c \in \mathcal{C} \setminus \mathcal{C}_D$ for period t.
$q_\phi^{lpt-,c}$,$q_\phi^{lpt+,c}$	gas from the linepack time-flexibility product of period $T(\phi)$ purchased by consumer $c \in \mathcal{C} \setminus \mathcal{C}_D$ that it actually uses at node ϕ, the earliest time authorized in the product's terms (resp. at subsequent nodes corresponding to the latest period authorized in the product's terms).
$q_\phi^{lpv,c}$	gas from the linepack volume-flexibility product of period $T(\phi)$ purchased by consumer $c \in \mathcal{C} \setminus \mathcal{C}_D$ that it actually uses at node ϕ.
q_ϕ^{c}	total gas used by consumer $c \in \mathcal{C} \setminus \mathcal{C}_D$ at node ϕ.
$q_{t,i}^{f,ISO}$	firm gas purchased (if negative) or sold (if positive) by the ISO for period t at node i.
$q_{t,i}^{lp,ISO}$	flexible gas (gas bundled with linepack flexibility service) purchased (if negative) or sold (if positive) by the ISO for period t at node i.
$q_{\psi,i}^{ISO}$	withdrawn (if positive) or injected gas quantity (if negative) at network node i anticipated by the ISO in its hypothetical dispatches for scenario node ψ.
$p_{\psi,i}$	pressure at network node i and ISO scenario node ψ.
$q_{\psi,i,a_{ij}}$, $q_{\psi,j,a_{ij}}$	flow in pipe a_{ij} at entry node i (resp. exit node j) at ISO scenario node ψ.
$m_{\psi,a_{ij}}$	linepack in pipe a_{ij} at ISO scenario node ψ.
$m_{\psi,a_{ij}}^{fin}$	final linepack in pipe a_{ij} at the end of the period of ISO node ψ.

Additional variables (LMP-F^{AdI} and LMP-F^{AdW} variants)

$\pi_{t,i}^{adi}$, $\pi_{t,i}^{adw}$	price of injection (resp. withdrawal) adjustment gas (gas conditional to ISO activation) at node i and period t.
$q_t^{ad,c}$, $q_t^{ad,d}$	adjustment gas (gas conditional to ISO activation) purchased by consumer c (resp. sold by supplier d) for period t.
q_χ^{c}	total gas quantity that consumer $c \in \mathcal{C}_D$ expects to withdraw at node χ.
q_ω^{d}	total gas quantity that supplier d expects to inject at node ω.
$q_{t,i}^{adi}$	injection adjustment gas (gas conditional to ISO activation) purchased by the ISO from suppliers for period t at node i.
$q_{t,i}^{adw}$	withdrawal adjustment gas (gas conditional to ISO activation) sold by the ISO to consumers for period t at node i.
$q_{\psi,i}^{adit-}$, $q_{\psi,i}^{adit+}$	gas from the injection time-adjustment products of period $T(\psi)$ sold by suppliers at node i that is activated by the ISO (i.e. asked to be injected) in its hypothetical dispatch at node ψ, the earliest time authorized in the product's terms (resp. at subsequent nodes corresponding to the latest period authorized in the product's terms).

$q_{\psi,i}^{adiv}$	gas from the injection volume-adjustment products of period $T(\psi)$ sold by suppliers at node i that is activated by the ISO (i.e. asked to be injected) in its hypothetical dispatch at node ψ.
$q_{\psi,i}^{adwt-}$, $q_{\psi,i}^{adwt+}$	gas from the withdrawal time-adjustment products of period $T(\psi)$ purchased by consumers at node i that is activated by the ISO (i.e. asked to be withdrawn) in its hypothetical dispatch at node ψ, the earliest time authorized in the product's terms (resp. at subsequent nodes corresponding to the latest period authorized in the product's terms).

Variables (Integrated utility model)

q_{ϕ}^{c}, q_{ϕ}^{d}	gas withdrawn (resp. injected) by consumer c (resp. supplier d) at scenario node ϕ.
$p_{\phi,i}$	pressure at network node i and scenario node ϕ.
$q_{\phi,i,a_{ij}}$, $q_{\phi,j,a_{ij}}$	flow in pipe a_{ij} at entry node i (resp. exit node j) at scenario node ϕ.
$m_{\phi,a_{ij}}$	linepack in pipe a_{ij} at scenario node ϕ.
$m_{\phi,a_{ij}}^{fin}$	final linepack in pipe a_{ij} at the end of the period of node ϕ.

VI.2.5 Transient network representation

Since the trade-offs that occur in the allocation of transport and flexibility services originate in gas networks physics, we adopt a detailed network representation for our models. Each network node is associated to a pressure level $p_{t,i}$ at each period. In order to account for linepack storage, the ability to store gas in pipes, we use a transient flow modeling. It implies that the flow entering a pipe a_{ij} at node i over a given period t, denoted $q_{t,i,a_{ij}}$, can differ from the flow of gas leaving this pipe at node j over that same period, denoted $q_{t,j,a_{ij}}$. Since pipeline gas flows inherit the tremendous complexity of fluid mechanics, it is not possible to describe them through simple analytical formulations in most cases. From a variety of numerical formulations (Banda and Herty 2008, Domschke et al. 2011, Mahlke et al. 2007, Modisette and Modisette 2001, Moritz 2007, Osiadacz 1984, Zavala 2014), some isothermal transient representations can be adapted to obtain simplified but realistic formulations (Kelling et al. 2000, Westphalen 2004), that are suitable for economic models (Keyaerts 2012, Midthun 2007). Following (Read et al. 2012), we use here a somewhat similar transient formulation based on a finite difference scheme and a Weymouth-type flow equation. We differ in making it coarser by considering large time steps (typically hours) and much larger space steps (directly the length of the pipe). The flows obtained using this approximated flow formula are not suitable to conduct technical analysis. However, as our formulation retains the fundamental dynamics ruling gas flow and linepack storage, it is sufficient to observe the impact of technical constraints on market organization. Its improved tractability also allows us to implement complex models and makes our economic analysis possible.

Our formulations of the three main relationships that play a role in transient pipeline flows are described hereafter. The first one is the flow formula, that states that the gas flow through a pipe depends on the difference of the squared inlet and outlet pressure:

$$\left(\frac{q_{t,i,a_{ij}}+q_{t,j,a_{ij}}}{2}\right)^2 \leq K_{a_{ij}}^{flow} \cdot \left({p_{t,i}}^2 - {p_{t,j}}^2\right) \qquad \forall t \in \mathcal{T}, \forall a_{ij} \in \mathcal{A} \qquad (1)$$

where $K_{a_{ij}}^{flow}$ is a constant depending on the pipe length, its diameter $d_{a_{ij}}$ and physical constants, which is detailed in Appendix C. This equation is originally an equality in (Read et al. 2012) and thus a non-convex relationship. While a linearization approach could be taken as in (Midthun et al. 2009, Read et al. 2012), we just use it in its inequality form, as suggested in (Keyaerts 2012). Assuming that flows cannot change direction[48], i.e. that flow variables are positive, this equation becomes convex. The flows obtained remain realistic as long as this constraint is binding, i.e. as long as there is no incentive to increase pressure without increasing the corresponding flow[49].

The second relationship translates for each period the pressure levels at the inlet and outlet of a pipe into a mass of gas inside the pipe, the linepack, $m_{t,a_{ij}}$:

$$m_{t,a_{ij}} = K_{a_{ij}}^{lp} \cdot \frac{p_{t,i}+p_{t,j}}{2} \qquad \forall t \in \mathcal{T}, \forall a_{ij} \in \mathcal{A} \qquad (2)$$

where $K_{a_{ij}}^{lp}$ is another constant depending on the pipe length, its diameter $d_{a_{ij}}$ and physical constants. It is also detailed in Appendix C. Other authors that apply formulations similar to finite difference scheme to space steps as large as the length the pipe suggest to adapt this relationship to better account for the pressure decrease shape gas in such a space interval (Kelling et al. 2000, Keyaerts 2012, Midthun 2007). While we recognize the value of this alteration to obtain more accurate flow predictions, we believe that this is not necessary in our case, as it does not fundamentally change flow patterns, and would only add non-convexity issues to our already challenging mathematical programs.

Finally, the mass balance equation calculates the next period linepack in a pipe by adding the incoming mass of gas during a period to the initial the mass of gas while subtracting the mass of gas leaving the pipe. The inertia and storage possibilities of pipeline gas transport stem from that intertemporal relationship:

$$m_{t+1,a_{ij}} - m_{t,a_{ij}} = q_{t,i,a_{ij}} - q_{t,j,a_{ij}} \qquad \forall t \in \mathcal{T}, \forall a_{ij} \in \mathcal{A}. \qquad (3)$$

[48] This is the case for most real pipelines, as reversing flows requires adapted infrastructures.

[49] In our auction set-up, such situations can appear when the network operator can offer more firm or flexible products if it is able to reduce flow while keeping a large pressure decrease at some point. Since our model is aimed at unveiling trends in network management rather than forecasting actual dispatches, this is not an issue. It can be seen as the result of the use of network elements not described in our formulation, such as pressure reducers and valves. We also compared the results of our convex formulation with a non-convex model using the flow formula in its equality form. While the results differed slightly, the trends presented in Section VI.5 were not changed.

The maximum flow through a pipe is limited by the upper pressure limits $P_{t,i}^{max}$, stemming from security concerns, and by the lower pressure limits $P_{t,i}^{min}$, which are often part of contractual delivery conditions:

$$
\begin{aligned}
& p_{t,i} \geq P_i^{min} && \forall t \in \mathcal{T}, \forall i \in \mathcal{N} \\
& p_{t,i} \leq P_i^{max} && \forall t \in \mathcal{T}, \forall i \in \mathcal{N}.
\end{aligned} \tag{4}
$$

While many components, such as valves, pressure regulators and compressors, are essential to control flows in practice, they only increase the operational range of a network. Since we focus here on market design, the impact that those technical limits can have on market performance is more relevant than the precise conditions under which such limits arise. Based on this observation and given the combinatorial and non-convexity issues that modeling them would imply, we simply omit them and model a bare network. However, these could be added (Pepper et al. 2012, Read et al. 2012, Schmidt et al. 2012).

VI.2.6 General assumptions

Locational marginal pricing was initially defined as the division of the energy commodity into a multiplicity of spatially and timely differentiated products, exchanged on multiple markets corresponding to each node of the network and linked by a transporter that could move the commodity between nodes (Caramanis et al. 1982). Although LMP has been implemented in liberalized energy markets through centralized auctions, those are just a mechanism used to set nodal prices. The same optimal prices could theoretically result from dynamic market interactions at each node. Therefore, we choose to model each auction round as an equilibrium problem where suppliers and consumers exchange gas at each node for each period and where the ISO can purchase and sell gas at every node, and move it freely using gas pipes as long as network flow constraints are respected. We assume that such markets are perfectly competitive (market participants are price takers) and we model no strategic interactions. Although such interactions may appear, they are beyond the scope of this study focused on the potential of explicit flexibility allocation.

VI.3 Models of the gas market

In this section, we describe the formulations of the LMP-T and LMP-F models, as well as their variants. We also introduce the formulation of the ideal integrated utility benchmark, leading to an optimal network allocation.

VI.3.1 Basic successive LMP auctions

We first focus on the LMP-T auction design where only firm products are offered. This design features successive auctions over the gas day $(t_0, \dots, t, \dots, t_n)$. At least one auction must be run at the beginning of the day, right before period t_0 to allocate gas over the whole day. A new auction can then be run later in the day before a given period t, to reallocate gas over the remaining periods $(t, \dots, t_n)$. Depending on regulatory choices, this process can be repeated multiple times (the more auction rounds, the more possibility for market participants to adjust to new demand conditions).

We start by presenting a generic problem describing one of these auctions. We assume that this auction round occurs just before node ϕ and after uncertainty for this node has been lifted. This auction allocates gas from the period associated to node ϕ to the last period of the gas day. We also assume that this is not the first auction of the day and that at least a previous auction took place at one of the ancestor nodes of node ϕ, denoted ϕ'. Hence, each player takes the clearing results of node ϕ''s auction as parameters, for the remaining periods between the period associated to node ϕ and the last period of the gas day. This auction is an equilibrium problem composed of the optimization problems of gas suppliers, consumers and of the ISO. We then introduce the relations between all the auction rounds occurring over the gas day.

The suppliers' optimization problems

In a given auction round occurring at scenario node ϕ, producer d located at a network node i_d solves its individual program where it sells gas at the market price $\pi_{t,i_d}^{f,\phi}$ for all the periods to come so as to maximize its expected profit. Its profit is sum of the revenues from gas sale at this auction round minus the procurement cost associated to sum of the quantity of gas sold at this auction round and the quantity sold in previous auction rounds. We assume that it faces constant production or import costs[50] $C^d(\cdot)$:

$$\max_{q_t^{f,d,\phi}, q_t^{d,\phi}} \sum_{t \in \mathcal{T} | t \geq T(\phi)} \pi_{t,i_d}^{f,\phi} q_t^{f,d,\phi} - \sum_{t \in \mathcal{T} | t \geq T(\phi)} \int_0^{q_t^{d,\phi}} C^d(y) dy \tag{5}$$

$$\text{s.t. } q_t^{d,\phi} = q_t^{d,\phi'} + q_t^{f,d,\phi} \qquad \forall t \in \mathcal{T}.$$

The constraint states that the total quantity of gas that a supplier committed to inject over all the auction rounds up to node ϕ's auction is the sum of the following quantities:

- the total quantity of gas that it had committed to inject in all previous auctions (positive),
- the quantity that it trades in node ϕ's auction (positive or negative).

The consumers' optimization problems

Similarly, each consumer c located at a node i_c tries to maximize its expected benefit of gas consumption minus its procurement costs for all the periods of the horizon to come taking the market price as fixed. This time, the benefit of consuming gas $B_\phi^c(\cdot)$ may be uncertain for some network users:

[50] This keeps our model and its extensions clear, but the uncertainty of procurement costs could easily be introduced.

$$\max_{q_t^{f,c,\phi}, q_t^{c,\phi}} \sum_{\phi'' \in \Phi | \phi \leq \phi''} \left[\theta_{\phi''} \int_0^{q_{T(\phi'')}^{c,\phi}} B_{\phi''}^c(y) dy \right] - \sum_{t \in \mathcal{T} | t \geq T(\phi)} \pi_{t,i_c}^{f,\phi} q_t^{f,c,\phi} \tag{6}$$
$$\text{s.t. } q_t^{c,\phi} = q_t^{c,\phi'} + q_t^{f,c,\phi} \qquad \forall t \in \mathcal{T} .$$

As for suppliers, the constraint states that the total quantity of gas that a consumer committed to withdraw over all the auction rounds up to node ϕ's auction is the sum of the following quantities:

- the total quantity of gas that it had committed to withdraw in all previous auctions (positive),
- the quantity that it trades in node ϕ's auction (positive or negative).

The ISO's optimization problem

The ISO is represented as an independent market participant purchasing gas at some nodes and selling it at others over all remaining periods so as to maximize its profit. Since gas can only be moved from a node to another if physical network flows are feasible, the trades of the ISO must be compatible with the dispatch computed through network constraints. As stated in the first of these constraints, dispatched quantities are the result of the commitments made up to the previous auction round at node ϕ' and of the new quantities cleared in node ϕ's auction round:

$$\max_{\substack{q_{t,i}^{f,ISO,\phi}, q_{t,i,a_{ij}}^{\phi}, q_{t,j,a_{ij}}^{\phi} \\ p_{t,i}^{\phi}, m_{t,a_{ij}}^{\phi}, m_{a_{ij}}^{fin,\phi}}} \sum_{t \in \mathcal{T} | t \geq T(\phi)} \sum_{i \in \mathcal{N}} \pi_{t,i}^{f,\phi} q_{t,i}^{f,ISO,\phi}$$

$$\begin{aligned}
\text{s.t. } & q_{t,i}^{f,ISO,\phi} + q_{t,i}^{f,ISO,\phi'} + \sum_{j \in \mathcal{N} | a_{ij} \in \mathcal{A}} q_{t,i,a_{ij}}^{\phi} = \sum_{j \in \mathcal{N} | a_{ji} \in \mathcal{A}} q_{t,i,a_{ji}}^{\phi} && t \in \mathcal{T} | t \geq T(\phi), \forall i \in \mathcal{N} \\
& p_{t,i}^{\phi} \geq P_i^{min} && t \in \mathcal{T} | t \geq T(\phi), \forall i \in \mathcal{N} \\
& p_{t,i}^{\phi} \leq P_i^{max} && t \in \mathcal{T} | t \geq T(\phi), \forall i \in \mathcal{N} \\
& \left(\frac{q_{t,i,a_{ij}}^{\phi} + q_{t,j,a_{ij}}^{\phi}}{2} \right)^2 \leq K_{a_{ij}}^{flow} \cdot \left({p_{t,i}^{\phi}}^2 - {p_{t,j}^{\phi}}^2 \right) && t \in \mathcal{T} | t \geq T(\phi), \forall a_{ij} \in \mathcal{A} \\
& m_{t,a_{ij}}^{\phi} = K_{a_{ij}}^{lp} \cdot \frac{p_{t,i}^{\phi} + p_{t,j}^{\phi}}{2} && t \in \mathcal{T} | t \geq T(\phi),\ \forall a_{ij} \in \mathcal{A} \\
& m_{t+1,a_{ij}}^{\phi} - m_{t,a_{ij}}^{\phi} = q_{t,i,a_{ij}}^{\phi} - q_{t,j,a_{ij}}^{\phi} && t \in \mathcal{T} \backslash \{t_n\} | t \geq T(\phi), \forall a_{ij} \in \mathcal{A} \\
& m_{a_{ij}}^{fin,\phi} - m_{t_n,a_{ij}}^{\phi} = q_{t_n,i,a_{ij}}^{\phi} - q_{t_n,j,a_{ij}}^{\phi} && \forall a_{ij} \in \mathcal{A} \\
& m_{T(\phi),a_{ij}}^{\phi} = M_{a_{ij}}^{ini,\phi} && \forall a_{ij} \in \mathcal{A} \\
& \textstyle\sum_{a_{ij} \in \mathcal{A}} m_{a_{ij}}^{fin,\phi} \geq \sum_{a_{ij} \in \mathcal{A}} M_{a_{ij}}^{ini} ,
\end{aligned} \tag{7}$$

where the three last equations enforce initial and final conditions for linepack. More specifically, the initial condition sets a given level of linepack for each pipe separately. The final condition only imposes that the total linepack in the network at the end of the day be at least equal to the total linepack at the beginning of the gas day.

Market clearing conditions

In our equilibrium representation, separate markets take place at each node and each period for which gas is to be allocated. The market clearing conditions for these markets state that transactions are balanced, i.e. that the quantity bought by consumers and possibly the ISO is equal to the quantity sold by suppliers and potentially the ISO, at each node and for each period:

$$q_{t,i}^{f,ISO,\phi} + \sum_{d \in \mathcal{D}/i_d = i} q_t^{f,d,\phi} - \sum_{c \in \mathcal{C}/i_c = i} q_t^{f,c,\phi} = 0 \qquad \left(\pi_{t,i}^{f,\phi}\right) \qquad t \in \mathcal{T} | t \geq T(\phi), \forall i \in \mathcal{N}. \tag{8}$$

Written this way, these conditions mean that offer meets demand at node i and for period t at gas price $\pi_{t,i}^{f,\phi}$. This is coherent with our perfect competition assumption, for which economic theory states that the price in each market $\pi_{t,i}^{f,\phi}$ is the dual variable of the balance constraint of this market.

The model of a given auction occurring at a node ϕ is made of previously stated problems of the consumers, suppliers and of the ISO, as well as of the market clearing conditions. The model of the first auction of the day can easily be obtained from this formulation by stating that the quantities traded in previous auctions are null.

Link between successive auctions

We assume that multiple auctions are scheduled over the day, and examine the links between two successive auctions. The first is scheduled at node ϕ' and the next one is scheduled at node ϕ (not necessarily immediately following node ϕ'). The ISO must check that the new dispatch resulting from the auction at node ϕ is feasible given the network state at the beginning of this auction. This state is inherited from the dispatch resulting from the previous auction that occurred at node ϕ'. In our network representation, this means that the linepack taken into account by the ISO for the first period of node ϕ's auction must be equal to the one resulting from the dispatch decisions of node ϕ''s auction, i.e. the linepack of node ϕ as calculated in the clearing of node ϕ''s auction round:

$$M_{a_{ij}}^{ini,\phi} = m_{T(\phi'),a_{ij}}^{\phi'} \qquad \forall a_{ij} \in \mathcal{A}. \tag{9}$$

For the first auction of the day, the initial linepack is $M_{a_{ij}}^{ini}$, the linepack inherited from the dispatch of the previous day:

$$M_{a_{ij}}^{ini,\phi_0} = M_{a_{ij}}^{ini} \qquad \forall a_{ij} \in \mathcal{A}. \tag{10}$$

To sum-up, the overall problem of the LMP-T model (representing the succession of firm transport auctions over the gas day) is made of the assembly of the problems of all auction rounds, and of the definitions of the initial linepack for each auction.

VI.3.2 Daily LMP auction with 2-periods time-flexibility products

The LMP-F model describing a single daily auction with the offer of flexibility services keeps the same structure as each of the auction rounds appearing in the LMP-T model. The only differences come from the arrival of flexibility products in the objective functions of the agents, and the amendment of the network feasibility check performed by the ISO. We first present a full version of the model where only 2-periods time-flexibility services are offered, denoted LMP-F_{2T}, and explain in the next subsection how volume-flexibility and 3-periods time-flexibility products, as well as the offer of flexibility services by other players than the ISO can be introduced.

The suppliers' optimization problem

Since we do not study the offer of injection-flexibility products, the problems of suppliers are unchanged, but for the removal of the auction round upper index (only a single daily auction is run):

$$\max_{q_t^{f,d}} \sum_{t\in\mathcal{T}} \pi_{t,i_d}^{f} q_t^{f,d} - \sum_{t\in\mathcal{T}} \int_0^{q_t^{f,d}} C^d(y)dy. \tag{11}$$

The consumers' optimization problem

Consumers still maximize their expected benefit of gas consumption minus their procurement costs for all the periods of the gas day. However, they can now purchase firm at price π_{t,i_c}^{f} and flexible withdrawal services at price π_{t,i_c}^{lp}. Moreover, their consumption at a given scenario node q_ϕ^c once uncertainty is lifted now depends on how they decide to use the flexible services they purchased at the beginning of the day:

$$\begin{aligned}
&\max_{\substack{q_\phi^c, q_t^{f,c}, q_t^{lp,c} \\ q_t^{lpt-,c}, q_t^{lpt+,c}}} \sum_{\phi\in\Phi} \left[\theta_\phi \int_0^{q_\phi^c} B_\phi^c(y)dy\right] - \sum_{t\in\mathcal{T}} \left[\pi_{t,i_c}^{f} q_t^{f,c} + \pi_{t,i_c}^{lp} q_t^{lp,c}\right] \\
&\text{s.t. } q_\phi^{lpt-,c} + q_\phi^{lpt+,c} = q_{\mathcal{T}(\phi)}^{lpt,c} && \forall\phi\in\Phi |\, \mathcal{T}(\phi) \neq t_n \\
&q_{\phi_0}^c = q_{t_0}^{f,c} + q_{\phi_0}^{lpt-,c} \\
&q_\phi^c = q_{\mathcal{T}(\phi)}^{f,c} + q_\phi^{lpt-,c} + q_{\phi_f}^{lpt+,c} && \forall\phi\in\Phi |\, \mathcal{T}(\phi) \neq t_0,\ \mathcal{T}(\phi) \neq t_n \\
&q_\phi^c = q_{t_n}^{f,c} + q_{\phi_f}^{lpt+,c} && \forall\phi\in\Phi \,|\, \mathcal{T}(\phi) = t_n\,.
\end{aligned} \tag{12}$$

The constraints of this problem specify the terms of use of the flexibility services:

- The first one states the rule applying to a quantity of gas $q_{\mathcal{T}(\phi)}^{lpt,c}$ that was purchased through 2-periods time-flexible services for the period corresponding to a given node ϕ. When uncertainty is lifted for node ϕ, the owner must decide which fraction of the gas $q_{\phi}^{lpt-,c}$ it wishes to withdraw early, directly at node ϕ, and which fraction $q_{\phi}^{lpt+,c}$ it wishes to use later, during the following period.

- The remaining constraints just compute the total quantity of gas q_{ϕ}^{c} withdrawn by a consumer at each node ϕ. It is the sum of the gas purchased through firm services for the corresponding period $q_{\mathcal{T}(\phi)}^{f,c}$, and of the gas coming from early use of flexible services for this period $q_{\phi}^{lpt-,c}$ or from late use of flexible services of the previous period $q_{\phi_f}^{lpt+,c}$.

The ISO optimization problem

The ISO can now buy and sell firm products at each node and for each period at a price $\pi_{t,i}^{f}$, and sell flexible products at a price $\pi_{t,i}^{f}$. Its profit is rewritten accordingly. By valuing simultaneously firm and flexible products, it can strike the right balance between settling on a single dispatch to maximize transport capacity and managing the linepack in way that enables the network to cope with multiple dispatches possibilities. However, as stated before, it must ensure that all the hypothetical dispatches resulting from the different use possibilities of all sold flexible products are feasible. Thus, the network feasibility constraints are now enforced along another scenario tree Ψ developed by the ISO[51]:

$$\max_{\substack{q_{\psi,i}^{ISO},q_{t,i}^{f,ISO},q_{t,i}^{lp,ISO}\\ q_{\psi,i,a_{ij}},q_{\psi,j,a_{ij}}\\ p_{\psi,i},m_{\psi,a_{ij}},m_{\psi,a_{ij}}^{fin}}} \sum_{i\in\mathcal{N}} \pi_{t,i}^{f} q_{t,i}^{f,ISO} + \sum_{i\in\mathcal{N}} \pi_{t,i}^{lp} q_{t,i}^{lp,ISO}$$

$$\begin{aligned}
\text{s.t. } & q_{\psi_0,i}^{ISO} = q_{t_0,i}^{f,ISO} + R_{\psi_0}^{lpt-}\, q_{t_0,i}^{lp,ISO} && \forall i \in \mathcal{N} \\
& q_{\psi,i}^{ISO} = q_{\mathcal{T}(\psi),i}^{f,ISO} + R_{\psi,i}^{lpt-}\, q_{\mathcal{T}(\psi),i}^{lp,ISO} + R_{\psi_f,i}^{lpt+}\, q_{\mathcal{T}(\psi_f),i}^{lp,ISO} && \forall \psi \in \Psi \mid \mathcal{T}(\psi) \neq t_0,\ \mathcal{T}(\psi) \neq t_n,\ \forall i \in \mathcal{N} \\
& q_{\psi,i}^{ISO} = q_{\mathcal{T}(\psi),i}^{f,ISO} + R_{\psi_f,i}^{lpt+}\, q_{\mathcal{T}(\psi_f),i}^{lp,ISO} && \forall \psi \in \Psi \mid \mathcal{T}(\psi) = t_n,\ \forall i \in \mathcal{N} \\
& q_{\psi,i}^{ISO} + \sum_{j\in\mathcal{N}|a_{ij}\in\mathcal{A}} q_{\psi,i,a_{ij}} = \sum_{j\in\mathcal{N}|a_{ji}\in\mathcal{A}} q_{\psi,i,a_{ji}} && \forall \psi \in \Psi,\ \forall i \in \mathcal{N} \\
& p_{\psi,i} \geq P_i^{min} && \forall \psi \in \Psi,\ \forall i \in \mathcal{N} \\
& p_{\psi,i} \leq P_i^{max} && \forall \psi \in \Psi,\ \forall i \in \mathcal{N} \\
& \left(\frac{q_{\psi,i,a_{ij}} + q_{\psi,j,a_{ij}}}{2}\right)^2 \leq K_{a_{ij}}^{flow} \cdot \left({p_{\psi,i}}^2 - {p_{\psi,j}}^2\right) && \forall \psi \in \Psi,\ \forall a_{ij} \in \mathcal{A}
\end{aligned} \tag{13}$$

[51] This tree depends on the type of flexibility products used, on the number of periods for which they are offered, and on the way the ISO chooses to account for "all the possible use" of a given flexibility product.

$$m_{\psi,a_{ij}} = K_{a_{ij}}^{lp} \cdot \frac{p_{\psi,i}+p_{\psi,j}}{2} \qquad \forall\psi \in \Psi\ ,\ \forall a_{ij} \in \mathcal{A}$$

$$m_{\psi\prime,a_{ij}} - m_{\psi,a_{ij}} = q_{\psi,i,a_{ij}} - q_{\psi,j,a_{ij}} \qquad \forall(\psi,\psi') \in \Psi\backslash\{\psi_0\} \times \Psi\ |\psi = \psi'_f\ ,\ \forall a_{ij} \in \mathcal{A}$$

$$m_{\psi,a_{ij}}^{fin} - m_{\psi,a_{ij}} = q_{\psi,i,a_{ij}} - q_{\psi,j,a_{ij}} \qquad \forall\psi \in \Psi\ |\mathrm{T}(\psi) = t_n\ ,\forall a_{ij} \in \mathcal{A}$$

$$m_{\psi_0,a_{ij}} = M_{a_{ij}}^{ini} \qquad \forall a_{ij} \in \mathcal{A}$$

$$\textstyle\sum_{a_{ij}\in\mathcal{A}} m_{\psi,a_{ij}}^{fin} \geq \sum_{a_{ij}\in\mathcal{A}} M_{a_{ij}}^{ini} \qquad \forall\psi \in \Psi|T(\psi) = t_n\ .$$

The first three constraints of the problem compute the set of hypothetical withdrawal and injection quantities of which the ISO must verify network feasibility. For each type of flexibility product offered, the ISO defines what are the possible use cases it needs to include in dispatch tests. Since those products are defined by allocated quantities, the real use possibilities are infinite: in our example, consumers can split the quantity allocated by a 2-periods time-flexibility product between early and late withdrawals in an infinite number of ways. Consequently, the ISO should pick only a finite set of use possibilities that are representative of the impact of a product on the network. For our 2-periods time-flexibility product, those are the two cases when consumers use all of the gas early ($q_{\phi}^{lpt+,c} = 0$), or all of the gas late ($q_{\phi}^{lpt+,c} = 0$).

Once the use patterns to be tested have been chosen for each type of flexibility product, a scenario tree is defined by the ISO to reflect all the combinations of those uses over a gas day. The hypothetical withdrawal and injection schedules to be verified can then be calculated for each network node by adding up the gas allocated through firm products and the gas from flexible products hypothetically used at each scenario node. In our 2-periods time-flexibility example, the hypothetical quantity of gas $q_{\psi,i}^{ISO}$ withdrawn or injected at a node i and at scenario node ψ is the sum of the gas purchased through firm services for the corresponding period $q_{t,i}^{f,ISO}$, and of the gas hypothetically coming from early use of flexible services for this period $R_{\psi,i}^{lpt-}\ q_{T(\psi),i}^{lp,ISO}$ or from late use of flexible services of the previous period $R_{\psi_f,i}^{lpt+}\ q_{T(\psi_f),i}^{lp,ISO}$. Parameters $R_{\psi,i}^{lpt-}$ and $R_{\psi',i}^{lpt+}$ can be either equal to zero or unity, to reflect whether flexible gas is completely withdrawn early ($R_{\psi,i}^{lpt-} = 1$) or late ($R_{\psi,i}^{lpt+} = 1$)[52]. The remaining constraints are the usual network constraint indexed by the nodes of the ISO's scenario tree.

Market clearing conditions

Once again, our equilibrium representation of an LMP auction features multiple gas markets at each node and for each period. In this LMP-F_{2T} model, separate markets for firm products and flexible

[52] This adequately represents the whole range of impacts on network dispatch from time-flexibility products if only a single consumer is present at each node. To account for multiple consumers, those parameters should take a few more different values.

products take place simultaneously. All those markets are balanced, which is stated by the following market clearing conditions:

$$q_{t,i}^{f,ISO} + \sum_{d\in\mathcal{D}/i_d=i} q_t^{f,d} - \sum_{c\in\mathcal{C}/i_c=i} q_t^{f,c} = 0 \qquad (\pi_{t,i}^{f}) \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N} \qquad (14)$$

$$q_{t,i}^{lp,ISO} - \sum_{c\in\mathcal{C}/i_c=i} q_t^{lp,c} = 0 \qquad (\pi_{t,i}^{lp}) \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}. \qquad (15)$$

More specifically, the offer and demand of firm products at node i and for period t meet at a price $\pi_{t,i}^{f}$, while the offer and demand of flexible products at node i and for period t meet at a price $\pi_{t,i}^{lp}$.

The LMP-F$_{2T}$ model is made of the suppliers', consumers' and ISO's problems and of the market clearing conditions.

VI.3.3 Various flexibility and adjustment products for daily LMP auctions

The LMP-F$_{2T}$ model presented above can be adapted to account for the offer of any type of flexible products. We first present hereafter the two other flexibility products that will be used to derive our numerical results.

3-periods time-flexibility products

We stated previously that the owner of a 2-periods time-flexibility product purchased for a given period t was allowed to withdraw gas at this period or during the following one $t+1$. We can define a 3-periods time-flexibility product, that allows consumers that purchase it for a given period t to withdraw gas at this period or two periods afterwards, at $t+2$. Since the required changes are only minimal and to keep this section concise, we present the LMP-F$_{3T}$ model featuring these products in Appendix A.

Volume-flexibility products

Some consumers might face a risk of not being able to use the gas they purchased at the beginning of the day. In this case, volume-flexibility products would be more adapted than time-flexibility products. Such products can be introduced in a LMP-F$_V$ model, by replacing the constraints of the consumers' problems of model LMP-F$_{2T}$ by the following ones:

$$q_{\phi}^{lpv,c} \leq q_{T(\phi)}^{lp,c} \qquad \forall \phi \in \Phi \qquad (16)$$

$$q_{\phi}^{c} = q_{T(\phi)}^{f,c} + q_{\phi}^{lpv,c} \qquad \forall \phi \in \Phi\,, \qquad (17)$$

which state that the total quantity of gas q_{ϕ}^{c} withdrawn or injected by a consumer c at node ϕ is the sum of the gas purchased through firm services for the corresponding period $q_{T(\phi)}^{f,c}$, and of the gas coming from the use of flexible services for this period $q_{\phi}^{lpv,c}$. Moreover, the latter can be lower than the quantity of gas actually purchased through flexible services $q_{T(\phi)}^{lp,c}$.

Similar changes have to be included in the ISO's problem to reflect the new terms of use of volume-flexibility products. The three first constraints of this problem can be replaced by the following one:

$$q_{\psi,i}^{ISO} = q_{T(\psi),i}^{f,ISO} + R_{\psi,i}^{lpv}\ q_{T(\psi),i}^{lp,ISO} \qquad \forall\psi \in \Psi, \forall i \in \mathcal{N}. \tag{18}$$

The parameter $R_{\psi,i}^{lpv}$ can be equal to zero or unity, to let the ISO test the feasibility of network dispatches where the gas associated to volume-flexibility products is either not withdrawn at all or entirely withdrawn by consumers[53]. For the sake brevity, the full LMP-F_V model featuring volume-flexibility products is presented in Appendix A.

Injection adjustment products

Suppliers can be allowed to sell injection adjustment products to the ISO. We describe here the LMP-F_{2T}^{AdI} model where time-flexibility products and corresponding injection time-adjustment products are available. Only the problems of the suppliers and of the ISO differ from the LMP-F_{2T} model presented above.

The program of the suppliers can be reformulated as follows:

$$\max_{q_\omega^d, q_t^{f,d}, q_t^{ad,d}} \sum_{t\in\mathcal{T}} \left[\pi_{t,i_d}^{f} q_t^{f,d} + \pi_{t,i_d}^{adi} q_t^{ad,d}\right] - \sum_{\omega\in\Omega^d} \left[\theta_\omega \int_0^{q_\omega^d} C^d(y)dy\right]$$

$$\text{s.t. } q_{\omega_0}^d = q_{t_0}^{f,d} + R_\omega^{adt-,d} q_{t_0,i}^{ad,d} \tag{19}$$

$$q_\omega^d = q_{T(\omega)}^{f,d} + R_{\omega,i}^{adt-,d}\ q_{T(\omega),i}^{ad,d} + R_{\omega_f,i}^{adt+,d}\ q_{T(\omega_f),i}^{ad,d} \qquad \forall\omega \in \Omega^d | \mathcal{T}(\omega) \neq t_0, \mathcal{T}(\omega) \neq t_n$$

$$q_\omega^d = q_{t_n}^{f,d} + R_{\omega_f,i}^{lpt+,d}\ q_{T(\omega_f),i}^{ad,d} \qquad \forall\omega \in \Omega^d \,|\, \mathcal{T}(\omega) = t_n\,.$$

where the constraints of the problem compute the set of hypothetical quantity injected by the supplier depending on how the ISO activates the quantity of injection adjustment products it sold $q_{t,i}^{ad,d}$. This is similar to the computation realized by the ISO for its robust network feasibility check of flexibility products. For each type of adjustment product offered, individual suppliers define what are the possible activation cases that they need to include in their profit expectation. They should pick a finite set of those possible activations which is representative of the impact of each adjustment product on their expected profit. For our 2-periods time-adjustment product, we arbitrarily choose in our numerical cases to consider the two cases when the ISO requires that all of the gas must be injected either early or late.

Once the use patterns to be tested have been chosen for each type of adjustment product, a scenario tree Ω^d is defined by each supplier d to reflect all the combinations of those uses over a gas

[53] Once again, this adequately represents the whole range of impacts on network dispatch from volume-flexibility products if only a single consumer is present at each node. To account for multiple consumers, this parameter should take a few more different values.

day. The hypothetical injection schedules to be verified can then be calculated by adding up the gas sold through firm products and the activation of the gas sold through adjustment products. In our 2-periods time-adjustment example, the hypothetical quantity of gas q^{d}_{ω} injected at scenario node ω is the sum of the gas sold through firm services for the corresponding period $q^{f,d}_{T(\omega)}$, and of the gas hypothetically coming from the activation of adjustment services for this period $R^{adt-,d}_{\omega,i}\ q^{ad,d}_{T(\omega),i}$ or from late activation of adjustment services of the previous period $R^{adt+,d}_{\omega_f,i}\ q^{ad,d}_{T(\omega_f),i}$. Parameters $R^{adt-,d}_{\omega,i}$ and $R^{adt+,d}_{\omega,i}$ can be either equal to zero or unity, to reflect whether adjustment gas is completely injected early ($R^{adt-,d}_{\omega,i} = 1$) or late ($R^{adt+,d}_{\omega,i} = 1$). The objective function is the expected surplus of the supplier, made of the revenues from selling firm gas, and from selling injection adjustment products at price π^{adi}_{t,i_d} minus the gas procurement costs.

The program of the ISO must also be reformulated to account for the purchase and use of injection adjustment products. Its objective function is now the sum of the costs and revenues from purchasing and selling firm gas, of the revenues from selling flexibility products and of the costs of purchasing injection adjustment products:

$$\sum_{i\in\mathcal{N}}\pi^{f}_{t,i}q^{f,ISO}_{t,i} + \sum_{i\in\mathcal{N}}\pi^{lp}_{t,i}q^{lp,ISO}_{t,i} - \sum_{i\in\mathcal{N}}\pi^{adi}_{t,i}q^{adi}_{t,i} \tag{20}$$

where $q^{adi}_{t,i}$ is the total quantity of gas purchased through injection adjustment products at a node. The constraints of the problem remain the same, except for the three constraints used to compute the hypothetical quantities injected and withdrawn at each node, that must be replaced by the following ones:

$$\begin{aligned}
& q^{adit-}_{\psi,i} + q^{adit+}_{\psi,i} = q^{adi}_{T(\psi),i} && \forall\psi\in\Psi|\,\mathcal{T}(\psi)\neq t_n, \forall i\in\mathcal{N}\\
& q^{ISO}_{\psi_0,i} = q^{f,ISO}_{t_0,i} + R^{lpt-}_{\psi_0}\,q^{lp,ISO}_{t_0,i} - q^{adit-}_{\psi_0,i} && \forall i\in\mathcal{N}\\
& q^{ISO}_{\psi,i} = q^{f,ISO}_{T(\psi),i} + R^{lpt-}_{\psi,i}\ q^{lp,ISO}_{T(\psi),i} + R^{lpt+}_{\psi_f,i}\ q^{lp,ISO}_{T(\psi_f),i} && \forall\psi\in\Psi|\,\mathcal{T}(\psi)\neq t_0,\ \mathcal{T}(\psi)\neq\\
& \qquad - q^{adit-}_{\psi,i} - q^{adit+}_{\psi_f,i} && t_n\ , \forall i\in\mathcal{N}\\
& q^{ISO}_{\psi,i} = q^{f,ISO}_{T(\psi),i} + R^{lpt+}_{\psi_f,i}\ q^{lp,ISO}_{T(\psi_f),i} - q^{adit+}_{\psi_f,i} && \forall\psi\in\Psi|\,\mathcal{T}(\psi)= t_n\ , \forall i\in\mathcal{N}\ .
\end{aligned} \tag{21}$$

Those constraint describe decisions similar to the those faced by consumers purchasing flexibility products:

- The first one states the rules for triggering the injection of a quantity of gas purchased through adjustment products by the ISO $q^{adi}_{T(\psi),i}$ for the period corresponding to a given node ψ. When uncertainty is lifted for node ψ and based on the use of flexibility products by consumers, the ISO must decide which fraction of the gas $q^{adit-}_{\psi,i}$ it wishes to ask suppliers to inject early, directly at node ψ, and which fraction $q_{\psi,i}^{adit+}$ it wishes to ask suppliers to inject later, during the following period.

- The remaining constraints just compute the total quantity of gas $q_{\psi,i}^{ISO}$ hypothetically injected or withdrawn at each network and for each scenario node ψ. The contributions of firm products and flexible products are calculated as in the basic LMP-F_{2T} model, and added up to the gas linked to early activation of adjustment products for this period $q_{\psi,i}^{adit-}$ or to the late activation of the adjustment products of the previous period $q_{\psi_f,i}^{adit+}$.

Finally, in our equilibrium representation, a market clearing condition must be added to account for the trade of injection adjustment products:

$$q_{t,i}^{adi} - \sum_{d \in \mathcal{D} / i_d = i} q_t^{ad,d} = 0 \qquad \left(\pi_{t,i}^{adi}\right) \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}. \tag{22}$$

This LMP-F_{2T}^{AdI} can easily be adapted to account for volume-adjustment injection products. The corresponding model LMP-F_V^{AdI} is presented in Appendix A.

Withdrawal adjustment products

A similar reformulation leads to model LMP-F_{2T}^{AdW} which accounts for the possible introduction of withdrawal adjustment products. Not to clutter developments further, this model and its assumptions are presented in Appendix A.

VI.3.4 Integrated utility benchmark

Benchmark model

We finally present the problem of an ideal integrated utility, denoted I, which maximizes the expected welfare and can adjust its withdrawal and injection decisions over the gas day as uncertainty is lifted (full recourse model):

$$\max_{\substack{q_\phi^c, q_\phi^d \\ q_{\phi,i,a_{ij}}, q_{\phi,j,a_{ij}} \\ p_{\phi,i}, m_{\phi,a_{ij}}, m_{\phi,a_{ij}}^{fin}}} \sum_{\phi \in \Phi} \theta_\phi \sum_{c \in \mathcal{C}} \int_0^{q_\phi^c} B_\phi^c(y) dy - \sum_{\phi \in \Phi} \theta_\phi \sum_{d \in \mathcal{D}} \int_0^{q_\phi^d} C^d(y) dy$$

s.t.

$$\sum_{c \in \mathcal{C} \mid i_c = i} q_\phi^c + \sum_{j \in \mathcal{N} \mid a_{ij} \in \mathcal{A}} q_{\phi,i,a_{ij}} = \sum_{d \in \mathcal{D} \mid i_d = i} q_\phi^d + \sum_{j \in \mathcal{N} \mid a_{ji} \in \mathcal{A}} q_{\phi,i,a_{ji}} \qquad \forall \phi \in \Phi, \forall i \in \mathcal{N} \tag{23}$$

$$p_{\phi,i} \geq P_i^{min} \qquad \forall \phi \in \Phi, \forall i \in \mathcal{N}$$

$$p_{\phi,i} \leq P_i^{max} \qquad \forall \phi \in \Phi, \forall i \in \mathcal{N}$$

$$\left(\frac{q_{\phi,i,a_{ij}}+q_{\phi,j,a_{ij}}}{2}\right)^2 \leq K_{a_{ij}}^{flow} \cdot \left({p_{\phi,i}}^2 - {p_{\phi,j}}^2\right) \qquad \forall \phi \in \Phi, \forall a_{ij} \in \mathcal{A}$$

$$m_{\phi,a_{ij}} = K_{a_{ij}}^{lp} \cdot \frac{p_{\phi,i}+p_{\phi,j}}{2} \qquad \forall \phi \in \Phi, \forall a_{ij} \in \mathcal{A}$$

$$m_{\phi',a_{ij}} - m_{\phi,a_{ij}} = q_{\phi,i,a_{ij}} - q_{\phi,j,a_{ij}} \qquad \forall (\phi,\phi') \in \Phi \backslash \{\phi_0\} \times \Phi \,|\phi = {\phi'}_f, \forall a_{ij} \in \mathcal{A}$$

$$m_{\phi,a_{ij}}^{fin} - m_{\phi,a_{ij}} = q_{\phi,i,a_{ij}} - q_{\phi,j,a_{ij}} \qquad \forall \phi \in \Phi \,|\mathrm{T}(\phi) = t_n, \forall a_{ij} \in \mathcal{A}$$

$$m_{\phi_0,a_{ij}} = M_{a_{ij}}^{ini} \qquad \forall a_{ij} \in \mathcal{A}$$

$$\sum_{a_{ij}\in\mathcal{A}} m_{\phi,a_{ij}}^{fin} \geq \sum_{a_{ij}\in\mathcal{A}} M_{a_{ij}}^{ini} \qquad \forall \phi \in \Phi | T(\phi) = t_n .$$

The constraints of this problem are the usual network constraints which ensure that all the dispatch schedules that may be chosen during the day are feasible. One can also note that the final linepack condition imposes that at least as much gas is injected in the network as is withdrawn over a gas day. The resulting gas allocation and dispatches are the best possible under our perfect information assumption. They will be used as a benchmark in our numerical analyses.

Welfare comparison

In order to be able to compare the performance of the models presented above with this benchmark, we briefly detail how to calculate this welfare for each model.

The expected total welfare over the gas day can be easily computed in the case of the ideal integrated utility as the value of the objective function of this optimization program. This computation is almost as simple for LMP-F models without adjustment products, as the total expected welfare can be obtained as the sum of the objective functions of the programs of the suppliers, of the consumers and of the ISO.

The total expected welfare can be derived *ex post* for the LMP-T model. For each node ϕ, the consumers' benefits and suppliers' costs must be calculated with the final quantities $q_t^{c,\phi'}$ and $q_t^{d,\phi'}$ obtained in the last auction round allocating gas for this node (i.e. such that the period of auction following node ϕ''s auction is later that $T(\phi)$). These final benefits and costs are then weighted with the probability corresponding to their scenario node θ_ϕ.

As for LMP-F models with adjustment products, the total expected welfare is also the result of an *ex post* calculation. For each node ϕ, the flow decisions of the consumers and suppliers depend on the activation of the adjustment products by the ISO. Therefore, the final flow decisions must be extracted from the hypothetical dispatch of the ISO that matches with the use of flexibility products chosen by consumers for this scenario node[54].

Since those *ex post* calculations are simple to derive, but tedious and specific to each situation, we do not detail them here.

VI.3.5 Model properties and solutions

Locational marginal pricing models

The LMP-T model is composed of successive equilibrium problems, one for each auction round. The LMP-F model is a single equilibrium problem. For each equilibrium problem, the first order conditions (KKT) of the problems of all market players (consumers, suppliers and the ISO) are written and concatenated as a single complementarity problem, which can then be solved. Assuming linear inverse demand and offer functions, we note that each of these problems (i.e. prices being fixed) taken separately is convex. Economic variables are bounded (assuming a strictly decreasing inverse demand function and a strictly increasing inverse offer function, exchanged quantities and prices are bounded) while network variables are also bounded, thanks to the enforcement of minimum and maximum pressure constraints. Given that our complementarity constraints feature regular functions in this bounded space, the standard fixed-point theorem can be applied to prove that a solution to this equilibrium problem exists.

So as to compute numerical results from these equilibrium problems, the KKT conditions of the optimization problems were derived and reformulated into complementarity problems using the EMP framework (Ferris et al. 2009) directly in GAMS environment (GAMS Development Corporation 2016). The PATH solver was then used to solve those complementarity problems.

The ideal integrated utility problem

Given the assumption on demand and offer functions and thanks to the convexity of the network representation used, this non-linear optimization problem is convex and can be solved, for example, using the KNITRO solver in GAMS.

VI.4 Numerical cases

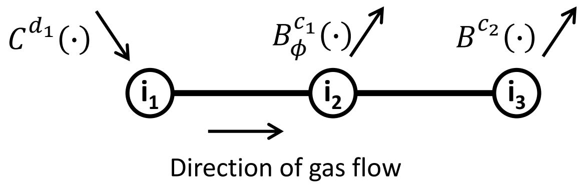
We use numerical cases to illustrate the complexity of the LMP-F auction design, due to the variety of products it requires, and to assess its performance compared to basic LMP-T auctions. Given the complexity of the equilibrium models presented above, we only derived those quantitative insights on simple test cases. The characteristics of these test cases are introduced thereafter.

[54] The final dispatch could also be simulated *ex post* for each scenario, with firm, flexibility and adjustment products allocation being fixed. The use of flexibility products would be chosen independently by each consumer to maximize their benefits in a given scenario, while the activation of adjustment products would result from a feasibility assessment of network constraints by a neutral ISO based on the consumers decisions regarding flexibility products.

VI.4.1 Network structure, users location and time horizon

We study a simple network made of a linear three-nodes pipeline, as illustrated in Figure 4, featuring three network users located at separate nodes. A supplier is placed at the first node of the pipe, a consumer with an uncertain demand at the intermediary node and a consumer with deterministic demand at the end node. Those consumers can represent, for instance, a CCGT and an industrial facility. This pipe is made of three identical, 75km-long sections with a diameter D=0.4m or D=0.7m depending on the test case. We also assume that the initial network linepack $M_{a_{ij}}^{ini}$ is the result of a previous steady state operation which is detailed in Appendix B. Maximum and minimum pressure are also reported there.

Figure 4: Structure of the network and location of network users



We analyze the allocation of network transport services through gas markets for three one-hour periods in all our test cases but one. When three-periods time-flexibility products are studied the time horizon studied extends to four one-hour periods.

VI.4.2 Demand and offer parameters

We assume that the inverse demand and offer functions are linear. For consumer c_1, the uncertainty of demand is modelled through the intercept of the inverse demand function. Consumer c_1's demand and supplier d_1's offer are deterministic. While we tuned demand and offer parameters to obtain realistic flows a network of this size, the monetary value of prices is arbitrary.

We study three rather simple demand cases, that we believe are representative of the basic types of demand uncertainty. For all test cases the demand of consumer c_1 is at a deterministic medium level for the first period and uncertain afterwards. We assume that this uncertainty is lifted just after the first period and simultaneously for all following periods. Thus, at the beginning of the second period, market players can adjust their behaviour for whole of the remaining time horizon. Consequently, LMP-T auctions are modelled as the succession of two auction rounds, one before the first period and another before the second period. This is illustrated for the three-periods test cases in Figure 6.

Finally, since the first period is deterministic and uncertainty is revealed only before the second period, flexibility products associated to the first period would have no value for network users in LMP-F auctions. Hence, we simplify our model by offering flexibility and adjustment products for the second period and the followings only.

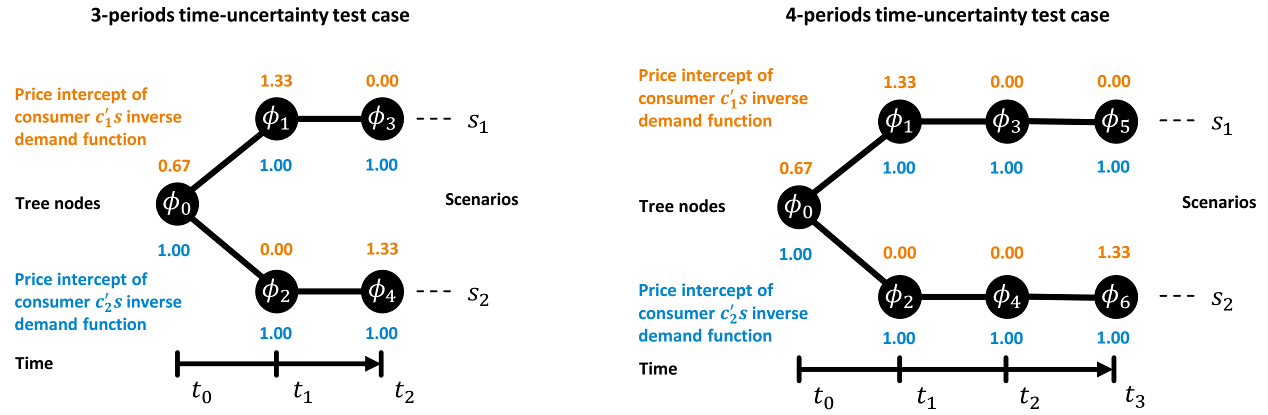
The uncertainty of demand for the following periods is different in each test case:

- in the 3-periods time-uncertainty case, the time of consumer c_1's demand peak is unknown, but its magnitude certain. Its demand is expected to be high for the either the second or the third period of the time horizon.
- in the 3-periods volume-uncertainty case, only the magnitude of consumer c_1's demand peak is unknown. Its demand is expected to be either high or low for the second period of the time horizon and always low afterwards.
- in the 4-periods time-uncertainty case, uncertainty is similar to the 3-periods time-uncertainty scenario, excepted for the fact that the time-uncertainty on consumer c_1's demand's peak is wider. Its demand is expected to be high for the either the second or the fourth period of the time horizon.

The values of the intercept of the inverse demand functions of consumer c_1 and c_2 are illustrated in Figure 5. The intercept of the inverse offer function is arbitrarily assumed to be null. In all scenarios, the absolute value of the slopes of the inverse demand and offer functions are arbitrarily chosen identical and constant, at a level of 5 of our arbitrary monetary units/MSCM.

While the nodal representation introduced in Section VI.2 is highly convenient to formulate our mathematical models, we identify results using the classical and more natural period and scenario indications. These indices are easily linked and the corresponding the demand scenario are indicated for each test case in Figure 5. In each test case, all scenarios have equal probabilities.

Figure 5: Structure of demand uncertainty for numerical test cases



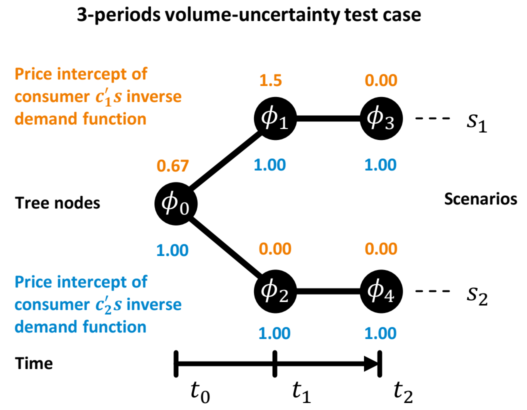
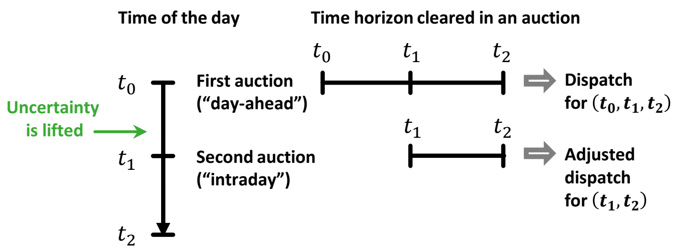


Figure 6: specific LMP-T implementation for our 3-periods cases



VI.4.3 Anticipation of the use of flexibility and adjustment products

We detail the scenarios anticipated by the ISO regarding the use of flexibility products by consumer c_1 in its hypothetical dispatch in Appendix B. The scenarios anticipated by supplier s_1 and consumer c_2 concerning the use of adjustment products by the ISO in their respective profit expectations can be constructed as suggested in the definition of adjustment products.

VI.4.4 Solving procedure

The solutions of the models are computed using a 10^{-12} solver precision.

VI.5 Results

In this section, we apply the models described in Section VI.3 to the test cases presented in Section VI.4 and draw conclusions regarding the complexity and the efficiency of the explicit allocation of gas network flexibility.

VI.5.1 Explicit allocation requires a complex design of products

We first show that to be efficient, an explicit allocation of network flexibility through LMP auctions requires a complex set of flexibility products and a corresponding offer of adjustment products. We also illustrate that while the diversity of flexibility products is an absolute necessity in this framework, adjustment products are only useful when network congestion arise.

Offering a variety of flexibility products is necessary

In this subsection we focus on flexibility products and consider an LMP-F auction where adjustment products are not offered. The network allocation resulting from the offer of 2-periods and 3-periods time-flexibility products as well as volume-flexibility products is computed for the three cases of consumer c_1's demand uncertainty that were introduced in Section VI.4 (i.e. 2-periods and 3-periods time uncertainty, and volume uncertainty). We ensure that the network at stake is not subject to congestions by assuming that it features a large diameter (D=0.7m). In each case, the expected welfare obtained is listed in Table 1 and compared to the expected welfare resulting from the optimal allocation of the ideal integrated utility. The objective value of the programs of the supplier, the consumers and the ISO are also presented in this table.

Table 1: Performance of LMP-F_{2T}, LMP-F_V and LMP-F_{3T} designs under 2-periods and 3-periods time uncertainty of demand or under volume uncertainty of demand

Demand uncertainty	2-periods time uncertainty			Volume uncertainty			3-periods time uncertainty		
Market model	LMP-F_{2T}	LMP-F_V	I	LMP-F_{2T}	LMP-F_V	I	LMP-F_{2T}	LMP-F_{3T}	I
Welfare	0.2093	0.1575	0.2093	0.1582	0.1604	0.1923	0.2070	0.2618	0.2618
∟ Relative difference with I	(0.0%)	(24.7%)	–	(17.7%)	(16.6%)	–	(20.9%)	(0.0%)	–
Supplier s_1's objective	0.1172	0.0993	–	0.0942	0.0990	–	0.1220	0.1440	–
Consumer c_1's objective	0.0499	0.0041	–	0.0060	0.0071	–	0.0048	0.0538	–
Consumer c_2's objective	0.0422	0.0541	–	0.0580	0.0543	–	0.0802	0.0640	–
ISO's objective	0.0000	0.0000	–	0.0000	0.0000	–	0.0000	0.0000	–

Note: this table compares the welfare, as well as suppliers', consumers' and ISO's objectives for various demand cases and network allocation frameworks. For each demand case, the relative difference between the welfare of the LMP-F framework and the welfare of the ideal integrated utility benchmark is shown in brackets.

As expected, these first results confirm that offering a type of flexibility product that corresponds to the type of demand uncertainty faced by consumers yields best results. Conversely, offering ill-adapted flexibility products creates a welfare loss. For instance, when consumer c_1 experiences a 2-periods time uncertainty, the welfare resulting from the offer of 2-periods time-flexibility products is about 25% higher than when only volume-flexibility products are available. As shown in Figure 7, purchasing a single time-flexibility product for the second period allows consumer c_1 to schedule its consumption in each scenario such that it occurs when its willingness to pay is at the highest (i.e for the second period in the first scenario, or for the third period in the second scenario). On the opposite, its consumption remains limited when only volume-flexibility products are offered. This is due to the fact that no matter whether it purchases gas for the second period through firm products or volume-flexibility products, that gas will be only consumed 50% of the time (in the first scenario). Hence, purchasing gas for the second period is less profitable, and consumer c_1's consumption remains low. The same result is obtained for 3-periods time-uncertainty, where 3-periods time-flexibility products are more cost-efficient than 2-periods time-flexibility products and allow the consumer to increase its consumption according to the realization of its willingness to pay, as shown in Figure 8.

Figure 7: Quantity consumed by consumer c_1 for a 2-periods time-uncertain demand in LMP-F$_{2T}$ or LMP-F$_V$ auction designs

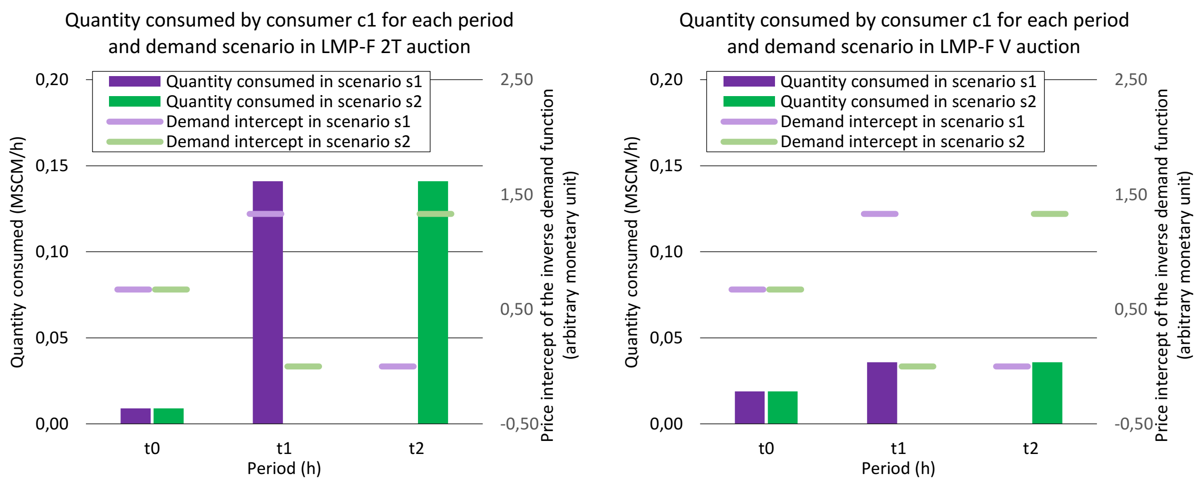
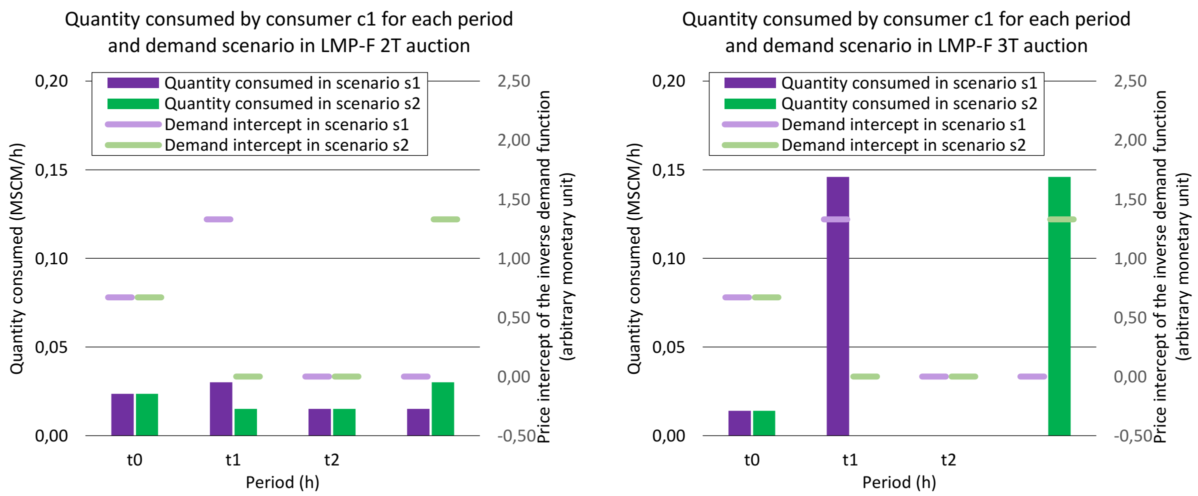
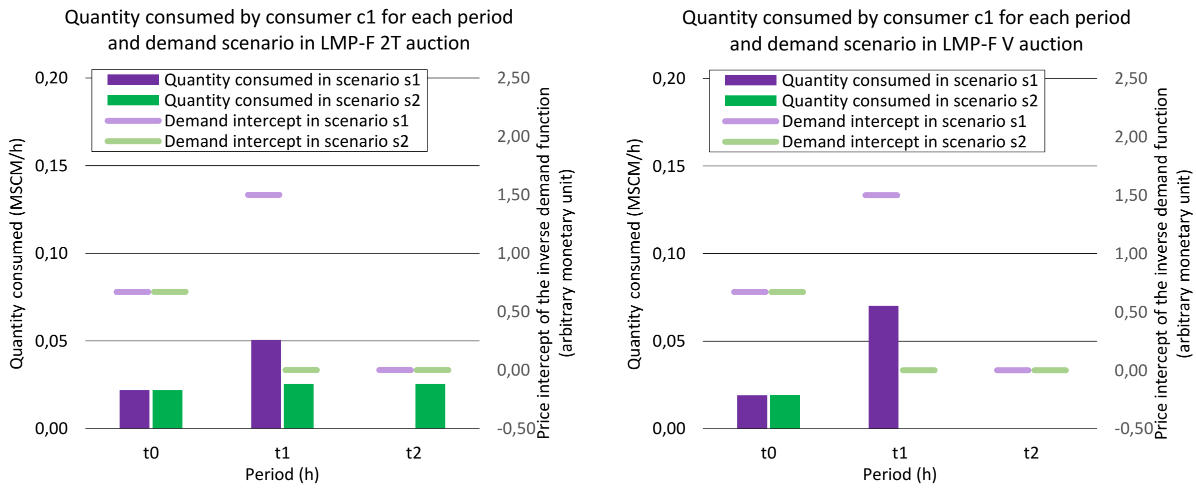


Figure 8: Quantity consumed by consumer c_1 for a 3-periods time-uncertain demand in LMP-F$_{2T}$ or LMP-F$_{3T}$ auction designs



Similarly, volume-flexibility products are more adapted to the need of the consumer c_1 when it faces volume uncertainty, than time-flexibility products. This time however, this is due to the losses it incurs when purchasing time-flexibility products. According to Figure 9, volume-flexibility products allow consumer c_1 to increase its consumption in the second period of the first scenario when its willingness to pay is high, while it can avoid consuming in the second scenario when its willingness to pay is negative. On the contrary, the gas obtained through a time-flexibility product must always be used. Hence if consumer c_1 purchases gas for the second period through firm of time-flexibility products expecting to harvest the benefits of consumption in the first scenario when its willingness to pay is high, it will still have to withdraw that gas in the second scenario when its willingness to pay is negative for both the second and third period. Hence, it will incur losses and as shown on Figure 9, it is better off reducing its consumption altogether. This leads to a less efficient dispatch.

Figure 9: Quantity consumed by consumer c_1 for a volume-uncertain demand in LMP-F$_{2T}$ or LMP-F$_V$ auction designs



Relative importance of adjustment products

We now study the relevance of adjustment products in the LMP-F framework. We assume that consumer c_1 faces either a 2-periods time uncertainty or a volume uncertainty. In each case we compute the dispatches resulting from the offer of the adjustment products available for that type of uncertainty (i.e. either both withdrawal and injection time-adjustment products for time uncertainty or injection volume-adjustment products only for volume uncertainty of demand). We compare these dispatches to those resulting from the LMP-F auction without adjustment products, and with the usual ideal integrated-utility benchmark. These comparisons are presented for two different networks. The first one has a large diameter (D=0.7m) while the other has a smaller diameter (D=0.4m). This allows us to compare the benefits of adjustment products when the network is congested and when it is not. We recall that when network congestion appears in an LMP framework, prices differ spatially and the ISO collects a revenue from transport services, also called congestion cost. Hence, the presence of congestion can be confirmed by observing the objective function of the ISO. A positive ISO's objective means that the network faces congestion, at least for one pipe and for one period. The results are listed in Table 2 for time uncertainty of consumer c_1's demand and Table 3 for volume uncertainty.

Table 2: Performance of LMP-F_{2T}, LMP-F_{2T}^{AdI}, LMP-F_{2T}^{AdW} and LMP-F_{2T}^{AdIW} designs under 2-periods time-uncertainty of demand

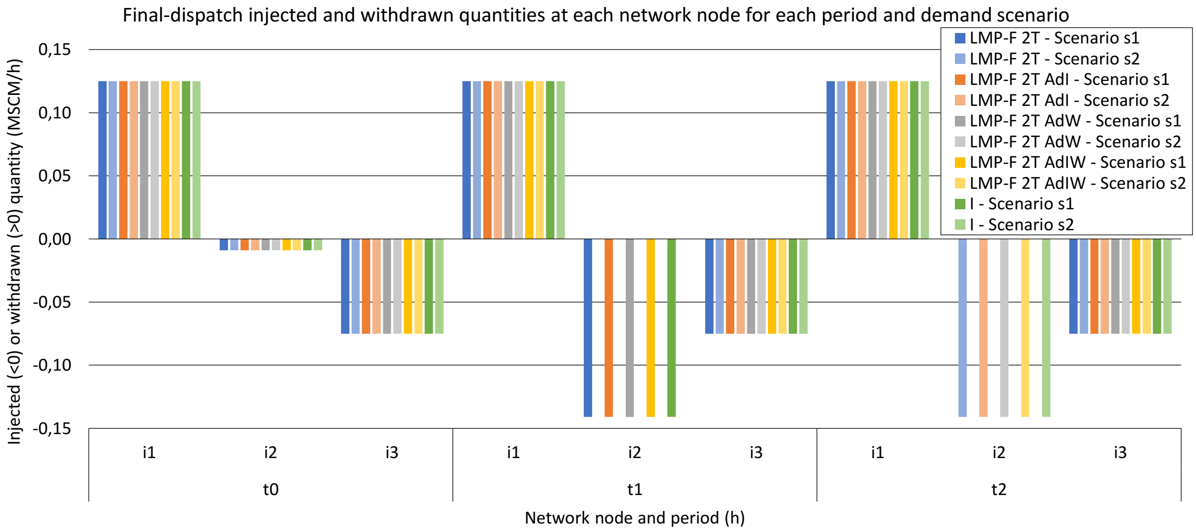
Network size	D=0.7m					D=0.4m				
Market model	LMP-F_{2T}	LMP-F_{2T}^{AdI}	LMP-F_{2T}^{AdW}	LMP-F_{2T}^{AdIW}	I	LMP-F_{2T}	LMP-F_{2T}^{AdI}	LMP-F_{2T}^{AdW}	LMP-F_{2T}^{AdIW}	I
Welfare	0.2093	0.2093	0.2093	0.2093	0.2093	0.20513	0.20527	0.20526	0.20537	0.20544
∟ Relative difference with I	(0.00%)	(0.00%)	(0.00%)	(0.00%)	–	(0.15%)	(0.08%)	(0.08%)	(0.03%)	–
Supplier s_1's objective	0.1172	0.1172	0.1172	0.1172	–	0.09321	0.09458	0.09474	0.09572	–
Consumer c_1's objective	0.0499	0.0499	0.0499	0.0499	–	0.04118	0.04167	0.04244	0.04275	–
Consumer c_2's objective	0.0422	0.0422	0.0422	0.0422	–	0.03154	0.03201	0.03198	0.03232	–
ISO's objective	0.0000	0.0000	0.0000	0.0000	–	0.03920	0.03701	0.03610	0.03458	–

Note: this table compares the welfare, as well as suppliers', consumers' and ISO's objectives for various demand cases, pipe diameters, and network allocation frameworks. For each demand case, the relative difference between the welfare of the LMP-F framework and the welfare of the ideal integrated utility benchmark is shown in brackets.

When comparing the welfare of LMP-F auctions with or without adjustment products, two situations can appear. When the network diameter is large and no congestion appears, the benefit from introducing adjustment products is null. On the contrary, when the network diameter is smaller and congestion arises, adjustment products marginally improve the performance of the LMP-F design.

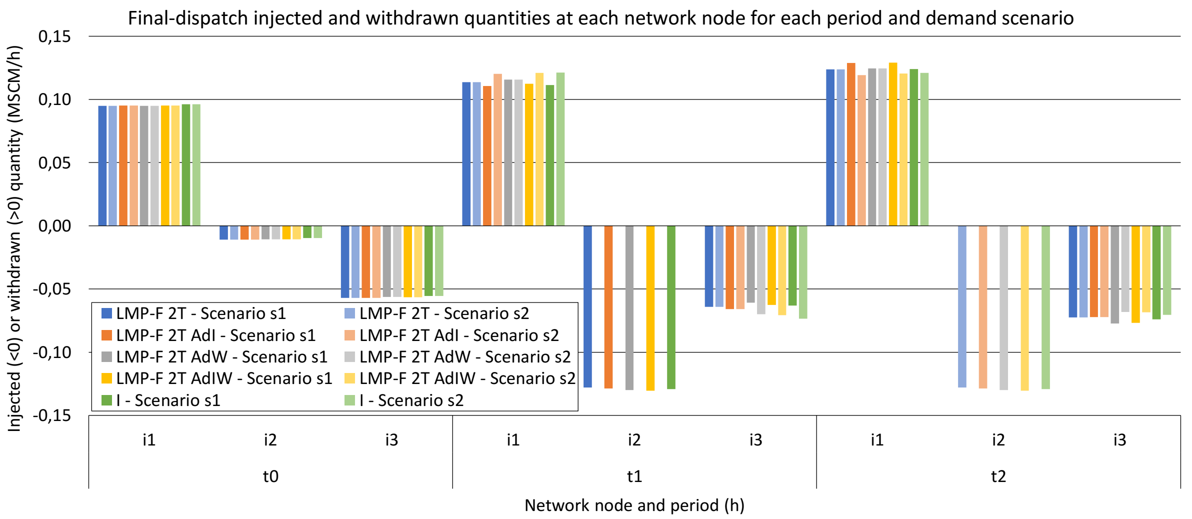
Figure 10 shows the dispatches of each network allocation design, for the two scenarios of consumer c_1's demand when the network is not congested. We can see that in such a situation, the injection schedule of supplier d_1 and the withdrawal schedule of consumer c_2 do not depend on the demand scenario of consumer c_1. Thus, adjustment products, which allow the ISO to gain control over the injection or withdrawal schedule of some network users to better tune the dispatch to the use of the flexibility products purchased by others, are not relevant in this case.

Figure 10: Network use for various adjustment-products offering for a non-congested network (D=0.7m) under time-uncertainty of demand



When the network is congested though, Figure 11 shows that ideal integrated utility adapts the injection schedule of supplier d_1 and the withdrawal schedule of consumer c_2 to the consumer c_1's demand scenario. Doing so, it can partially relieve congestion and offer more capacity to satisfy the needs of consumer c_1 in each of its demand scenario. Looking at the dispatch of the various LMP-F auction designs also presented, it is easy to see that adjustment products allow the ISO to adapt the injection and withdrawal schedules of supplier d_1 or consumer c_2, or both, to the consumer c_1's demand scenario and to lean toward the ideal dispatch. One can note that in our test case, this improvement in only marginal though. The magnitude of the benefits provided by adjustment products may be different for other network or demand cases.

Figure 11: Network use for various adjustment-products offering for a congested network (D=0.4m) under time-uncertainty of demand



These results also highlight that even with the introduction of the adapted adjustment products and their availability to all network users, the ISO may not be able to fully reach the ideal dispatch when congestion arises. While the remaining efficiency gap is fairly low for time-uncertainty of demand, at 0.03%, it is much higher for volume uncertainty, remaining at 15.8%. This is the result of the definition of volume-flexibility products in the LMP-F design. A network user purchasing volume-flexibility products wishes to be able not to use the gas quantity it is entitled to under certain circumstances. However, the ISO has no way to assess when this gas will actually be used, nor the probability at which this is likely to happen. Thus, even if the gas linked to a volume-flexibility product might not be used, the robust network-feasibility check performed by the ISO forces it to clear a corresponding injection quantity. Volume-adjustment products, which may in theory allow the ISO to reduce injection when not needed, also suffer from a similar flaw. When a supplier sells a volume-adjustment product to the ISO, it has no guarantee that the ISO will not use this gas, nor any estimate of the frequency at which it may do so. This is by design, as those products must allow for the ISO to decide on their activation without any constraint, to match the freedom offered to the buyers of flexibility products. In this context, the supplier has no choice but to charge the adjustment product at least at the price of firm gas, as it may incur the related procurement costs when it is activated. Compared to more agile management of the ideal integrated utility, the gas purchased through volume-flexibility is paid even when it is not consumed in the LMP-F design, which induces a sub-optimal resource allocation and creates important welfare losses. This result is new and quite important: it highlights that while the LMP-F design may improve the allocation of network flexibility, the associated concept of probabilistic network allocation introduces some limitations that may be detrimental to overall market performance, depending on the type of uncertainty faced by consumers.

Table 3: Performance of LMP-$\mathrm{F_V}$ and LMP-$\mathrm{F_V^{AdI}}$ designs under volume-uncertainty of demand

Network size	D=0.7m			D=0.4m		
Market model	LMP-$\mathrm{F_V}$	LMP-$\mathrm{F_V^{AdI}}$	I	LMP-$\mathrm{F_V}$	LMP-$\mathrm{F_V^{AdI}}$	I
Welfare	0.1604	0.1604	0.1923	0.1571	0.1578	0.1874
∟ Relative difference with I	(16.6%)	(16.6%)	–	(16.2%)	(15.8%)	–
Supplier s_1's objective	0.0990	0.0990	–	0.0833	0.0856	–
Consumer c_1's objective	0.0071	0.0071	–	0.0078	0.0077	–
Consumer c_2's objective	0.0543	0.0543	–	0.0441	0.0452	–
ISO's objective	0.0000	0.0000	–	0.0220	0.0194	–

Note: this table compares the welfare, as well as suppliers', consumers' and ISO's objectives for various demand cases, pipe diameters, and network allocation frameworks. For each demand case, the relative difference between the welfare of the LMP-F framework and the welfare of the ideal integrated utility benchmark is shown in brackets.

In this subsection, we showed that diversified flexibility products are always necessary in an LMP-F design, while adjustment products may only be useful when congestion arises. More importantly, we showed that the practical implementation of explicit allocation of network flexibility in an LMP setting may create limitations that decrease its overall efficiency. This calls for a comparison of this market design with other market-based allocations of gas network capacity.

VI.5.2 Is explicit flexibility allocation more efficient?

We now compare the performance of the LMP-F design (that explicitly allocates network flexibility) with the usual LMP-T framework (that features successive auctions in which only firm products are traded). So as to compare the efficiency of the LMP-T design with the best possible implementation of the LMP-F auction, we assume that adapted adjustment products are always available in the LMP-F variants examined thereafter. Once again, we separate the cases of congested and non-congested networks. Results are presented for the same time- and volume-uncertain demand test cases as previously, and respectively listed in Table 4 and Table 5.

Table 4: Performance of LMP-T and LMP-$\mathrm{F}_{2\mathrm{T}}^{\mathrm{AdIW}}$ designs under 2-periods time-uncertainty of demand

Network size	D=0.7 m			D=0.4 m		
Market model	LMP-T	LMP-$\mathrm{F}_{2\mathrm{T}}^{\mathrm{AdIW}}$	I	LMP-T	LMP-$\mathrm{F}_{2\mathrm{T}}^{\mathrm{AdIW}}$	I
Welfare	0.2070	0.2093	0.2093	0.2045	0.2054	0.2054
∟ Relative difference with I	(1.11%)	(0.00%)	–	(0.45%)	(0.03%)	–
Supplier s_1's objective	0.0951	0.1172	–	0.0786	0.0957	–
Consumer c_1's objective	0.0502	0.0499	–	0.0485	0.0427	–
Consumer c_2's objective	0.0617	0.0422	–	0.0507	0.0323	–
ISO's objective	0.0000	0.0000	–	0.0266	0.0346	–
Supplier s_1's expected cost	-0.1198	-0.1172	-0.1172	-0.0942	-0.0957	-0.0945
Consumer c_1's expected benefit	0.1465	0.1437	0.1437	0.1397	0.1377	0.1363
Consumer c_2's expected benefit	0.1802	0.1828	0.1828	0.1590	0.1633	0.1636

Note: this table compares the welfare, as well as suppliers', consumers' and ISO's objectives for various demand cases, pipe diameters, and network allocation frameworks. For each demand case, the relative difference between the welfare of the LMP-F framework and the welfare of the ideal integrated utility benchmark is shown in brackets. We also report the expected total cost incured by suppliers and the expected total benefits received by consumers. These values are calculated ex post as the costs and benefits really allocated, taking into account the actual activation of adjustment products by the ISO in LMP-F auctions. As for LMP-T auctions, the final dispatched quantities are used to calculated objectives, costs and benefit values.

We observe that the performance of the LMP-F design heavily depends on the type of demand uncertainty it addresses. When the demand is time-uncertain, the LMP-F design yields a better network allocation than LMP-T auctions, no matter the congestion status of the network. The difference with LMP-T auctions remains small though, at 1.11% for a non-congested network, and

0.44% for a congested network[55], while the performance of both designs is very close or identical to that of the optimal dispatch. When demand is volume-uncertain though, these auction systems perform worse than the ideal integrated utility, and exhibit welfare losses of a least 6.63%. But this time, the LMP-F design performs much worse than LMP-T auctions, with an additional welfare loss of 10.6% and 9.83% in non-congested and congested networks respectively. This is again the result of the limitations associated to the nature of volume-flexibility products, which imposes to pay for gas even when it is not used. In the next paragraphs, we explain the better performance of the LMP-F design in handling time-uncertain demand.

Table 5: Performance of LMP-T and LMP-F_V^{AdI} designs under volume-uncertainty of demand

Network size	D=0.7 m			D=0.4 m		
Market model	LMP-T	LMP-F_V^{AdI}	I	LMP-T	LMP-F_V^{AdI}	I
Welfare	0.1794	0.1604	0.1923	0.1750	0.1578	0.1874
∟ Relative difference with I	(6.69%)	(16.57%)	–	(6.63%)	(15.81%)	–
Supplier s_1's objective	0.0932	0.0990	–	0.0772	0.0856	–
Consumer c_1's objective	0.0246	0.0071	–	0.0225	0.0077	–
Consumer c_2's objective	0.0617	0.0543	–	0.0513	0.0452	–
ISO's objective	0.0000	0.0000	–	0.0240	0.0194	–
Supplier s_1's expected cost	-0.1005	-0.0990	-0.1039	-0.0763	-0.0856	-0.0783
Consumer c_1's expected benefit	0.0805	0.0584	0.1000	0.0749	0.0582	0.0927
Consumer c_2's expected benefit	0.1995	0.2010	0.1961	0.1765	0.1852	0.1730

Note: this table compares the welfare, as well as suppliers', consumers' and ISO's objectives for various demand cases, pipe diameters, and network allocation frameworks. For each demand case, the relative difference between the welfare of the LMP-F framework and the welfare of the ideal integrated utility benchmark is shown in brackets. We also report the expected total cost incured by suppliers and the expected total benefits received by consumers. These values are calculated ex post as the costs and benefits really allocated, taking into account the actual activation of adjustment products by the ISO in LMP-F auctions. As for LMP-T auctions, the final dispatched quantities are used to calculated objectives, costs and benefit values.

We now focus on the test case that features a time-uncertain demand. The main feature of the LMP-F design is to found gas prices on the users' preferences for both gas transport and flexibility, and to do so at the same time, before dispatch, considering the whole gas day. On the contrary, in each LMP-T auction, prices are formed based on the needs for firm gas transport only. Figure 12 shows the nodal prices resulting from each LMP-T auction round and compare them with the prices of the LMP-F auction when the network is not congested. Given the absence of congestion, revealed by the dispatch pressures reported in Figure 14, prices are, as expected, identical for all nodes and all periods

[55] Due to the complexity of gas network physics, these results may vary for other demand or network cases, as shown in (Schewe and Schmidt 2015).

in each auction. In the LMP-F auction, prices already reflect the different consumption schedules that may occur in each demand scenario, even if the auction takes place before uncertainty is lifted. On the contrary, in the LMP-T auctions, prices exhibit a 20% increase from the first auction to the second auction[56] (which differs for each realization of consumers c_1's demand scenario). Due to the uncertainty of its willingness to pay for the second and third periods, consumer c_1 can only purchase firm products corresponding to the total amount of gas that it wishes to consume once uncertainty has been lifted. Therefore, prices change during the day and the ISO is informed of the real demand later than in the LMP-F auction.

Figure 12: Price of gas in LMP-T and LMP-F designs for a non-congested network (D=0.7m)

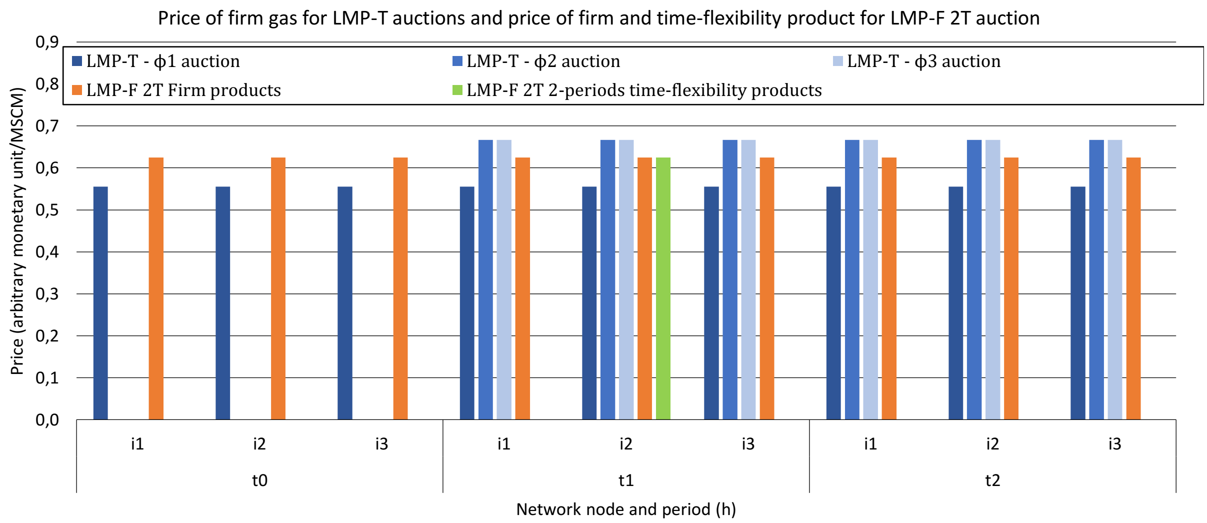
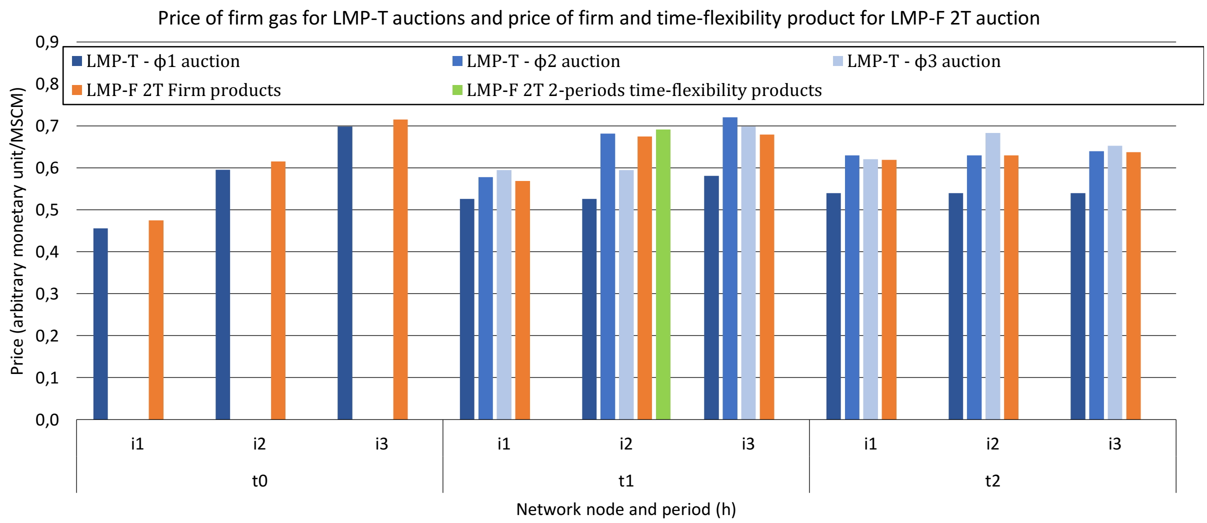


Figure 13: Price of gas in LMP-T and LMP-F designs for a congested network (D=0.4m)



[56] This price differential may be partially reduced by the introduction of convergence bidding (also called virtual bidding), which was not included in our model. This would not let users express flexibility needs though.

For a smaller pipe diameter, when congestion appears (see network pressures in Figure 15), prices differ over space and time in both systems, while the price difference between the two auctions rounds of the LMP-T design remains. The presence of this price change even in our perfect information framework confirms that some aspects of demand could not be traded in the first auction in this design.

Figure 14: Network pressure in LMP-T and LMP-F designs for a non-congested network (D=0.7m)

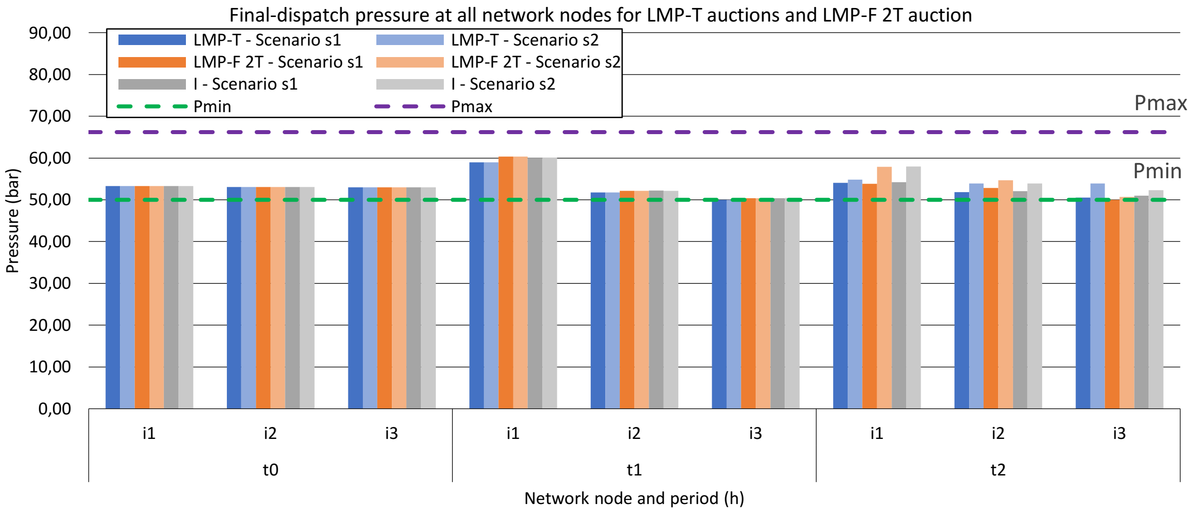
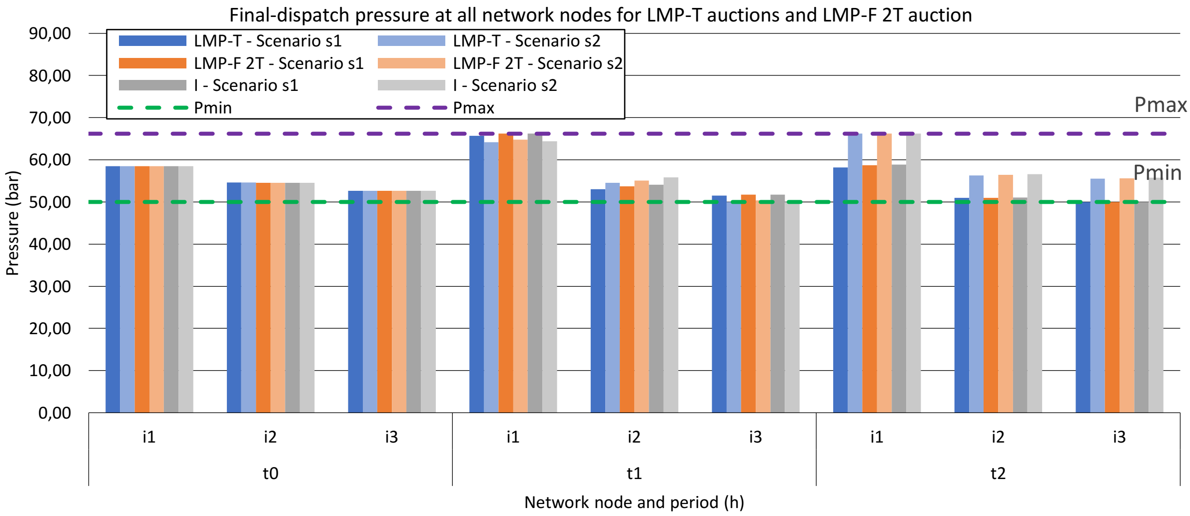


Figure 15: Network pressure in LMP-T and LMP-F designs for a congested network (D=0.4m)



As results show, this lack of information has negative consequences on welfare. At first sight though, the detailed objective and benefit values may appear counter-intuitive. Both in terms of profit and raw benefit (which can be seen as a value-weighted proxy for total consumption of a player over the day) the consumer that faces uncertainty is better off in the LMP-T scheme that is assumed to better handle flexibility. On the contrary, while its profit decreases, the other consumer is allowed to consume more gas in LMP-F, increasing its raw benefit. This stems from the balance of two opposite dynamics in network management between the LMP-T and LMP-F designs, the first one resulting from first period allocation, and the second from the allocation of the remaining periods.

Figure 16: Network use for LMP-T and LMP-F designs for a non-congested network (D=0.7m)

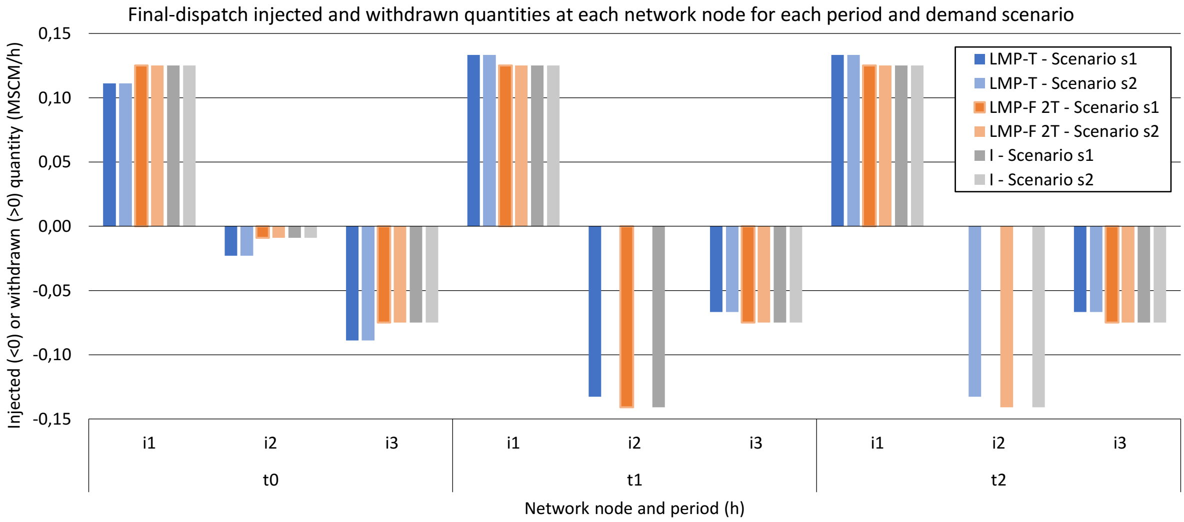
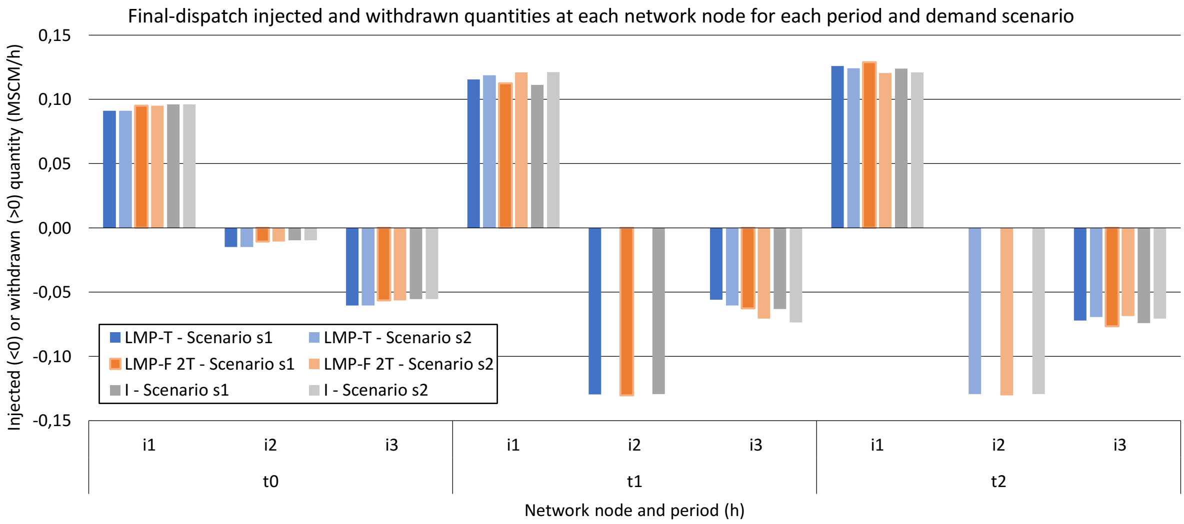


Figure 17: Network use for LMP-T and LMP-F designs for a congested network (D=0.4m)



Looking at quantities dispatched for the first period, presented in Figure 16 and Figure 17 for a non-congested and congested network respectively, each consumer is allocated a much larger consumption in the LMP-T design. Such a clearing is possible, because the ISO can optimize the network for a single dispatch, based the allocation of firm products only.

For the second and third periods though, it is the LMP-F design that allocates larger quantities to both players, compared to LMP-T auctions. This can be attributed to the better network management when flexibility is explicitly offered to the market. Figure 18 and Figure 19 present the overall linepack in the network for each design in a non-congested and congested network respectively. We observe that under an LMP-F design, the ISO is able to pack more gas during the first period (resulting in a higher linepack level in the second period) based on its better knowledge of future demand eventualities. Thanks to this network preparation, it can simultaneously offer more capacity to the consumer facing uncertain demand when its willingness to pay is high, and more capacity to the other consumer no matter the demand of consumer c_1. As seen previously, packing gas in the first period reduces the capacity available in the first period though.

On the contrary, in the LMP-T design, the linepack of the second period is the result of a dispatch that is based on a wrongly limited expression of users' needs (which can only imperfectly "internalize" their flexibility needs in firm products). This results in a network prepared so as to maximize capacity for a single flow pattern, which features a lower linepack level. When the actual demand is revealed in each scenario, and expressed by consumers in the intraday market, the ISO cannot accommodate as much deviation from its initial dispatch as would be economically optimal. Therefore, it can only offer less capacity to consumer c_1 when it reveals its true demand, and must reduce consumer c_2 consumption too much compared to optimality to cope with this planning change.

By allocating more gas to consumer c_1 at a time when its willingness to pay is low and reducing its consumption when it is high, the LMP-T design decreases the overall welfare. But from the individual point of view of each consumer, the gains obtained from the first-period allocation may be enough to compensate the losses in the next periods, explaining the initial counter-intuitive aspect of results.

We underline that these results differ from (Vazquez and Hallack 2013), who claimed the LMP-T and LMP-F design would be equivalent if liquid intraday markets existed. The early and more complete revealing of users' preferences allowed by the LMP-F design unlocks a better management of the technical intertemporal constraints of gas networks. In the LMP-T design, users cannot fully "internalize" their future preferences in firm products, which deprives the ISO of some information when allocating network capacities. Moreover, in the case of volume-uncertainty, those two market organizations differ even more, due to the limitations inherent to volume-flexibility products.

Figure 18: Overall network linepack in the LMP-T and LMP-F designs for a non-congested network (D=0.7m)

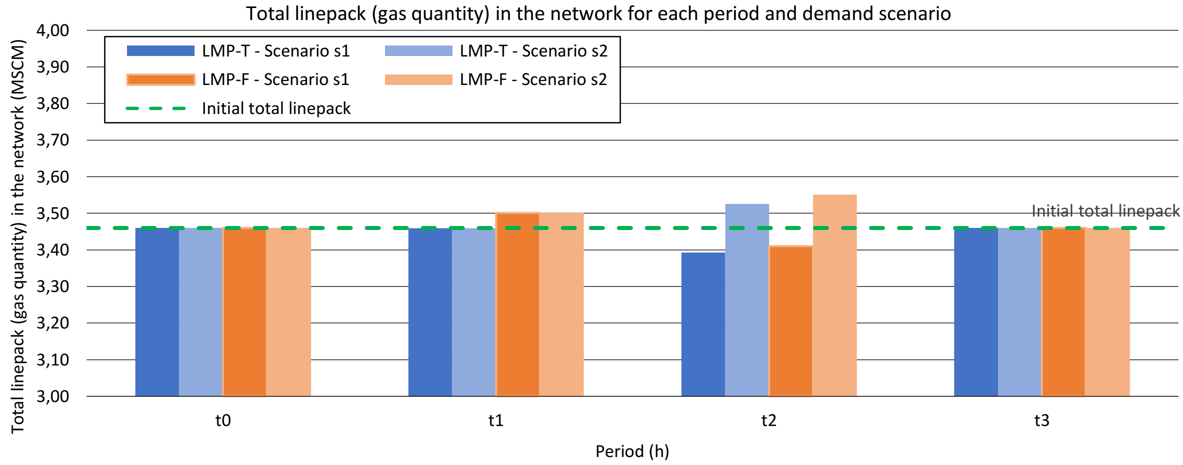
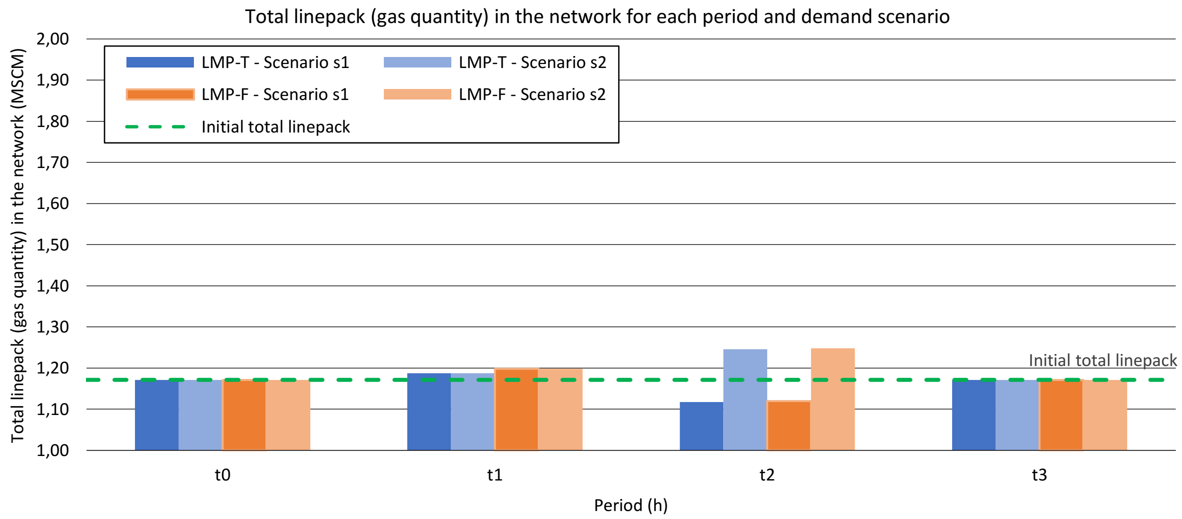


Figure 19: Overall network linepack in the LMP-T and LMP-F designs for a congested network (D=0.4m)



To complement our analysis of the better time-uncertainty management of the LMP-F design compared to LMP-T auctions, we finally show that the benefits may increase for other network configurations. In Table 6 we present the results of the LMP-T and LMP-F designs for a shorter pipe (each section is 30km long, instead of 75km in previous examples) and when the location of consumers c_1 and c_2 is reversed, as shown in Figure B-2 of Appendix B. The consumer that experiences demand uncertainty is now at the end node of the pipe, while the consumer with certain demand is at the middle node ($i_{c_1} = i_3$, $i_{c_2} = i_2$). While the difference between LMP-T and LMP-F designs remains almost identical for a non-congested network (LMP-T auctions fare 1.10% worse), the performance gap becomes much larger when this network is congested, reaching 9.76%.

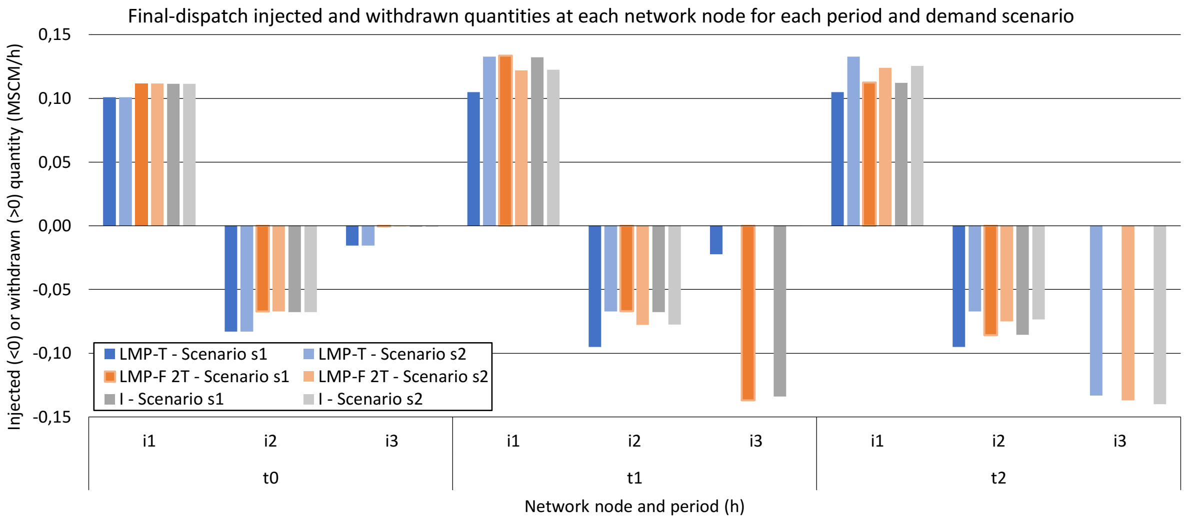
Table 6: Performance of LMP-T and LMP-F$_{2T}^{AdIW}$ designs under 2-periods time-uncertainty of demand for a shorter pipe (L=2x30km) and flipped consumers ($i_{c_1} = i_3$, $i_{c_2} = i_2$)

Network size	D=0.7 m			D=0.4 m		
Market model	LMP-T	LMP-F$_{2T}^{AdIW}$	I	LMP-T	LMP-F$_{2T}^{AdIW}$	I
Welfare	0.2068	0.2090	0.2090	0.1876	0.2079	0.2080
∟ Relative difference with I	(1.10%)	(0.00%)	–	(9.80%)	(0.02%)	–
Supplier s_1's objective	0.0909	0.1129	–	0.0879	0.1069	–
Consumer c_1's objective	0.0356	0.0499	–	0.0271	0.0470	–
Consumer c_2's objective	0.0587	0.0398	–	0.0585	0.0407	–
ISO's objective	0.0216	0.0065	–	0.0140	0.0133	–
Supplier s_1's expected cost	-0.1155	-0.1129	-0.1129	-0.0970	-0.1069	-0.1070
Consumer c_1's expected benefit	0.1461	0.1435	0.1435	0.0904	0.1356	0.1357
Consumer c_2's expected benefit	0.1762	0.1784	0.1784	0.1942	0.1793	0.1793

Note: this table compares the withdrawn flows, injection and withdrawal tariffs, injection and withdrawal capacity limits as well as total expected welfare for a single pipe under an Entry-Exit system where no cost-recovery constraint is enforced to that of the same network under the management of an integrated utility. The last column presents the relative difference between their total expected welfares.

As shown in Figure 20, an ISO running LMP-T auctions in this congested network drastically limits the consumption of consumer c_1 in the first scenario compared to when it allocates network capacity through an LMP-F auction. This comes from the fact that shorter pipes are more easily packed but also more easily depleted. Thus, the need to reserve linepack flexibility according to consumers' needs is more stringent. In the first scenario and under LMP-T auctions, the network cannot cope with a sudden demand increase in the second period, as the network was not prepared in the first period, while the LMP-F design is almost optimal by reserving linepack to accommodate this change. Conversely, in the second scenario, both auctions perform well. Since consumer c_1's demand increase occurs later, in the third period, the network can be prepared to handle it even through LMP-T auctions. The location of consumer c_1, further down the pipe, also limits the possibility for the ISO to compensate a low linepack by an increase in injection. In other words, it takes more time for gas to reach the consumer that needs it. A market design that lets the ISO be informed of possible future demand fluctuations can take actions in advance to handle these fluctuations. Finally, one can note that this time, the profits and benefits of consumers c_1 and c_2 are aligned with intuition, i.e. not only is the welfare higher in the LMP-F design, the profit and benefits of the consumer that faces demand uncertainty are also improved. This shows that the losses incurred in the LMP-T design by consumer c_1 for the second and third period, when demand uncertainty plays a role, are now more important than its gains from a biased allocation in the first period.

Figure 20: Network use for LMP-T and LMP-F designs for a shorter pipe (L=2x30km, D=0.4m) and flipped consumers ($i_{c_1} = i_3$, $i_{c_2} = i_2$)



To sum-up, we showed that the LMP-F design can improve dispatch when consumers experience time-uncertainty by better allocating network flexibility. However, it is less efficient than LMP-T auctions to handle the volume-uncertainty of demand. The performance improvements also largely depend on the magnitude of the impact the intertemporal network constraints. This magnitude is determined by the topology of the network, and by the characteristics of consumers' demand uncertainty. Thus, the opportunity of implementing an explicit allocation of network flexibility through locational marginal pricing must be carefully assessed considering the specificities of each market.

VI.6 Conclusion

Through a novel modeling of an LMP framework including flexibility products and a transient network representation, we showed that the motivation for explicit allocation of flexibility in gas system is real. Such an allocation can let the ISO better handle the intertemporal constraints that stem from gas network physics. However, the implementation of this model and the analysis of numerical test cases reveal that the performance gains heavily depend on the specific characteristics of the market, such as the topology of the network, its congestion state, and consumers' demand uncertainty profiles. Moreover, our detailed definition of differentiated products showed that a complex set of flexibility and adjustment products is required to harvest the full benefits of this explicit allocation, which may reduce market readability and transparency. The highly challenging network calculations associated to the clearing process of an LMP auction with flexibility products also question the feasibility of its implementation.

These results call for better allocating the flexibility potential of gas networks but also stress the need for thoroughly assessing the potential drawbacks of explicit mechanisms in each specific market before choosing whether to use them or not. In this context, further research to better understand the interaction of demand uncertainty and market-based gas network allocation on realistic networks is key.

VI.7 Acknowledgments

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VI.8 References

Arvesen ø., Medbø V, Fleten SE, Tomasgard A, Westgaard S (2013) Linepack storage valuation under price uncertainty. *Energy* 52:155–164.

Banda MK, Herty M (2008) Multiscale modeling for gas flow in pipe networks. *Math. Meth. Appl. Sci.* 31(8):915–936.

Boiteux M, Stasi P (1952) Sur la détermination des prix de revient de développement dans un système interconnecté de production-distribution. *Reprinted in G. Morlat et F.Bessière, 1971, "Vingt-cinq ans d'économie électrique," Paris Dunod, 361-400.*

Brijs T, Geth F, Siddiqui S, Hobbs BF, Belmans R (2016) Price-based unit commitment electricity storage arbitrage with piecewise linear price-effects. *Journal of Energy Storage* 7(Supplement C):52–62.

Brijs T, Huppmann D, Siddiqui S, Belmans R (2016) Auction-based allocation of shared electricity storage resources through physical storage rights. *Journal of Energy Storage* 7(Supplement C):82–92.

Caramanis MC, Bohn RE, Schweppe FC (1982) Optimal Spot Pricing: Practice and Theory. *IEEE Transactions on Power Apparatus and Systems* PAS-101(9):3234–3245.

Carter RG, Rachford HHJ (2003) Optimizing Line-Pack Management to Hedge Against Future Load Uncertainty. *PSIG Annual Meeting*.

De Wolf D, Smeers Y (2000) The gas transmission problem solved by an extension of the simplex algorithm. *Management Science* 46(11):1454–1465.

Domschke P, Geißler B, Kolb O, Lang J, Martin A, Morsi A (2011) Combination of nonlinear and linear optimization of transient gas networks. *INFORMS Journal on Computing* 23(4):605–617.

Ferris MC, Dirkse SP, Jagla JH, Meeraus A (2009) An extended mathematical programming framework. *Computers & Chemical Engineering* 33(12):1973–1982.

GAMS Development Corporation (2016) *General Algebraic Modeling System (GAMS) Release 24.7.1* (Washington, DC, USA).

Hallack M, Vazquez M (2013) European Union regulation of gas transmission services: Challenges in the allocation of network resources through entry/exit schemes. *Utilities Policy* 25:23–32.

Kelling C, Reith K, Sekirnjak E (2000) A Practical Approach to Transient Optimization For Gas Networks. *PSIG Annual Meeting*.

Keyaerts N (2012) *Gas Balancing and Line-pack Flexibility. Concepts and Methodologies for Organizing and Regulating Gas Balancing in Liberalized and Integrated EU Gas Markets. (Gasbalancering en netwerkflexibiliteit. Concepten en methodologieën voor de organisatie en regulering van gasbalancering in vrijgemaakte en geïntegreerde EU gasmarkten.)*. (KU Leuven).

Keyaerts N, Hallack M, Glachant JM, D'haeseleer W (2011) Gas market distorting effects of imbalanced gas balancing rules: Inefficient regulation of pipeline flexibility. *Energy Policy* 39(2):865–876.

Lochner, Stefan (2009) Nodal prices, capacity valuation and investments in natural gas markets: Overview and analytical framework. *EWI working paper* (09 02).

Mahlke D, Martin A, Moritz S (2007) A simulated annealing algorithm for transient optimization in gas networks. *Math Meth Oper Res* 66(1):99–115.

Midthun KT (2007) *Optimization models for liberalized natural gas markets*. dissertation. (Norwegian University of Science and Technology).

Midthun KT, Bjorndal M, Tomasgard A (2009) Modeling optimal economic dispatch and system effects in natural gas networks. *The Energy Journal* 30(4):155–180.

Modisette JL, Modisette JP (2001) Transient And Succession-of-Steady-States Pipeline Flow Models. *PSIG Annual Meeting*.

Moritz S (2007) *A mixed integer approach for the transient case of gas network optimization*. (TU Darmstadt).

Osiadacz A (1984) Simulation of transient gas flows in networks. *Int. J. Numer. Meth. Fluids* 4(1):13–24.

Pepper W, Ring BJ, Read EG, Starkey SR (2012) Implementation of a Scheduling and Pricing Model for Natural Gas. Sorokin A, Rebennack S, Pardalos PM, Iliadis NA, Pereira MVF, eds. *Handbook of Networks in Power Systems II*. Energy Systems. (Springer Berlin Heidelberg), 3–35.

Pietsch U, Rachford HHJ, Carter RG (2001) Investigating Real-World Applications Of Transient Optimization. (Pipeline Simulation Interest Group).

Read EG, Ring BJ, Starkey SR, Pepper W (2012) An LP Based Market Design for Natural Gas. Sorokin A, Rebennack S, Pardalos PM, Iliadis NA, Pereira MVF, eds. *Handbook of Networks in Power Systems II*. Energy Systems. (Springer Berlin Heidelberg), 77–113.

Ruff LE (2012) Rethinking Gas Markets--and Capacity. *Economics of Energy & Environmental Policy* Volume 1(Number 3).

Schewe L, Schmidt M (2015) *The Impact of Physics on Pricing in Energy Networks* (Social Science Research Network, Rochester, NY).

Schmidt M, Steinbach MC, Willert BM (2012) High detail stationary optimization models for gas networks—Part 1: Model components. *IfAM Preprint* 94.

Schweppe FC, Caramanis MC, Tabors RD, Bohn RE (1988) *Spot Pricing of Electricity* (Springer US, Boston, MA).

Vazquez M, Hallack M (2013) Need and design of short-term auctions in the EU gas markets. *Energy Policy* 63:484–493.

Westphalen M (2004) *Anwendungen der stochastischen Optimierung im Stromhandel und Gastransport*. (University of Duisburg-Essen).

Zavala VM (2014) Stochastic optimal control model for natural gas networks. *Computers & Chemical Engineering* 64:103–113.

Appendix A – LMP-F model variants: 3-periods time-flexibility, volume-flexibility and adjustment products

We detail here the LMP-F for 3-periods time-flexibility products, volume-flexibility products, and adjustment products.

LMP-F$_{3T}$ model: LMP auction with 3-periods time-flexibility products

The suppliers' problems and market clearing conditions are identical to the LMP-F$_{2T}$ model. Therefore, we only detail the consumers' and ISO's problems of the LMP-F$_{3T}$ model.

The consumers' optimization problem

$$\max_{\substack{q_\phi^c, q_t^{f,c}, q_t^{lp,c} \\ q_t^{lpt-,c}, q_t^{lpt+,c}}} \sum_{\phi\in\Phi}\left[\theta_\phi \int_0^{q_\phi^c} B_\phi^c(y)dy\right] - \sum_{t\in\mathcal{T}}\left[\pi_{t,i_c}^f q_t^{f,c} + \pi_{t,i_c}^{lp} q_t^{lp,c}\right]$$

$$\begin{aligned}
\text{s.t. } & q_\phi^{lpt-,c} + q_\phi^{lpt+,c} = q_{T(\phi)}^{lpt,c} && \forall\phi\in\Phi \,|\, T(\phi)\neq t_n,\ T(\phi)\neq t_{n-1} \\
& q_\phi^c = q_{T(\phi)}^{f,c} + q_\phi^{lpt-,c} && \forall\phi\in\Phi \,|\, T(\phi)=t_0 \text{ or } T(\phi)=t_1 \\
& q_\phi^c = q_{T(\phi)}^{f,c} + q_\phi^{lpt-,c} + q_{(\phi_f)_f}^{lpt+,c} && \forall\phi\in\Phi \,|\, T(\phi)\neq t_0,\ T(\phi)\neq t_1,\ T(\phi)\neq t_{n-1},\ T(\phi)\neq t_n \\
& q_\phi^c = q_{T(\phi)}^{f,c} + q_{(\phi_f)_f}^{lpt+,c} && \forall\phi\in\Phi \,|\, T(\phi)=t_{n-1} \text{ or } T(\phi)=t_n\,.
\end{aligned} \tag{A-1}$$

The ISO's optimization problem

$$\max_{\substack{q_{\psi,i}^{ISO}, q_{t,i}^{f,ISO}, q_{t,i}^{lp,ISO} \\ q_{\psi,i,a_{ij}}, q_{\psi,j,a_{ij}} \\ p_{\psi,i}, m_{\psi,a_{ij}}, m_{\psi,a_{ij}}^{fin}}} \sum_{i\in\mathcal{N}} \pi_{t,i}^f q_{t,i}^{f,ISO} + \sum_{i\in\mathcal{N}} \pi_{t,i}^{lp} q_{t,i}^{lp,ISO}$$

$$\begin{aligned}
\text{s.t. } & q_{\psi,i}^{ISO} = q_{T(\psi),i}^{f,ISO} + R_\psi^{lpt-}\, q_{T(\psi),i}^{lp,ISO} && \forall\psi\in\Psi \,|\, T(\psi)=t_0 \text{ or } T(\psi)\neq t_1,\ \forall i\in\mathcal{N} \\
& q_{\psi,i}^{ISO} = q_{T(\psi),i}^{f,ISO} + R_{\psi,i}^{lpt-}\, q_{T(\psi),i}^{lp,ISO} + R_{\psi_f,i}^{lpt+}\, q_{T\left((\psi_f)_f\right),i}^{lp,ISO} && \forall\psi\in\Psi \,|\, T(\psi)\neq t_0,\ T(\psi)\neq t_1,\ T(\psi)\neq t_{n-1},\ T(\psi)\neq t_n, \forall i\in\mathcal{N} \\
& q_{\psi,i}^{ISO} = q_{T(\psi),i}^{f,ISO} + R_{\psi_f,i}^{lpt+}\, q_{T\left((\psi_f)_f\right),i}^{lp,ISO} && \forall\psi\in\Psi \,|\, T(\psi)=t_{n-1} \text{ or } T(\psi)=t_n,\ \forall i\in\mathcal{N} \\
& q_{\psi,i}^{ISO} + \sum_{j\in\mathcal{N}|a_{ij}\in\mathcal{A}} q_{\psi,i,a_{ij}} = \sum_{j\in\mathcal{N}|a_{ji}\in\mathcal{A}} q_{\psi,i,a_{ji}} && \forall\psi\in\Psi,\ \forall i\in\mathcal{N} \\
& p_{\psi,i} \geq P_i^{min} && \forall\psi\in\Psi,\ \forall i\in\mathcal{N} \\
& p_{\psi,i} \leq P_i^{max} && \forall\psi\in\Psi,\ \forall i\in\mathcal{N}
\end{aligned} \tag{A-2}$$

$$\left(\frac{q_{\psi,i,a_{ij}}+q_{\psi,j,a_{ij}}}{2}\right)^2 \leq K_{a_{ij}}^{flow} \cdot \left({p_{\psi,i}}^2 - {p_{\psi,j}}^2\right) \qquad \forall\psi \in \Psi, \forall a_{ij} \in \mathcal{A}$$

$$m_{\psi,a_{ij}} = K_{a_{ij}}^{lp} \cdot \frac{p_{\psi,i}+p_{\psi,j}}{2} \qquad \forall\psi \in \Psi, \forall a_{ij} \in \mathcal{A}$$

$$m_{\psi',a_{ij}} - m_{\psi,a_{ij}} = q_{\psi,i,a_{ij}} - q_{\psi,j,a_{ij}} \qquad \forall(\psi,\psi') \in \Psi\backslash\{\psi_0\} \times \Psi \mid \psi = {\psi'}_f, \forall a_{ij} \in \mathcal{A}$$

$$m_{\psi,a_{ij}}^{fin} - m_{\psi,a_{ij}} = q_{\psi,i,a_{ij}} - q_{\psi,j,a_{ij}} \qquad \forall\psi \in \Psi \mid \mathrm{T}(\psi) = t_n, \forall a_{ij} \in \mathcal{A}$$

$$m_{\psi_0,a_{ij}} = M_{a_{ij}}^{ini} \qquad \forall a_{ij} \in \mathcal{A}$$

$$\textstyle\sum_{a_{ij}\in\mathcal{A}} m_{\psi,a_{ij}}^{fin} \geq \sum_{a_{ij}\in\mathcal{A}} M_{a_{ij}}^{ini} \qquad \forall\psi \in \Psi \mid T(\psi) = t_n .$$

LMP-F$_V$ model: LMP-F auction with volume-flexibility products

Here also, the suppliers' problems and market clearing conditions are identical to the LMP-F$_{2T}$ model. Therefore, we only detail the consumers' and ISO's problems of the LMP-F$_V$ model.

The consumers' optimization problems

$$\max_{\substack{q_\phi^c, q_t^{f,c}, q_t^{lp,c} \\ q_t^{lpv,c}}} \sum_{\phi\in\Phi}\left[\theta_\phi \int_0^{q_\phi^c} B_\phi^c(y)dy\right] - \sum_{t\in\mathcal{T}}\left[\pi_{t,i_c}^f q_t^{f,c} + \pi_{t,i_c}^{lp} q_t^{lp,c}\right]$$

$$\text{s.t. } q_\phi^{lpv,c} \leq q_{T(\phi)}^{lp,c} \qquad \forall\phi \in \Phi \qquad \text{(A-3)}$$

$$q_\phi^c = q_{T(\phi)}^{f,c} + q_\phi^{lpv,c} \qquad \forall\phi \in \Phi$$

The ISO's optimization problem

$$\max_{\substack{q_{\psi,i}^{ISO}, q_{t,i}^{f,ISO}, q_{t,i}^{lp,ISO} \\ q_{\psi,i,a_{ij}}, q_{\psi,j,a_{ij}} \\ p_{\psi,i}, m_{\psi,a_{ij}}, m_{\psi,a_{ij}}^{fin}}} \sum_{i\in\mathcal{N}} \pi_{t,i}^f q_{t,i}^{f,ISO} + \sum_{i\in\mathcal{N}} \pi_{t,i}^{lp} q_{t,i}^{lp,ISO}$$

$$\text{s.t. } q_{\psi,i}^{ISO} = q_{T(\psi),i}^{f,ISO} + R_{\psi,i}^{lpv}\, q_{T(\psi),i}^{lp,ISO} \qquad \forall\psi \in \Psi, \forall i \in \mathcal{N}$$

$$q_{\psi,i}^{ISO} + \sum_{j\in\mathcal{N}|a_{ij}\in\mathcal{A}} q_{\psi,i,a_{ij}} = \sum_{j\in\mathcal{N}|a_{ji}\in\mathcal{A}} q_{\psi,i,a_{ji}} \qquad \forall\psi \in \Psi, \forall i \in \mathcal{N} \qquad \text{(A-4)}$$

$$p_{\psi,i} \geq P_i^{min} \qquad \forall\psi \in \Psi, \forall i \in \mathcal{N}$$

$$p_{\psi,i} \leq P_i^{max} \qquad \forall\psi \in \Psi, \forall i \in \mathcal{N}$$

$$\left(\frac{q_{\psi,i,a_{ij}}+q_{\psi,j,a_{ij}}}{2}\right)^2 \leq K_{a_{ij}}^{flow} \cdot \left({p_{\psi,i}}^2 - {p_{\psi,j}}^2\right) \qquad \forall\psi \in \Psi, \forall a_{ij} \in \mathcal{A}$$

$$m_{\psi,a_{ij}} = K_{a_{ij}}^{lp} \cdot \frac{p_{\psi,i}+p_{\psi,j}}{2} \qquad \forall\psi \in \Psi, \forall a_{ij} \in \mathcal{A}$$

$$m_{\psi',a_{ij}} - m_{\psi,a_{ij}} = q_{\psi,i,a_{ij}} - q_{\psi,j,a_{ij}} \qquad \forall(\psi,\psi') \in \Psi\backslash\{\psi_0\} \times \Psi \;|\psi = \psi'_f\,, \forall a_{ij} \in \mathcal{A}$$

$$m^{fin}_{\psi,a_{ij}} - m_{\psi,a_{ij}} = q_{\psi,i,a_{ij}} - q_{\psi,j,a_{ij}} \qquad \forall\psi \in \Psi \;|\mathrm{T}(\psi) = t_n\,, \forall a_{ij} \in \mathcal{A}$$

$$m_{\psi_0,a_{ij}} = M^{ini}_{a_{ij}} \qquad \forall a_{ij} \in \mathcal{A}$$

$$\textstyle\sum_{a_{ij}\in\mathcal{A}} m^{fin}_{\psi,a_{ij}} \geq \sum_{a_{ij}\in\mathcal{A}} M^{ini}_{a_{ij}} \qquad \forall\psi \in \Psi | T(\psi) = t_n$$

LMP-F^{AdI}_{2T} model: LMP auctions with injection 2-periods adjustment products

The consumers' problems are identical to the LMP-F_{2T} model. Therefore, we only detail the suppliers' and ISO's problems as well as the market clearing conditions of the LMP-F^{AdI}_{2T} model.

Suppliers' optimization problem

$$\max_{q^d_\omega, q^{f,d}_t, q^{ad,d}_t} \sum_{t\in\mathcal{T}} \left[\pi^f_{t,i_d} q^{f,d}_t + \pi^{adi}_{t,i_d} q^{ad,d}_t\right] - \sum_{\omega\in\Omega^d} \left[\theta_\omega \int_0^{q^d_\omega} C^d(y)dy\right]$$

$$\text{s.t. } q^d_{\omega_0} = q^{f,d}_{t_0} + R^{adt-,d}_{\omega} q^{ad,d}_{t_0,i} \tag{A-5}$$

$$q^d_\omega = q^{f,d}_{T(\omega)} + R^{adt-,d}_{\omega,i}\, q^{ad,d}_{T(\omega),i} + R^{adt+,d}_{\omega_f,i}\, q^{ad,d}_{T(\omega_f),i} \qquad \forall\omega \in \Omega^d | \mathcal{T}(\omega) \neq t_0,\, \mathcal{T}(\omega) \neq t_n$$

$$q^d_\omega = q^{f,d}_{t_n} + R^{lpt+,d}_{\omega_f,i}\, q^{ad,d}_{T(\omega_f),i} \qquad \forall\omega \in \Omega^d \;| \mathcal{T}(\omega) = t_n\,.$$

The ISO's optimization problem

$$\max_{\substack{q^{ISO}_{\psi,i}, q^{f,ISO}_{t,i}, q^{lp,ISO}_{t,i} \\ q^{adi}_{t,i}, q^{adit-}_{\psi,i}, q^{adit+}_{\psi,i} \\ q_{\psi,i,a_{ij}}, q_{\psi,j,a_{ij}} \\ p_{\psi,i}, m_{\psi,a_{ij}}, m^{fin}_{\psi,a_{ij}}}} \sum_{i\in\mathcal{N}} \pi^f_{t,i} q^{f,ISO}_{t,i} + \sum_{i\in\mathcal{N}} \pi^{lp}_{t,i} q^{lp,ISO}_{t,i} - \sum_{i\in\mathcal{N}} \pi^{adi}_{t,i} q^{adi}_{t,i}$$

$$\text{s.t. } q^{adit-}_{\psi,i} + q^{adit+}_{\psi,i} = q^{adi}_{T(\psi),i} \qquad \forall\psi \in \Psi | \mathcal{T}(\psi) \neq t_n, \forall i \in \mathcal{N}$$

$$q^{ISO}_{\psi_0,i} = q^{f,ISO}_{t_0,i} + R^{lpt-}_{\psi_0}\, q^{lp,ISO}_{t_0,i} - q^{adit-}_{\psi_0,i} \qquad \forall i \in \mathcal{N}$$

$$q^{ISO}_{\psi,i} = q^{f,ISO}_{T(\psi),i} + R^{lpt-}_{\psi,i}\, q^{lp,ISO}_{T(\psi),i} + R^{lpt+}_{\psi_f,i}\, q^{lp,ISO}_{T(\psi_f),i} - q^{adit-}_{\psi,i} - q^{adit+}_{\psi_f,i} \qquad \forall\psi \in \Psi | \mathcal{T}(\psi) \neq t_0,\, \mathcal{T}(\psi) \neq t_n\,, \forall i \in \mathcal{N} \tag{A-6}$$

$$q^{ISO}_{\psi,i} = q^{f,ISO}_{T(\psi),i} + R^{lpt+}_{\psi_f,i}\, q^{lp,ISO}_{T(\psi_f),i} - q^{adit+}_{\psi_f,i} \qquad \forall\psi \in \Psi | \mathcal{T}(\psi) = t_n\,, \forall i \in \mathcal{N}\,.$$

$$q^{ISO}_{\psi,i} + \sum_{j\in\mathcal{N}|a_{ij}\in\mathcal{A}} q_{\psi,i,a_{ij}} = \sum_{j\in\mathcal{N}|a_{ji}\in\mathcal{A}} q_{\psi,i,a_{ji}} \qquad \forall\psi \in \Psi\,, \forall i \in \mathcal{N}$$

$$p_{\psi,i} \geq P^{min}_i \qquad \forall\psi \in \Psi\,, \forall i \in \mathcal{N}$$

$$p_{\psi,i} \leq P^{max}_i \qquad \forall\psi \in \Psi\,, \forall i \in \mathcal{N}$$

$$\left(\frac{q_{\psi,i,a_{ij}}+q_{\psi,j,a_{ij}}}{2}\right)^2 \leq K_{a_{ij}}^{flow} \cdot \left({p_{\psi,i}}^2 - {p_{\psi,j}}^2\right) \qquad \forall \psi \in \Psi, \forall a_{ij} \in \mathcal{A}$$

$$m_{\psi,a_{ij}} = K_{a_{ij}}^{lp} \cdot \frac{p_{\psi,i}+p_{\psi,j}}{2} \qquad \forall \psi \in \Psi, \forall a_{ij} \in \mathcal{A}$$

$$m_{\psi',a_{ij}} - m_{\psi,a_{ij}} = q_{\psi,i,a_{ij}} - q_{\psi,j,a_{ij}} \qquad \forall (\psi,\psi') \in \Psi \backslash \{\psi_0\} \times \Psi \,|\psi = {\psi'}_f, \forall a_{ij} \in \mathcal{A}$$

$$m_{\psi,a_{ij}}^{fin} - m_{\psi,a_{ij}} = q_{\psi,i,a_{ij}} - q_{\psi,j,a_{ij}} \qquad \forall \psi \in \Psi \,|\mathrm{T}(\psi) = t_n, \forall a_{ij} \in \mathcal{A}$$

$$m_{\psi_0,a_{ij}} = M_{a_{ij}}^{ini} \qquad \forall a_{ij} \in \mathcal{A}$$

$$\sum\nolimits_{a_{ij}\in\mathcal{A}} m_{\psi,a_{ij}}^{fin} \geq \sum\nolimits_{a_{ij}\in\mathcal{A}} M_{a_{ij}}^{ini} \qquad \forall \psi \in \Psi | T(\psi) = t_n .$$

Market clearing conditions

$$q_{t,i}^{f,ISO} + \sum_{d\in\mathcal{D}/i_d=i} q_t^{f,d} - \sum_{c\in\mathcal{C}/i_c=i} q_t^{f,c} = 0 \qquad (\pi_{t,i}^f) \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N} \tag{A-7}$$

$$q_{t,i}^{lp,ISO} - \sum_{c\in\mathcal{C}/i_c=i} q_t^{lp,c} = 0 \qquad (\pi_{t,i}^{lp}) \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}. \tag{A-8}$$

$$q_{t,i}^{adi} - \sum\nolimits_{d\in\mathcal{D}/i_d=i} q_t^{ad,d} = 0 \qquad (\pi_{t,i}^{adi}) \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}. \tag{A-9}$$

LMP-F_V^{AdI} model: LMP auction with injection volume-adjustment products

This time, the consumers' problems are identical to the LMP-F_V model. Therefore, we only detail the suppliers' and ISO's problems as well as the market clearing conditions of the LMP-F_V^{AdI} model.

The suppliers' optimization problems

$$\max_{q_\omega^d, q_t^{f,d}, q_t^{ad,d}} \sum_{t\in\mathcal{T}} \left[\pi_{t,i_d}^f q_t^{f,d} + \pi_{t,i_d}^{adw} q_t^{ad,d}\right] - \sum_{\omega\in\Omega^d} \left[\theta_\omega \int_0^{q_\omega^d} C^d(y)dy\right] \tag{A-10}$$

s.t. $q_\omega^d = q_{T(\omega)}^{f,d} + R_{\omega,i}^{adv,d}\, q_{T(\omega),i}^{ad,d} \qquad \forall \omega \in \Omega^d$

The ISO's optimization problem

$$\max_{\substack{q_{\psi,i}^{ISO}, q_{t,i}^{f,ISO}, q_{t,i}^{lp,ISO} \\ q_{t,i}^{adi}, q_{\psi,i}^{adiv} \\ q_{\psi,i,a_{ij}}, q_{\psi,j,a_{ij}} \\ p_{\psi,i}, m_{\psi,a_{ij}}, m_{\psi,a_{ij}}^{fin}}} \sum_{i\in\mathcal{N}} \pi_{t,i}^f q_{t,i}^{f,ISO} + \sum_{i\in\mathcal{N}} \pi_{t,i}^{lp} q_{t,i}^{lp,ISO} - \sum_{i\in\mathcal{N}} \pi_{t,i}^{adi} q_{t,i}^{adi} \tag{A-11}$$

s.t. $q_{\psi,i}^{adiv} \leq q_{T(\psi),i}^{adiv} \qquad \forall \psi \in \Psi, \forall i \in \mathcal{N}$

$$q_{\psi,i}^{ISO} = q_{T(\psi),i}^{f,ISO} + R_{\psi,i}^{lpv}\, q_{T(\psi),i}^{lp,ISO} - q_{\psi,i}^{adiv} \qquad \forall \psi \in \Psi, \forall i \in \mathcal{N}$$

$$q_{\psi,i}^{ISO} + \sum_{j\in\mathcal{N}|a_{ij}\in\mathcal{A}} q_{\psi,i,a_{ij}} = \sum_{j\in\mathcal{N}|a_{ji}\in\mathcal{A}} q_{\psi,i,a_{ji}} \qquad \forall\psi \in \Psi, \forall i \in \mathcal{N}$$

$$p_{\psi,i} \geq P_i^{min} \qquad \forall\psi \in \Psi, \forall i \in \mathcal{N}$$

$$p_{\psi,i} \leq P_i^{max} \qquad \forall\psi \in \Psi, \forall i \in \mathcal{N}$$

$$\left(\frac{q_{\psi,i,a_{ij}}+q_{\psi,j,a_{ij}}}{2}\right)^2 \leq K_{a_{ij}}^{flow} \cdot \left({p_{\psi,i}}^2 - {p_{\psi,j}}^2\right) \qquad \forall\psi \in \Psi, \forall a_{ij} \in \mathcal{A}$$

$$m_{\psi,a_{ij}} = K_{a_{ij}}^{lp} \cdot \frac{p_{\psi,i}+p_{\psi,j}}{2} \qquad \forall\psi \in \Psi, \forall a_{ij} \in \mathcal{A}$$

$$m_{\psi\prime,a_{ij}} - m_{\psi,a_{ij}} = q_{\psi,i,a_{ij}} - q_{\psi,j,a_{ij}} \qquad \forall(\psi,\psi') \in \Psi\backslash\{\psi_0\} \times \Psi \,|\psi = {\psi'}_f, \forall a_{ij} \in \mathcal{A}$$

$$m_{\psi,a_{ij}}^{fin} - m_{\psi,a_{ij}} = q_{\psi,i,a_{ij}} - q_{\psi,j,a_{ij}} \qquad \forall\psi \in \Psi \,|\mathrm{T}(\psi) = t_n, \forall a_{ij} \in \mathcal{A}$$

$$m_{\psi_0,a_{ij}} = M_{a_{ij}}^{ini} \qquad \forall a_{ij} \in \mathcal{A}$$

$$\textstyle\sum_{a_{ij}\in\mathcal{A}} m_{\psi,a_{ij}}^{fin} \geq \sum_{a_{ij}\in\mathcal{A}} M_{a_{ij}}^{ini} \qquad \forall\psi \in \Psi|T(\psi) = t_n$$

Market clearing conditions

$$q_{t,i}^{f,ISO} + \sum_{d\in\mathcal{D}/i_d=i} q_t^{f,d} - \sum_{c\in\mathcal{C}/i_c=i} q_t^{f,c} = 0 \qquad (\pi_{t,i}^f) \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N} \tag{A-12}$$

$$q_{t,i}^{lp,ISO} - \sum_{c\in\mathcal{C}/i_c=i} q_t^{lp,c} = 0 \qquad (\pi_{t,i}^{lp}) \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}. \tag{A-13}$$

$$q_{t,i}^{adi} - \sum_{d\in\mathcal{D}/i_d=i} q_t^{ad,d} = 0 \qquad (\pi_{t,i}^{adi}) \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}. \tag{A-14}$$

LMP-F_{2T}^{AdW}: LMP auction with withdrawal 2-periods adjustment products

We first introduce the specific formulations of this model. Since the suppliers' problems are identical to the LMP-F_{2T} model, we then only detail consumers' and ISO's problems as well as the market clearing conditions of the LMP-F_{2T}^{AdW} model.

Product introduction

Consumers can be allowed to purchase withdrawal adjustment products to the ISO. To avoid any unnecessary complexity at this stage, we assume that only the consumers with a deterministic demand function $B^c(\cdot)$, belonging to the subset $\mathcal{C}_D \subset \mathcal{C}$, can offer those services, i.e. consumers who would not see any benefit in purchasing flexibility products themselves[57]. In the following LMP-F_{2T}^{AdW} model where time-flexibility and the corresponding time-adjustment withdrawal products are available, only

[57] To keep our models as clear as possible, we remove the possibility to bid for flexibility products from the problem of those players.

the problems of this consumers subset $\mathcal{C}_D$ and of the ISO differ from the LMP-F2T or LMP-FV models presented above.

The objective function of the consumers $c \in \mathcal{C}_D$ can be reformulated as follows:

$$\max_{q_\chi^c, q_t^{f,c}, q_t^{ad,c}} \sum_{\chi \in \mathrm{X}^c} \left[\theta_\chi \int_0^{q_\chi^c} B^c\,(y)dy\right] - \sum_{t \in \mathcal{T}} \left[\pi_{t,i_c}^{f} q_t^{f,c} + \pi_{t,i_c}^{adw} q_t^{ad,c}\right]$$

$$\text{s.t. } q_{\chi_0}^c = q_{t_0}^{f,c} + R_{\chi_0}^{adt-,c} q_{t_0,i}^{ad,c} \qquad \text{(A-15)}$$

$$q_\chi^c = q_{T(\chi)}^{f,c} + R_{\chi,i}^{adt-,c}\, q_{T(\chi),i}^{ad,c} + R_{\chi_f,i}^{adt+,c}\, q_{T(\chi_f),i}^{ad,c} \qquad \forall \chi \in \mathrm{X}^c | \mathcal{T}(\chi) \neq t_0,\ \mathcal{T}(\chi) \neq t_n$$

$$q_\chi^c = q_{t_n}^{f,c} + R_{\chi_f,i}^{lpt+,c}\, q_{T(\chi_f),i}^{ad,c} \qquad \forall \chi \in \mathrm{X}^c\,|\,\mathcal{T}(\chi) = t_n\,.$$

where the constraints of the problem compute the set of hypothetical quantity withdrawn by the consumer depending on how the ISO activates the quantity of adjustment products it sold $q_{t,i}^{ad,c}$. This is similar to the computation realized by the ISO for its robust network feasibility check of flexibility products. For each type of adjustment product offered, individual consumers define what are the possible activation cases that they need to include in their profit expectation. They should pick a finite set of those possible activations which is representative of the impact of each type of adjustment products on their expected profit. For our 2-periods time-flexibility product, we arbitrarily choose in our numerical cases to consider the two cases when the ISO asks consumers to withdraw all of the gas linked to adjustment products either early or late.

Once the use patterns to be tested have been chosen for each type of adjustment products, a scenario tree X^c is defined by each consumer c to reflect all the combinations of those activations over a gas day. The hypothetical withdrawal schedules to taken into account can then be calculated by adding up the gas purchased through firm products and the gas purchased through adjustment products. In our 2-periods time-flexibility example, the hypothetical quantity of gas q_χ^c withdrawn at scenario node χ is the sum of the gas purchased through firm services for the corresponding period $q_{T(\chi)}^{f,c}$, and of the gas hypothetically coming from the early activation of adjustment services for this period $R_{\chi,i}^{adt-,c}\, q_{T(\chi),i}^{ad,c}$ or from late activation of adjustment services of the previous period $R_{\chi_f,i}^{adt+,c}\, q_{T(\chi_f),i}^{ad,c}$. Parameters $R_{\chi,i}^{adt-,c}$ and $R_{\chi,i}^{adt+,c}$ can be either equal to zero or unity, to reflect whether the ISO requires to withdraw all of the adjustment gas early ($R_{\chi,i}^{adt-,c} = 1$) or late ($R_{\chi,i}^{adt+,c} = 1$). The objective function is the expected surplus of the consumer, which is the expected benefit of gas consumption minus the cost of purchasing firm gas, added up to the cost of purchasing withdrawal adjustment products at price π_{t,i_c}^{adw}.

The program of the ISO must also be reformulated to account for the sale and activation of withdrawal adjustment products. Its objective function is now the sum of the costs and revenues from purchasing and selling firm gas and of the revenues from selling of flexibility and withdrawal adjustment products:

$$\sum_{i\in\mathcal{N}} \pi_{t,i}^{f} q_{t,i}^{f,ISO} + \sum_{i\in\mathcal{N}} \pi_{t,i}^{lp} q_{t,i}^{lp,ISO} + \sum_{i\in\mathcal{N}} \pi_{t,i}^{adw} q_{t,i}^{adw} \tag{A-16}$$

where $q_{t,i}^{adw}$ is the total quantity of gas sold through withdrawal adjustment products at a node. The constraints of the problem remain the same, except for the three constraints used to compute the hypothetical quantities injected and withdrawn at each node, that must be replaced by the following ones:

$$\begin{aligned}
&q_{\psi,i}^{adwt-} + q_{\psi,i}^{adwt+} = q_{T(\psi),i}^{adw} && \forall\psi \in \Psi |\, \mathcal{T}(\psi) \neq t_n, \forall i \in \mathcal{N} \\
&q_{\psi_0,i}^{ISO} = q_{t_0,i}^{f,ISO} + R_{\psi_0}^{lpt-}\, q_{t_0,i}^{lp,ISO} + q_{\psi_0,i}^{adwt-} && \forall i \in \mathcal{N} \\
&q_{\psi,i}^{ISO} = q_{T(\psi),i}^{f,ISO} + R_{\psi,i}^{lpt-}\, q_{T(\psi),i}^{lp,ISO} + R_{\psi_f,i}^{lpt+}\, q_{T(\psi_f),i}^{lp,ISO} + && \forall\psi \in \Psi |\, \mathcal{T}(\psi) \neq t_0,\, \mathcal{T}(\psi) \neq \\
&q_{\psi,i}^{adwt-} + q_{\psi_f,i}^{adwt+} && t_n\,, \forall i \in \mathcal{N} \\
&q_{\psi,i}^{ISO} = q_{T(\psi),i}^{f,ISO} + R_{\psi_f,i}^{lpt+}\, q_{T(\psi_f),i}^{lp,ISO} + q_{\psi_f,i}^{adwt+} && \forall\psi \in \Psi |\, \mathcal{T}(\psi) = t_n\,, \forall i \in \mathcal{N}\,.
\end{aligned} \tag{A-17}$$

These constraints describe decisions similar to those faced by consumers purchasing flexibility products:

- The first one states the rules for the activation of a quantity of gas sold through adjustment products by the ISO $q_{T(\psi),i}^{adwt}$ for the period corresponding to a given node ψ. When uncertainty is lifted for node ψ and based on the use of flexibility products by some consumers, the ISO must decide which fraction of the gas $q_{\psi,i}^{adwt-}$ it wishes to ask other consumers to withdraw early, directly at node ψ, and which fraction $q_{\psi,i}^{adwt+}$ it wishes to ask other consumers to withdraw later, during the following period.
- The remaining constraints just compute the total quantity of gas $q_{\psi,i}^{ISO}$ hypothetically injected or withdrawn at each network and for each scenario node ψ. The contributions of firm products and flexible products are calculated as in the simple LMP-F_{2T} model, and added up to the gas linked to early activation of adjustment products for this period $q_{\psi,i}^{adwt-}$ or from the late activation of the adjustment products of the previous period $q_{\psi_f,i}^{adwt+}$.

Finally, in our equilibrium representation, a market clearing condition must be added to account for the trade of withdrawal adjustment products:

$$q_{t,i}^{adw} - \sum_{c\in\mathcal{C}_D/i_c=i} q_t^{ad,c} = 0 \qquad (\pi_{t,i}^{adw}) \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}. \tag{A-18}$$

Complete model

Suppliers' problems are identical to that of the LMP-F$_{2T}$ model, as well as the problems of consumers with uncertain demand $c \in \mathcal{C} \setminus \mathcal{C}_D$.

As presented above, the optimization problems of consumers with deterministic demand $c \in \mathcal{C}_D$ are formulated as follows:

$$\max_{q_\chi^c, q_t^{f,c}, q_t^{ad,c}} \sum_{\chi \in \mathrm{X}^c} \left[\theta_\chi \int_0^{q_\chi^c} B^c\,(y)dy\right] - \sum_{t \in \mathcal{T}} \left[\pi_{t,i_c}^f q_t^{f,c} + \pi_{t,i_c}^{adw} q_t^{ad,c}\right]$$

$$\begin{aligned}
\text{s.t. } & q_{\chi_0}^c = q_{t_0}^{f,c} + R_{\chi_0}^{adt-,c} q_{t_0,i}^{ad,c} && \\
& q_\chi^c = q_{\mathcal{T}(\chi)}^{f,c} + R_{\chi,i}^{adt-,c}\, q_{\mathcal{T}(\chi),i}^{ad,c} + R_{\chi_f,i}^{adt+,c}\, q_{\mathcal{T}(\chi_f),i}^{ad,c} && \forall \chi \in \mathrm{X}^c | \mathcal{T}(\chi) \neq t_0,\ \mathcal{T}(\chi) \neq t_n \\
& q_\chi^c = q_{t_n}^{f,c} + R_{\chi_f,i}^{lpt+,c}\, q_{\mathcal{T}(\chi_f),i}^{ad,c} && \forall \chi \in \mathrm{X}^c\, | \mathcal{T}(\chi) = t_n\,.
\end{aligned} \tag{A-19}$$

The optimization problem of the ISO is:

$$\max_{\substack{q_{\psi,i}^{ISO}, q_{t,i}^{f,ISO}, q_{t,i}^{lp,ISO} \\ q_{t,i}^{adw}, q_{\psi,i}^{adwt-}, q_{\psi,i}^{adwt+} \\ q_{\psi,i,a_{ij}}, q_{\psi,j,a_{ij}} \\ p_{\psi,i}, m_{\psi,a_{ij}}, m_{\psi,a_{ij}}^{fin}}} \sum_{i \in \mathcal{N}} \pi_{t,i}^f q_{t,i}^{f,ISO} + \sum_{i \in \mathcal{N}} \pi_{t,i}^{lp} q_{t,i}^{lp,ISO} - \sum_{i \in \mathcal{N}} \pi_{t,i}^{adw} q_{t,i}^{adw}$$

$$\begin{aligned}
\text{s.t. } & q_{\psi,i}^{adwt-} + q_{\psi,i}^{adwt+} = q_{\mathcal{T}(\psi),i}^{adw} && \forall \psi \in \Psi | \mathcal{T}(\psi) \neq t_n, \forall i \in \mathcal{N} \\
& q_{\psi_0,i}^{ISO} = q_{t_0,i}^{f,ISO} + R_{\psi_0}^{lpt-} q_{t_0,i}^{lp,ISO} + q_{\psi_0,i}^{adwt-} && \forall i \in \mathcal{N} \\
& q_{\psi,i}^{ISO} = q_{\mathcal{T}(\psi),i}^{f,ISO} + R_{\psi,i}^{lpt-}\, q_{\mathcal{T}(\psi),i}^{lp,ISO} + && \forall \psi \in \Psi | \mathcal{T}(\psi) \neq t_0,\ \mathcal{T}(\psi) \neq \\
& R_{\psi_f,i}^{lpt+}\, q_{\mathcal{T}(\psi_f),i}^{lp,ISO} + q_{\psi,i}^{adwt-} + q_{\psi_f,i}^{adwt+} && t_n\,, \forall i \in \mathcal{N} \\
& q_{\psi,i}^{ISO} = q_{\mathcal{T}(\psi),i}^{f,ISO} + R_{\psi_f,i}^{lpt+}\, q_{\mathcal{T}(\psi_f),i}^{lp,ISO} + q_{\psi_f,i}^{adwt+} && \forall \psi \in \Psi | \mathcal{T}(\psi) = t_n\,, \forall i \in \mathcal{N}\,. \\
& q_{\psi,i}^{ISO} + \sum_{j \in \mathcal{N} | a_{ij} \in \mathcal{A}} q_{\psi,i,a_{ij}} = \sum_{j \in \mathcal{N} | a_{ji} \in \mathcal{A}} q_{\psi,i,a_{ji}} && \forall \psi \in \Psi\,, \forall i \in \mathcal{N} \\
& p_{\psi,i} \geq P_i^{min} && \forall \psi \in \Psi\,, \forall i \in \mathcal{N} \\
& p_{\psi,i} \leq P_i^{max} && \forall \psi \in \Psi\,, \forall i \in \mathcal{N} \\
& \left(\frac{q_{\psi,i,a_{ij}} + q_{\psi,j,a_{ij}}}{2}\right)^2 \leq K_{a_{ij}}^{flow} \cdot \left({p_{\psi,i}}^2 - {p_{\psi,j}}^2\right) && \forall \psi \in \Psi\,, \forall a_{ij} \in \mathcal{A} \\
& m_{\psi,a_{ij}} = K_{a_{ij}}^{lp} \cdot \frac{p_{\psi,i} + p_{\psi,j}}{2} && \forall \psi \in \Psi\,,\ \forall a_{ij} \in \mathcal{A} \\
& m_{\psi',a_{ij}} - m_{\psi,a_{ij}} = q_{\psi,i,a_{ij}} - q_{\psi,j,a_{ij}} && \forall (\psi, \psi') \in \Psi \backslash \{\psi_0\} \times \Psi\, | \psi = {\psi'}_f\,,\ \forall a_{ij} \in \mathcal{A}
\end{aligned} \tag{A-20}$$

$$m_{\psi,a_{ij}}^{fin} - m_{\psi,a_{ij}} = q_{\psi,i,a_{ij}} - q_{\psi,j,a_{ij}} \qquad \forall \psi \in \Psi \,|\mathrm{T}(\psi) = t_n , \forall a_{ij} \in \mathcal{A}$$

$$m_{\psi_0,a_{ij}} = M_{a_{ij}}^{ini} \qquad \forall a_{ij} \in \mathcal{A}$$

$$\sum_{a_{ij}\in\mathcal{A}} m_{\psi,a_{ij}}^{fin} \geq \sum_{a_{ij}\in\mathcal{A}} M_{a_{ij}}^{ini} \qquad \forall \psi \in \Psi|T(\psi) = t_n \, .$$

The market clearing conditions can be stated as follows:

$$q_{t,i}^{f,ISO} + \sum_{d\in\mathcal{D}/i_d=i} q_t^{f,d} - \sum_{c\in\mathcal{C}/i_c=i} q_t^{f,c} = 0 \qquad \left(\pi_{t,i}^{f}\right) \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N} \tag{A-21}$$

$$q_{t,i}^{lp,ISO} - \sum_{c\in\mathcal{C}\backslash\mathcal{C}_D/i_c=i} q_t^{lp,c} = 0 \qquad \left(\pi_{t,i}^{lp}\right) \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}. \tag{A-22}$$

$$q_{t,i}^{adw} - \sum_{c\in\mathcal{C}_D/i_c=i} q_t^{ad,c} = 0 \qquad \left(\pi_{t,i}^{adw}\right) \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}. \tag{A-23}$$

Appendix B – Numerical case studies

Initial pipeline state

We assume that the initial pipeline state is the result of a steady-state operation for the following injection and withdrawals:

$$q^{d_1} = 0.08333\ \mathrm{MSCM/h}$$

$$q^{c_1} = q^{c_2} = q^{d_1}/2 = 0.04166\ \mathrm{MSCM/h}$$

$$p_{i_3} = 53\ \mathrm{bar}$$

Pressure limits

Maximum and minimum pressures are assumed to be identical for all nodes. There values were inspired by (De Wolf and Smeers 2000) for the Belgian interconnection point with France Blaregnies:

$$P_i^{max} = 66.2\ \mathrm{bar} \qquad \forall i \in \mathcal{N} \tag{B-1}$$

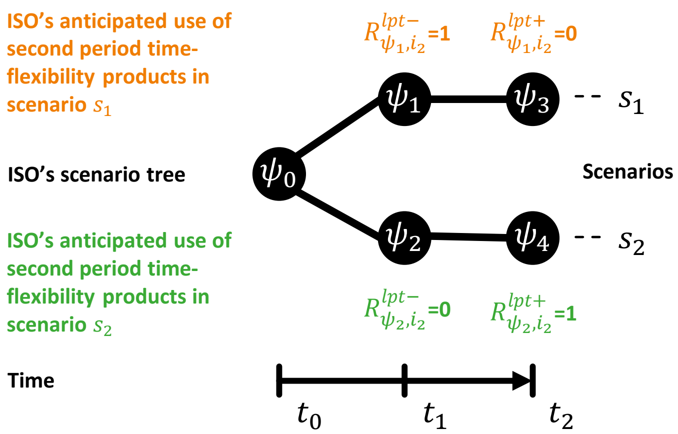
$$P_i^{min} = 50\ \mathrm{bar} \qquad \forall i \in \mathcal{N} \tag{B-2}$$

Scenario trees of ISO's anticipation of flexibility-products use

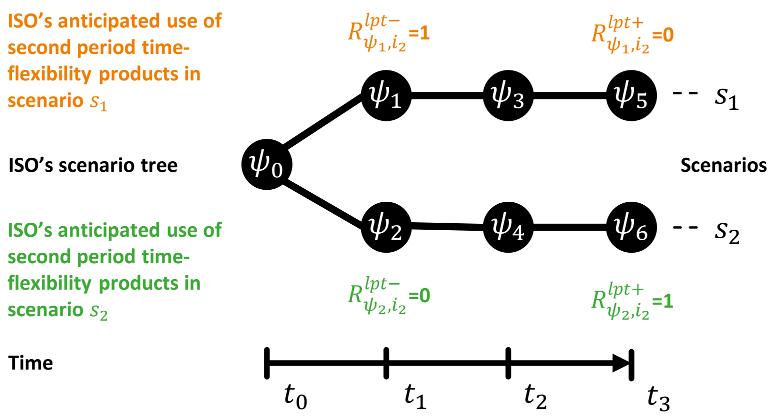
In the following figures, we present the scenario trees used by the ISO to take into account the possible uses of flexibility products by consumer c_1 in its hypothetical dispatches are presented. The value of the anticipation parameters appears above and under each node.

Figure B-1 : ISO's anticipation scenario trees and corresponding parameters for each test case

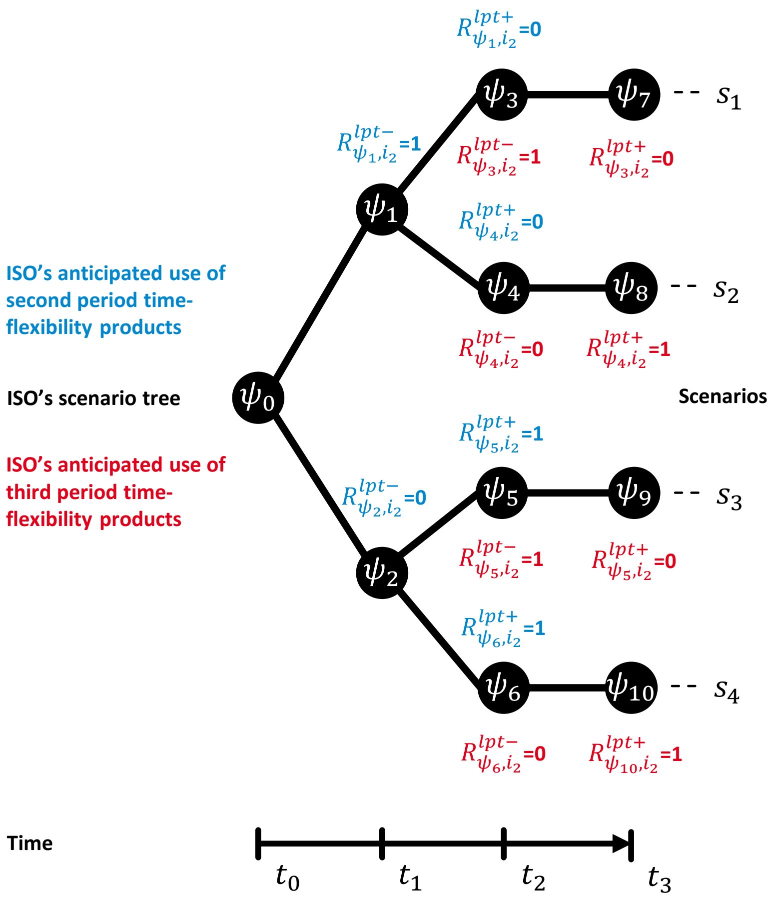
2-periods time flexibility products in a 3-periods test case



3-periods time-flexibility products in a 4-periods test case

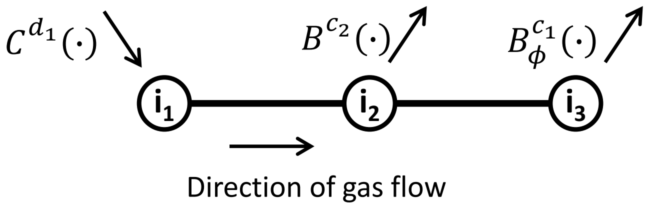


2-periods time-flexibility products in a 4-periods test case



Other network cases

Figure B-2 : Pipe topology and location of network users when consumers c_1 and c_2 are flipped.



Appendix C – Constants

Flow equation

The constant used in our Weymouth-based transient flow representation for gas flow measured in MSCM/h (million standard cubic meters per hour) is expressed as follows:

$$K_{a_{ij}}^{flow} = 1.66797 \cdot 10^{-16} \cdot 10^{18} \frac{1}{\lambda_{a_{ij}}} \cdot \frac{{D_{a_{ij}}}^5}{GTZL_{a_{ij}}} \tag{C-13}$$

where the friction coefficient $\lambda_{a_{ij}}$ is:

$$\frac{1}{\lambda_{a_{ij}}} = \left[2 \log\left(3.7 \frac{D_{a_{ij}}}{\varepsilon}\right)\right]^2 \quad .$$

$L_{a_{ij}}$ is the length of the pipe in meters, $D_{a_{ij}}$ its diameter in meters, and the other parameters take the following values chosen identical to (De Wolf and Smeers 2000):

- $G = 0.6106$ is the specific gravity of gas,
- $T = 281.15$ K is the assumed mean flow temperature,
- $Z = 0.8$ (dimensionless) is the gas compressibility factor at the mean flow temperature,
- $\varepsilon = 5 \cdot 10^{-5}$ m is the pipe rugosity.

Linepack equation

The constant used in the linepack equation for linepack expressed in MSCM (million standard cubic meters) (Pepper et al. 2012) is detailed thereafter:

$$K_{a_{ij}}^{lp} = 1.316 \cdot 10^{-4} \frac{\pi \left(D_{a_{ij}}/2\right)^2 L_{a_{ij}}}{GZR} \tag{C-14}$$

where $L_{a_{ij}}$ is the length of the pipe in meters, $D_{a_{ij}}$ its diameter in meters, and the other parameters take the following values:

- $G = 0.6106$ is the specific gravity of gas (dimensionless),
- $Z = 0.8$ (dimensionless) is the gas compressibility factor at the mean flow temperature,
- $R = 0.5183$ J K^{-1} g^{-1} is the ideal gas constant.

Concluding remarks

All these chapters converge toward the need to use economic modelling in combination with technical representations of the gas pipeline network to address the economic challenges at stake in this industry. However, given the complexity of the resulting models, Chapter IV and Chapter V also call for more research toward applying these models to large scale, realistic networks and markets. Finally, they show that regarding regulation, it is important to strike the right balance between the theoretical optimum and implementation realities.

Chapter VII. Résumé en français

VII.1 Contexte

En tant que combustible fossile peu émetteur de CO_2, le gaz est souvent présenté comment un élément essentiel à transition énergétique réussie. Tandis que cela semble permis par ses ressources abondantes, celles-ci sont toutefois éloignées des principaux sites de consommations. Les champs de production russes, norvégiens ou algériens, entre autres, sont situées à des milliers de kilomètres des régions européennes qui constituent leur principal débouché. Par conséquent, il est souvent nécessaire d'organiser le transport du gaz naturel sur de longues distances, soit sous sa forme gazeuse à travers des gazoducs, soit sous une forme liquéfiée (GNL) à l'aide de méthaniers. Quand les volumes à transporter sont récurrents et importants, et doivent l'être sur des distances moyennes, le transport par gazoduc reste la solution la plus efficace.

De telles infrastructures présentent cependant une forte intensité capitalistique. La construction d'un nouveau pipeline représente souvent un investissement de l'ordre de plusieurs milliards d'euros. D'autre part, une fois construite, ces installations offrent une capacité de transport plus ou moins invariable. Dès le développement du projet, le diamètre du tuyau envisagé défini presque totalement sa future capacité maximale. Cette irréversibilité est à l'origine d'un certain nombre de défis auxquels doit faire face le système gazier lorsqu'il est soumis à une demande variable.

Toutes les commodités énergétiques sont exposées à un certain degré d'incertitude sur la demande, et le gaz naturel ne fait à ce titre pas exception. A long terme, la croissance économique et ses cycles ont une incidence sur les volumes liés à la consommation industrielle. A moyen terme, des phénomènes incertains, tels que les températures saisonnières, influencent quant à eux la consommation de gaz naturel des particuliers, notamment en ce qui concerne ses usages de chauffage domestique. A court terme enfin, les fluctuations météorologiques peuvent entrainer des fluctuations locales. La diversité des utilisateurs de gaz naturel, des installations industrielles aux centrales thermiques de production d'électricité, en passant par les particuliers et les professionnels à faible consommation, accentue encore cette incertitude de la demande.

Par ailleurs, la transition énergétique du secteur électrique se caractérise aujourd'hui par une introduction massive de renouvelables, par exemple en Chine ou encore en Europe afin de satisfaire les objectifs politiques pris dans le cadre de l'Union Européenne. Les renouvelables intermittents, comme les éoliennes et les panneaux solaires représente déjà une part non négligeable de la production et cette part sera selon toute vraisemblance amenée à croitre encore dans un futur proche. Ceci fait apparaitre d'importants défis pour assurer l'équilibre du système électrique. Pour diverses raisons, l'intermittence des renouvelables sera principalement compensée par la production des centrales thermiques conventionnelles, et en particulier à l'aide de cycles combinés gaz, du fait de leur grande flexibilité, mais aussi de leurs émissions en CO_2 plus limitées et de leur coût d'investissement modéré. Ainsi, on peut s'attendre à l'augmentation du couplage entre les systèmes électriques et gaziers au fur et à mesure que les renouvelables se développeront. Ceci entraine d'ores et déjà le transfert d'une part de cette incertitude vers le système gazier.

La combinaison de ces différents facteurs introduit un certain nombre de difficultés à tous les stages de développement et d'exploitation d'un réseau de gaz naturel. Lors de la conception d'abord, définir la taille d'un gazoduc est délicat. Les volumes transportés sont susceptibles d'augmenter au cours de sa phase d'exploitation (par exemple lorsqu'il aurait été construit pour acheminer de nouvelles ressources de gaz naturel), ou de décroitre (du fait de substitution entre énergies). Après la mise en service ensuite, le réseau doit être partagé entre tous ses utilisateurs. Alors que cette tâche est déjà complexe techniquement pour un opérateur intégré, elle devient un défi économique dans le contexte d'un marché libéralisé. Dans le second cas, les utilisateurs doivent pouvoir exprimer leurs besoins de transport d'une manière claire et en coordination avec les transactions économiques sur les marchés du gaz.

Les services offerts par les infrastructures de transport de gaz sont toutefois multiples et indissociables. Le marché ou l'opérateur intégré se doit d'attribuer ces différents services, comme le transport du gaz ou son stockage au sein du réseau. Du fait des mécanismes physiques régissant les flux gaziers, les possibilités d'injection et de soutirage sont spatialement et temporellement liées à travers le réseau. Des décisions prises à un certain moment peuvent impacter l'ensemble des utilisateurs pour les heures qui suivent. Ceci impose d'allouer les différents services offerts par le réseau de manière coordonnée, ce qui se fait naturellement dans le cas d'un opérateur intégré, mais devient bien moins aisé dans un contexte de marché.

Dès lors, il parait essentiel d'analyser ces questions d'attribution de capacité, d'investissement et d'organisation de marché à l'aide de modèles représentant la gestion du réseau et les décisions économiques de façon combinée. C'est l'objet de cette thèse, pour une sélection de problématiques précises appartenant à ce vaste champ de recherche. Ces questions concernent tous les acteurs du système gazier, des producteurs de gaz aux fournisseurs d'électricité, des gestionnaires de réseau aux consommateurs particuliers, en passant par les agences de régulations et de développement.

Les deux premiers chapitres adoptent une perspective de long terme : on cherche à évaluer l'efficacité de la réglementation du taux de rendement lorsqu'il s'agit d'inciter à la réalisation de projets d'infrastructures gazières dans des pays en développement. Une première contribution analytique présente le développement d'une représentation simplifiée du réseau de transport de gaz, de forme Cobb-Douglas. Inspiré par les projets d'acheminement de gaz naturel au Mozambique, celle-ci est ensuite utilisée pour évaluer dans quelles conditions il est possible pour une autorité de régulation de choisir un taux de rendement régulé qui améliore l'efficacité du système dans le cas où la demande réelle serait plus importante que la demande anticipée par la firme régulée.

A moyen terme ensuite, l'efficacité face à une demande de plus en plus variable de la structure tarifaire actuelle dite « entrée-sortie » pour l'accès au réseau européen est évaluée. Après avoir démontré l'existence d'inefficacités dans un tel système, celles-ci sont évaluées numériquement.

Enfin, la dernière contribution explore la possibilité d'offrir directement la flexibilité du réseau de transport de gaz à ses utilisateurs, dans le cadre d'enchères et du système de prix nodaux. Après avoir souligné la complexité d'un tel mécanisme, les limites à son efficacité sont présentées. A chaque fois, l'analyse repose sur la modélisation simultanée du réseau de transport de gaz (en régime statique ou transitoire) et des mécanismes économiques en jeu.

VII.2 Revue de littérature consacrée à la formulation des problèmes de transport par gazoduc

Ce premier chapitre offre une brève revue de la littérature consacrée à la formulation de problèmes de transport par gazoduc, ainsi qu'à ses applications en recherche opérationnelle et au sein de modèles économiques. Après un rappel des équations physique décrivant le transport de gaz par pipeline (Menon 2005), cette revue est organisée autour des trois types de représentation possibles (Brouwer et al. 2011).

La première consiste à représenter chaque tuyau comme un moyen de transport d'un capacité fixe. Cette représentation très simple est notamment utilisée dans des modèles économiques visant à représenter les marchés gaziers à grande échelle (Egging et al. 2008, Gabriel et al. 2005). Dans les cas où il s'avère nécessaire de prendre en compte des flux gaziers plus réalistes, une représentation en régime stationnaire peut être utilisée. Celle-ci suppose que les flux n'évoluent pas au cours du temps, et est utilisée aussi bien pour résoudre des problèmes techniques majeurs (Koch et al. 2015) ou pour analyser des questions économiques (Babonneau et al. 2012, De Wolf and Smeers 1993, Fodstad et al. 2015).

Pour prendre en compte les effets du stockage de gaz en conduite enfin, une représentation en régime transitoire est nécessaire. Plus complexe, celle-ci a été utilisée afin d'optimiser la gestion du réseau (Mahlke et al. 2007, Moritz 2007). Sous une forme simplifiée, elle peut également être intégrée au sein de modèles économiques (Midthun et al. 2009) destinés à analyser par exemple les modalités de la régulation européenne concernant l'équilibrage du réseau (Keyaerts et al. 2011), l'organisation du marché via des systèmes de prix nodaux (Read et al. 2012) ou l'impact des renouvelables intermittents électriques sur le réseau gazier (Qadrdan et al. 2010).

VII.3 La technologie du gazoduc, éléments destinés à la compréhension de la structure de coût et à la régulation du taux de rendement

Ce chapitre vise à établir une caractérisation microéconomique des relations physiques entre les niveaux d'intrants et les niveaux de production à l'œuvre dans un réseau de transport de gaz simple de type point à point. Celle-ci est ensuite utilisée afin de contribuer aux débats de politique publique liés à la régulation des réseaux de transport de gaz naturel.

On montre dans un premier temps qu'il est possible d'approximer les équations techniques du transport de gaz par pipeline. En supposant que les pressions à l'entrée et à la sortie d'un pipeline sont identiques, et que l'augmentation de pression due au compresseur à l'entrée de ce pipeline est limitée, il est possible de combiner les équations de flux en régime stationnaire de type Weymouth et une représentation de l'énergie consommée par les compresseurs en une fonction de production :

$$q_{a_{ij}} = \sqrt[3]{\frac{2(C_2 \cdot P_0)^2}{C_1 \cdot b \cdot L_{a_{ij}}}} {d_{a_{ij}}}^{16/9} {h_{a_{ij}}}^{1/3} \quad . \tag{R-1}$$

Celle-ci exprime le flux traversant le gazoduc en fonction de son diamètre et de la puissance utilisée par les compresseurs. En associant grossièrement le capital requis pour construire un pipeline à la masse de métal utilisée pour fabriquer les tubes qui le constituent, cette fonction de production peut être reformuler pour s'exprimer plus classiquement en fonction de deux intrants économiques que sont le capital requis et l'énergie :

$${q_{t,a_{ij},s}}^{H} = B_{a_{ij}} \cdot {k_{a_{ij}}}^{G} \cdot {e_{t,a_{ij},s}}^{1-G} \quad . \tag{R-2}$$

Cette fonction de production est de type Cobb-Douglas, et dans le cas du transport de gaz naturel, les coefficients G et H sont respectivement égaux à 8/11 et 9/11, et est plus adaptée à une application au sein de modèles économiques que celles présentées dans (Chenery 1949, Yépez 2008).

A l'aide de celle-ci, on montre dans un premier temps que la fonction de coût de long terme de ce type d'infrastructure vérifie les conditions pour caractériser cette industrie comme un monopole naturel. Dans un second temps, la fonction de coût de court terme est analysée. On peut ainsi souligner qu'une tarification au coût marginal du réseau de transport de gaz pourrait entrainer une impossibilité de recouvrer les coûts de réseau.

Enfin, cette fonction de production est introduite au sein d'un modèle classique de la régulation du taux de rendement (Callen et al. 1976, Klevorick 1971). Ce modèle bi-niveau fait intervenir une autorité de régulation qui fixe un taux de rendement maximal autorisé en anticipant la réaction d'une firme ensuite libre de fixer ses prix tant que le plafonnement de son taux de rendement est vérifié. Deux résultats importants peuvent être obtenus dans le cas du gaz naturel.

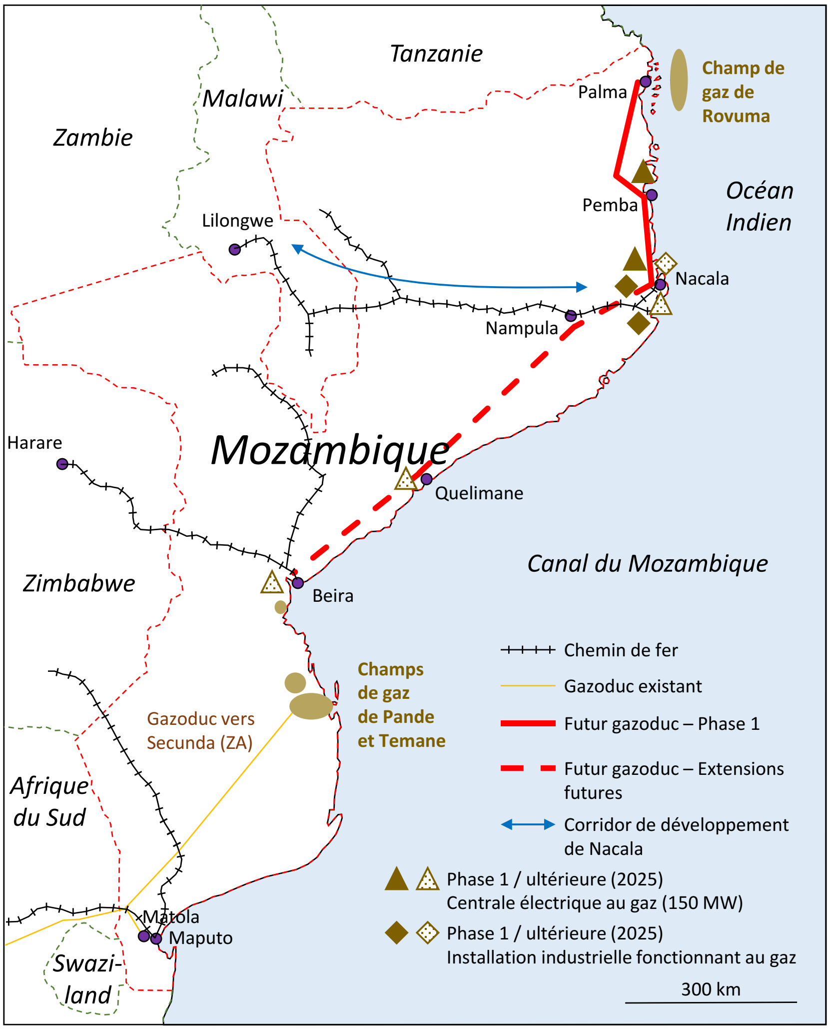
D’abord et de façon contre-intuitive, pour une certaine gamme d’élasticité de la demande, il n’est pas optimal de choisir un taux de rendement aussi proche du coût du capital que possible. Il peut être bénéfique d’octroyer à la firme un taux de rendement plus important. Ensuite, le taux de rendement optimal est borné. En s’appuyant sur des applications numériques réalisées pour différentes valeurs d’élasticité de la demande, on observe ensuite que cet outil de régulation permet de réduire de façon importante la rente de monopole, malgré son effet de surcapitalisation (Averch and Johnson 1962). On peut montrer enfin que la régulation du taux de rendement reste un outil efficace, même lorsque l’autorité de régulation choisi simplement le taux de rendement autorisé dans cet intervalle.

Ces travaux ont été réalisés en collaboration avec Olivier Massol. Ils ont été présentés au workshop annuel de la section étudiante de l’association française des économistes de l’énergie (FAEE) en Novembre 2016 et ont été soumis à la revue Utilities Policy sous forme de courte communication.

VII.4 De la bonne utilisation de la réglementation du taux de rendement pour inciter à l’investissement : le cas des infrastructures de transport de gaz dans des pays en développement.

Ce quatrième chapitre adopte une perspective de long terme en ce qui concerne l’incertitude de la demande. Il vise à évaluer l’efficacité de la réglementation du taux rendement lors de la réalisation de projets d’infrastructures gazières dans des pays en développement, en prenant en compte de potentielles évolutions de la demande après la mise en service. Cette contribution est inspirée par le cas du Mozambique, dont les importants gisements gaziers récemment découverts au nord pourraient servir non seulement à générer des revenus via l’export de gaz naturel liquéfié, mais aussi à développer l’économie locale. Dans le second cas, un gazoduc serait nécessaire pour acheminer le gaz des régions peu peuplées du Nord vers des villes de plus grande importance un peu plus au Sud, comme présenté en Figure 1.

Figure 1. Carte du projet de gazoduc mozambicain.



Afin de justifier la construction de telles infrastructures alors que la consommation de gaz naturel est pour le moment inexistante dans ces régions du Nord, les projets de développement envisagés prévoient de s'appuyer dans un premier temps sur quelques méga-projets industriels (ICF International 2012). La présence d'une source de gaz naturel pourrait ensuite permettre l'émergence d'une consommation domestique.

Du fait de l'irréversibilité et de la forte intensité capitalistique des infrastructures de transport de gaz par pipeline, il est plus efficace d'installer un pipeline surdimensionné initialement, afin d'absorber la future hausse de la demande attendue (Chenery 1952). Cependant, du fait d'un manque de capital disponible, le financement de tels projets est susceptible d'être réalisé via des sociétés privées étrangères, qu'il faudra convaincre de leur viabilité. Celles-ci souhaiteront probablement choisir un pipeline d'une capacité plus limitée, adaptée à des estimations de demande conservatrices liées uniquement aux méga-projets attendus.

En raison du caractère de monopole naturel de cette technologie, un cadre de régulation devra par ailleurs être adopté afin de limiter la rente de monopole. Cette régulation devra également être adaptée au contexte des pays en développement et présenter un mécanisme simple (Joskow 1999). Dans ce cadre, la réglementation du taux de rendement parait convenir. Celle-ci est connue notamment à travers l'un de ces biais nommé effet Averch-Johnson (Averch and Johnson 1962). Elle tendrait à favoriser l'utilisation du capital au détriment du travail dans les industries qui y sont soumises, déviant ainsi de l'utilisation optimale des intrants. On examine dans ce chapitre l'opportunité de tirer parti de ce possible biais afin d'inciter la firme privée à choisir un surdimensionnement initial de l'infrastructure afin de prendre en compte le développement de la demande attendu par les autorités.

VII.4.1 Méthodologie

La représentation simplifiée présentée dans le chapitre précédent est à nouveau utilisée au sein d'un modèle bi-niveau mettant en scène le couple autorité de régulation et entreprise régulée. On s'intéresse dans un premier temps à la situation *ex ante* où la firme régulée dimensionne l'infrastructure en se basant sur des estimations de demande conservatrices et en prenant en compte un taux de rendement attribué par l'autorité de régulation. La réaction de cette firme à une augmentation de la demande *ex post* est ensuite calculée lorsque le taux de rendement autorisé reste fixé à son niveau initial. Ces décisions sont ensuite comparées aux choix d'intrants (capital et énergie) qui seraient optimaux pour répondre à la demande *ex post*.

VII.4.2 Principaux résultats

On montre analytiquement qu'il est possible pour un régulateur de choisir un taux de rendement autorisé qui induit *ex ante* l'installation par la firme du capital optimal pour répondre à la demande *ex post*. Ceci n'est cependant possible que lorsque l'augmentation de la demande ne dépasse pas une certaine valeur seuil. A l'aide d'applications numériques on montre que le facteur d'augmentation de la demande doit rester relativement limité pour que cela soit possible dans le cas de valeurs d'élasticité de la demande réalistes pour le gaz naturel.

Enfin, étant donné que l'installation d'un pipeline surdimensionné pourrait induire une réduction du bien-être social *ex ante*, on discute de l'opportunité d'une telle stratégie. On prouve analytiquement que la valeur du facteur d'augmentation de la demande pour laquelle son application reste acceptable *ex ante* et permet d'obtenir une situation *ex post* optimale est bornée. Des applications numériques dans le cas du gaz naturel renseignent enfin sur l'étroitesse de cet intervalle pour des valeurs réalistes d'élasticité de la demande.

Ces travaux ont été réalisés en collaboration avec Olivier Massol. Ils ont été présentés à la 10ème et à la 11ème Annual Trans-Atlantic Infraday conference à Washington en Octobre 2015 et 2016, à la conférence INFORMS 2015 à Philadelphie et à la conférence annuelle de la section étudiante de l'association française des économistes de l'énergie (FAEE) en Novembre 2016.

VII.5 Evaluation de l'efficacité du système de tarification entrée-sortie européen

Le système de tarification entrée-sortie européen repose en partie sur des tarifs fixés à l'avance par le gestionnaire de réseau de transport. Il se fonde également sur un découplage entre les flux physiques et les transactions économiques.

Le marché gazier européen est divisé en larges zones (correspondant aux frontières nationales pour la plupart). D'un point de vue physique, afin de pouvoir soutirer du gaz à l'un des points du réseau à l'intérieur, le système entrée-sortie n'impose qu'une seule contrainte : acheter une capacité de sortie adaptée au volume et au moment souhaité. De même, afin de faire entrer du gaz à l'intérieur de cette zone, il suffit d'acquérir une capacité d'entrée appropriée. Ces obligations sont totalement indépendantes des transactions commerciales concernant la commodité gaz. Une place de marché virtuelle permet l'échanger au sein de chaque zone, sans qu'il soit nécessaire de posséder les capacités d'entrée ou de sortie correspondantes. Ainsi, en diminuant la spécificité des produits échangés et en limitant l'impact des aspects techniques d'allocation de réseau pour les participants, le marché est rendu plus liquide.

Du point de vue de l'opérateur de réseau cependant, ce système oblige à vérifier que chaque capacité d'entrée ou de sortie mise sur le marché pourra être honorée. Du fait des liens spatiaux et intertemporels forts résultants de la physique des flux gaziers, l'opérateur de réseau peut être contraint à réduire les capacités offertes afin de s'assurer de la faisabilité de chaque combinaison possible lors de leur utilisation.

D'autre part, la commercialisation de ces capacités est effectuée à travers des enchères, dont le prix plancher, appelé tarif de capacité, est fixé par l'opérateur de réseau. Bien que la législation européenne permette à ce prix de différer pour chaque entrée ou sortie, elle prévoit également qu'il soit fixé pour des périodes d'un an minimum (European Commission 2017). Ainsi, les incitations reçues par les utilisateurs quant aux coûts du réseau sont limitées. Ce découplage pourrait générer des inefficacités, à court terme en ce qui concerne les coûts variables de gestion du réseau (notamment le coût de l'énergie consommée pour faire fonctionner les compresseurs) et à long terme, en induisant des choix de réseau fondés sur des observations de flux non-optimaux (Hallack and Vazquez 2013, Hunt 2008, Rious and Hallack 2009). Dans le cas d'un système gazier fortement sollicité, par exemple pour répondre aux besoins variables de production d'électricité, ceci est d'autant plus probable.

Ainsi, ce système privilégie la liquidité des marchés à l'efficacité de l'utilisation des infrastructures. La littérature économique s'est jusqu'à présent naturellement tournée vers les aspects de fonctionnement des marchés tels que la liquidité (Miriello and Polo 2015), la sécurité d'approvisionnement (Abada and Massol 2011, Chaton et al. 2009), l'exercice de pouvoir de marché (Egging et al. 2008, Gabriel et al. 2005), ou les contrats de long terme (Abada et al. 2017, Creti and

Villeneuve 2004). Cependant il est important d'évaluer si ce choix d'organisation de marché est justifié par les magnitudes respectives des gains liés au meilleur fonctionnement des échanges et des pertes dues à une utilisation du réseau sous-optimale.

Cette contribution vise précisément à proposer une première évaluation quantitative des éventuelles pertes liées aux mécanismes d'attribution des capacités du réseau et à leur tarification.

VII.5.1 Méthodologie

Ce chapitre propose un modèle mathématique représentant le système entrée-sortie européen appliqué à une seule zone, qui associe une modélisation simplifiée du réseau de transport de gaz à un modèle multi-niveau des transactions économiques spécifiques à cette organisation de marché.

Au niveau supérieur, un régulateur ou un opérateur de réseau de transport fixe les capacités maximales offertes pour chaque point d'entrée et de sortie de la zone en question, ainsi que leur prix pour toute la période tarifaire considérée. Au niveau suivant, les fournisseurs et consommateurs échangent de manière compétitive sur le marché gazier en tenant compte de leur coût d'approvisionnement ou de leur bénéfice à consommer respectif, et du prix des capacités offertes par l'opérateur de réseau. Enfin, au dernier niveau, cet opérateur choisit la meilleure utilisation du réseau possible pour répondre au choix des décisions d'injection et de soutirage résultant des transactions sur le marché du gaz.

Au premier niveau, on suppose que l'opérateur de réseau anticipe les flux issus du marché gazier et les coûts de réseau engendrés par ceux-ci pour choisir les tarifs et les capacités qui maximisent le bien-être social. Afin d'examiner le meilleur système entrée-sortie théoriquement atteignable, ces tarifs et capacités peuvent être ici fixés librement, hors des méthodologies proposées habituellement par le législateur, qui peuvent elles aussi être à l'origine d'inefficacités.

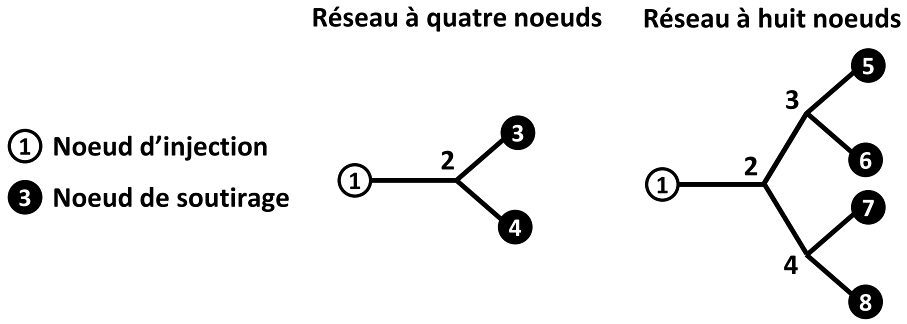
Tandis que les calculs analytiques sont réalisés à partir de ce modèle à trois niveaux, il possible de le simplifier en un modèle bi-niveau en appliquant une méthode similaire à celle présentée pour le marché électrique par (Grimm et al. 2016). Ce modèle est ensuite utilisé en combinaison avec la représentation approximée du réseau de transport de gaz présentée dans le second chapitre de cette thèse pour évaluer de façon numérique l'efficacité du système entrée-sortie. Du fait de la complexité des modèles utilisés, ces analyses sont effectuées dans le cas de réseaux simples.

VII.5.2 Principaux résultats

Les sources d'inefficacité du réseau de transport de gaz sont d'abord identifiées analytiquement. On montre ainsi que le système européen ne peut, en pratique, conduire à une allocation sur le marché gazier aussi efficace que celle proposée par une entité régulée verticalement intégrée. Ceci est vrai lorsque les tarifs sont fixés librement, lorsqu'une contrainte de recouvrement des coûts est imposée ou lorsque les capacités sont choisies séparément par l'opérateur de réseau.

Les résultats numériques démontrent que ces inefficacités sont faibles dans le cas de réseau simples, mais qu'elles dépendent du degré de variabilité de la demande de gaz, du choix des points d'entrée et de sortie du réseau et qu'elles sont susceptibles de croitre avec la taille du réseau. On présente en Table 1 à titre d'exemple les résultats obtenus lorsque la taille du réseau augmente, calculés avec les réseaux présentés en Figure 2.

Figure 2: Structure des réseaux à quatre et huit nœuds étudiés



On observe que la performance du système entrée-sortie est très proche de celle de l'opérateur intégré. Cependant, ces résultats sont obtenus pour des réseaux très simples, pour une durée d'une journée seulement, et dans le cas où trois scenarios de demande sont anticipés. D'autre part, les inefficacités du système entrée-sortie augmentent avec la taille du réseau. Ainsi, il est possible que les inefficacités soient bien plus importantes pour des zones de taille réelle.

Table 1: Influence de la taille du réseau sur l'efficacité du système entrée-sortie

$N_S = 3$		Flux			Tarif		Capacité		Bien-être social	
		$q^{with}_{t_1,i_3,s_1}$	$q^{with}_{t_1,i_3,s_2}$	$q^{with}_{t_1,i_3,s_3}$	$t^{inj}_{i_1}$	$t^{with}_{i_3}$	$k^{M,inj}_{i_1}$	$k^{M,with}_{i_3}$	w^T	$\frac{w_I^T - w_E^T}{w_I^T}$
$N_{\mathcal{N}} = 4$	Opérateur intégré (I)	13.44	10.17	6.84	–	–	–	–	780.647	
	Entrée-sortie (E)	13.44	10.24	6.76	0.46	1.01	26.95	13.44	780.618	**0.0038%**
		$q^{with}_{t_1,i_5,s_1}$	$q^{with}_{t_1,i_5,s_2}$	$q^{with}_{t_1,i_5,s_3}$	$t^{inj}_{i_1}$	$t^{with}_{i_5}$	$k^{M,inj}_{i_1}$	$k^{M,with}_{i_5}$	w^T	$\frac{w_I^T - w_E^T}{w_I^T}$
$N_{\mathcal{N}} = 8$	Opérateur intégré (I)	9.24	7.02	4.74	–	–	–	–	1050.372	
	Entrée-sortie (E)	9.24	7.10	4.66	0.45	1.79	36.94	9.24	1050.276	**0.0092%**

Ces travaux offrent pour la première fois une évaluation numérique des inefficacités d'attribution de capacités de réseau dans le cadre d'un système entrée-sortie. Bien que celles-ci soient faibles dans le cas de réseaux et d'hypothèses de demandes simples, elles semblent croitre avec la taille de la zone ou la variabilité de la demande. Dans un contexte d'augmentation de l'incertitude liée à la compensation de l'intermittence des renouvelables électriques, cette étude montre l'importance de poursuivre dans cette direction et de quantifier ces inefficacités pour des zones de taille réelle.

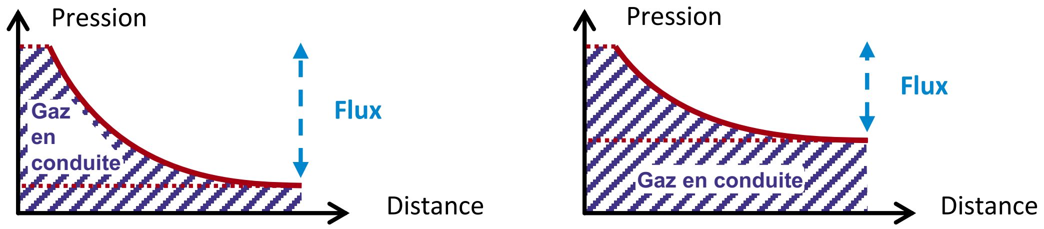
Ces travaux ont été présentés à différents stades de leur réalisation à la conférence EURO 2015 à Glasgow, au séminaire Young Energy Economists and Engineers Seminar (YEEES 2015) à Paris, au séminaire d'été de la section étudiante de l'association française des économistes de l'énergie (FAEE) en 2017 et à la conférence sur l'économie du gaz naturel « New Research Perspectives for a Rapidly-Changing World » organisée par la Chaire gaz, Mines, IFP, TSE, Dauphine à Paris en juin 2017.

VII.6 Mise aux enchères du stockage de gaz en conduite dans un système de prix nodaux pour le gaz

Afin de faire face au transfert d'incertitude du système électrique vers le système gazier par l'intermédiaire des centrales de production thermique, et lié à l'essor des renouvelables intermittents, il est crucial d'utiliser au mieux la flexibilité offerte par le réseau gazier. Cette dernière contribution examine la possibilité d'adapter la régulation concernant la gestion à court terme du réseau pour y parvenir.

La flexibilité du réseau gazier provient de la physique des flux gaziers, et plus précisément de la possibilité d'accumuler du gaz au sein du réseau en augmentant son niveau moyen de pression (Carter and Rachford 2003). Cependant, ce stockage de gaz en conduite se fait au détriment de la capacité de transport du réseau lorsque celui-ci atteint sa pression maximale. Comme le montre la Figure 3, le flux de gaz à travers un gazoduc dépend de la différence des niveaux de pression à ses extrémités. Lorsque la pression est augmentée afin de stocker du gaz, le flux transitant à travers le tuyau peut s'en trouver réduit si la pression maximale est atteinte au point d'injection. Cet arbitrage entre stockage et transport de gaz doit donc être pris en compte lors de l'allocation des capacités du réseau.

Figure 3: Arbitrage entre capacité de transport et stockage de gaz en conduite

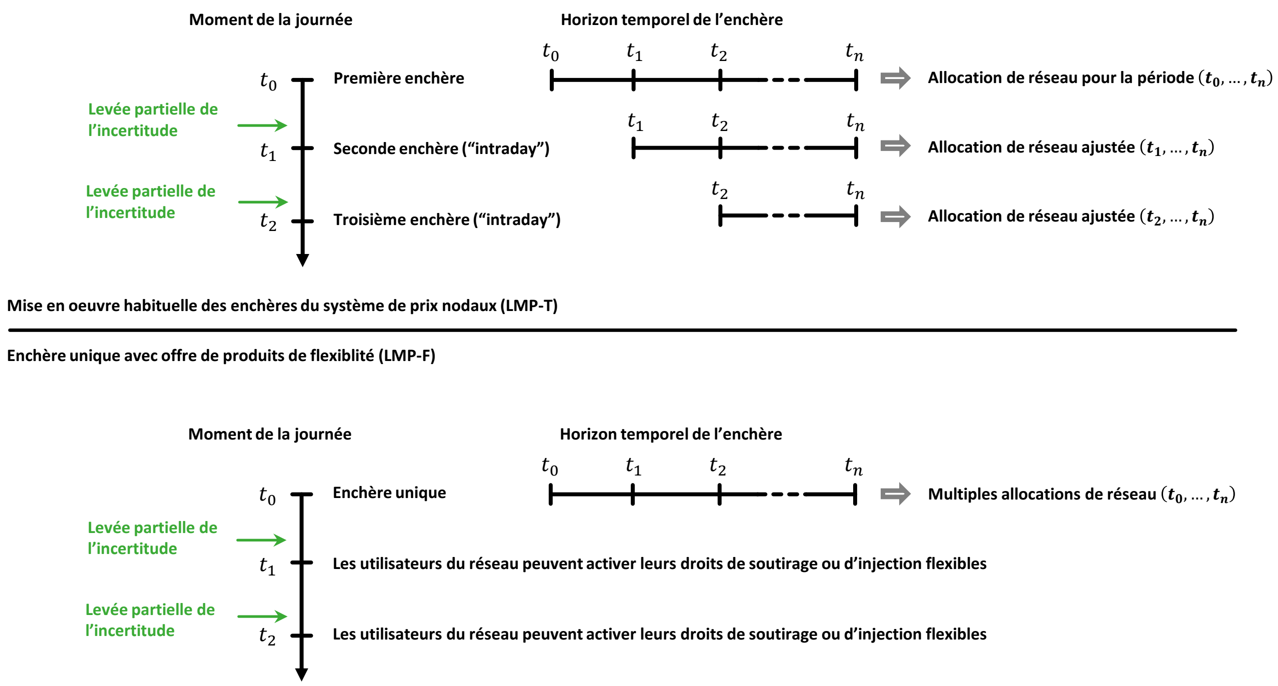


L'ensemble des formes d'organisations des marchés libéralisés du gaz laissent aujourd'hui le contrôle de cet arbitrage entre les mains de l'opérateur de réseau. Afin de proposer plus de flexibilité aux consommateurs qui pourraient en avoir besoin, ceux-ci ont recours à des dispositifs spécifiques au profit de ces utilisateurs ou autorisent un certain déséquilibre.

Plusieurs auteurs ont évoqués les inefficacités qui pourraient résulter d'une telle gestion du réseau (Hallack and Vazquez 2013, Keyaerts et al. 2011). Certains ont proposé pour y remédier la mise aux enchères directe du stockage de gaz en conduite afin d'améliorer la gestion de l'incertitude dans le cadre d'un système reposant sur les prix nodaux (Read et al. 2012, Vazquez and Hallack 2013). Ceux-ci proposent que des offres pour ce stockage soient soumises à l'opérateur de marché par les candidats à l'utilisation du réseau en même temps que des offres d'enchères pour les capacités fermes d'injection et de soutirage.

Le système de prix nodaux, notamment mis en œuvre pour la gestion de systèmes électriques libéralisés, n'a que récemment été appliqué au réseau gazier. Du fait du stockage de gaz en conduite, il est alors nécessaire d'intégrer une dimension intertemporelle à ce système. Dans sa mise en œuvre la plus simple, chaque enchère attribue des capacités de transport fermes pour l'ensemble de la journée gazière. Ces enchères peuvent ensuite être répétées au cours de la journée afin de permettre aux utilisateurs du réseau d'ajuster leurs offres en fonction de l'évolution des paramètres incertains auxquels ils font face. La séquence temporelle de cette organisation de marché est schématisée dans la partie supérieure de la Figure 4.

Figure 4: Structure des modèles d'enchères, avec et sans produits de flexibilité



Les propositions d'adapter cette organisation afin de proposer des produits de flexibilité sont jusqu'ici restées sommaire et les analyses associées qualitatives. Une première contribution de ce travail est de définir plus précisément l'organisation d'un tel marché et les produits de flexibilité nécessaires. Ainsi, une enchère unique attribuant à la fois les produits fermes et flexibles est proposée. Celle-ci est présentée dans la partie inférieure de la Figure 4. En particulier, le processus de résolution du marché effectué par l'opérateur de réseau est précisé. Pour chaque produit de flexibilité, celui-ci doit notamment s'assurer que toutes les utilisations possibles de ce produit par son acheteur conduisent à des états techniquement valides du réseau. Cette tâche, rendue particulièrement complexe par la physique du transport de gaz naturel, apparaît comme l'un des premiers obstacles à la mise en œuvre d'une telle organisation de marché.

VII.6.1 Méthodologie

Pour la première fois, il est également proposé un modèle quantitatif afin d'analyser les conséquences de ce système sur l'attribution des capacités du réseau et son efficacité.

Ce modèle représente le système d'enchères de prix nodaux sous la forme d'un équilibre dans lequel les consommateurs, les fournisseurs et l'opérateur de réseau échangent de façon compétitive des produits fermes ou flexibles à chaque nœud du réseau. L'opérateur est le seul à avoir la possibilité de faire circuler du gaz à travers le réseau, et donc à pouvoir acheter du gaz à un nœud pour le revendre à un autre. Cette modélisation est conforme à l'exposé initial du système de prix nodaux, décrit dans le contexte d'opérateurs intégrés (Boiteux and Stasi 1952, Caramanis et al. 1982, Schweppe et al. 1988).

Le modèle proposé tient compte de l'incertitude, et permet d'évaluer la performance d'un tel système d'enchère en le comparant à l'étalon de l'opérateur intégré et aux enchères traditionnelles proposant uniquement des produits fermes. Il inclue également une représentation simplifiée du réseau de transport en régime transitoire, afin de tenir compte des contraintes techniques de gestion du stockage de gaz en conduite, y compris au sein d'un processus de vérification robuste de la faisabilité des allocations du réseau dans le cas d'enchères avec produits de flexibilité.

Dans sa version d'enchères répétées de produits fermes uniquement, le modèle est nommé LMP-T. Dans le cas où il s'agit d'une seule enchère avec différents types de produits de flexibilité, celui-ci est dénoté LMP-F. Le problème résultant est d'une complexité importante, notamment du fait de la non-linéarité des équations de flux et du traitement de l'incertitude à l'aide de scénarios.

Des applications numériques sont ensuite réalisées à l'aide de ce modèle. Celles-ci se caractérisent par une structure temporelle simple à trois ou quatre périodes (la structure des enchères de prix nodaux traditionnelles est présentée dans un cas à trois périodes en Figure 5). Quant à l'incertitude examinée, elle peut être de deux types. Une incertitude temporelle d'abord, qui traduit l'impossibilité pour un consommateur de connaitre précisément la répartition dans le temps de ses besoins en gaz naturel. Elle est ici déclinée pour des intervalles de deux ou trois périodes. Une incertitude sur le volume de consommation ensuite, correspondant au cas où un utilisateur ne pourrait être certain de maitriser l'ampleur de sa consommation journalière.

Le réseau sous-jacent est lui aussi très simple, constitué de trois nœuds reliant un fournisseur et deux consommateurs de gaz naturel (voir Figure 6). Un de ces consommateurs présente une demande incertaine (pouvant représenter une centrale de production d'électricité par exemple), tandis que le second a une demande connue (comme c'est le cas pour de nombreux types d'installations industrielles).

Figure 5: Structure du modèle d'enchères traditionnelles dans un cas à trois périodes

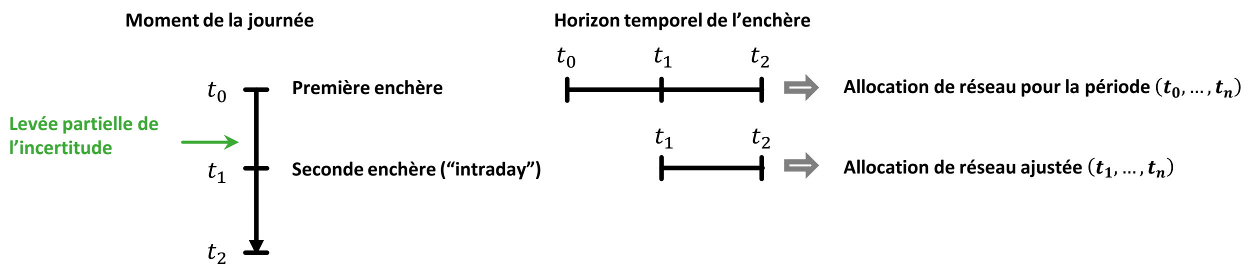
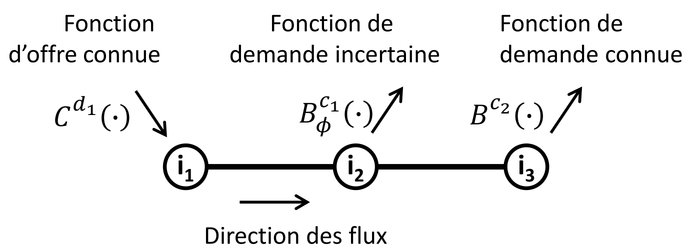


Figure 6: Structure du réseau utilisé pour les applications numériques



Pour chaque type d'incertitude, des produits de flexibilité adaptés sont introduits, tandis que les déclinaisons correspondantes du modèle sont nommées selon leurs caractéristiques. Dans le cas de produits de flexibilité temporelle à deux périodes (respectivement à trois périodes), le modèle est dénoté LMP-F_{2T} (respectivement LMP-F_{3T}). Dans le cas de produits de flexibilité en volume il est désigné par l'appellation LMP-F_V.

VII.6.2 Principaux résultats

Les résultats des applications numériques effectuées sur un réseau simple, pour diverses structures d'incertitudes et différents types de produits de flexibilité sont présentés dans la Table 2. Ceux-ci permettent de conclure qu'un tel système ne peut être efficace que lorsque le type de produits de flexibilité offert est adapté au type d'incertitude rencontrée par les utilisateurs du réseau. Ceci constitue un autre inconvénient pour cette organisation de marché : un nombre important de produits de flexibilité différents devraient être proposés pour assurer son efficacité, ce qui diminuerait d'autant la lisibilité du marché pour les participants.

Table 2: Performance des modèles d'enchère avec produits de flexibilité temporelle à deux périodes LMP-F_{2T}, à trois périodes LMP-F_{3T} et de flexibilité en volume LMP-F_V, pour une incertitude temporelle de la demande de deux ou trois périodes, ou une incertitude sur les volumes

Incertitude de la demande	Incertitude temporelle à deux périodes			Incertitude sur les volumes			Incertitude temporelle à trois périodes		
Modèle	LMP-F_{2T}	LMP-F_V	Opérateur intégré	LMP-F_{2T}	LMP-F_V	Opérateur intégré	LMP-F_{2T}	LMP-F_{3T}	Opérateur intégré
Bien-être social	0.2093	0.1575	0.2093	0.1582	0.1604	0.1923	0.2070	0.2618	0.2618
∟ Différence relative avec l'opérateur intégré	(0.0%)	(24.7%)	–	(17.7%)	(16.6%)	–	(20.9%)	(0.0%)	–

Ce système est ensuite comparé au modèle d'enchères de produits fermes traditionnelles. Les résultats correspondants sont présentés en Table 3 pour une incertitude temporelle et en Table 4 pour une incertitude sur les volumes. On observe que la mise aux enchères de la flexibilité du réseau n'apporte des gains que dans le cas d'une incertitude temporelle. D'autre part, ces gains sont relativement faibles pour les cas tests considérés. Ceux-ci ne sauraient cependant présager de l'importance de ces gains sur des réseaux réels.

Table 3: Performance du modèle d'enchères traditionnelles LMP-T et du modèle d'enchère avec produits de flexibilité et d'ajustement LMP-F_{2T}^{AdIW} pour une incertitude temporelle de la demande de deux périodes

Taille du gazoduc	D=0.7 m			D=0.4 m		
Modèle	LMP-T	LMP-F_{2T}^{AdIW}	Opérateur intégré	LMP-T	LMP-F_{2T}^{AdIW}	Opérateur intégré
Bien-être social	0.2070	0.2093	0.2093	0.2045	0.2054	0.2054
∟ Différence relative avec l'opérateur intégré	(1.11%)	(0.00%)	–	(0.45%)	(0.03%)	–

Dans le cas de l'incertitude sur les volumes, les caractéristiques intrinsèques des produits de flexibilité empêchent la gestion efficace de cette incertitude. La gestion par répétition d'enchères de produits fermes est alors plus adaptée.

Table 4: Performance du modèle d'enchères traditionnelles LMP-T T et du modèle d'enchère avec produits de flexibilité et d'ajustement LMP-F_V^{AdI} pour une incertitude sur les volumes

Taille du gazoduc	D=0.7 m			D=0.4 m		
Modèle	LMP-T	LMP-F_V^{AdI}	Opérateur intégré	LMP-T	LMP-F_V^{AdI}	Opérateur intégré
Bien-être social	0.1794	0.1604	0.1923	0.1750	0.1578	0.1874
∟ Différence relative avec l'opérateur intégré	(6.69%)	(16.57%)	–	(6.63%)	(15.81%)	–

Enfin, cette comparaison est déclinée dans le cas de réseaux présentant ou non des congestions. Lorsque le diamètre du gazoduc est faible (0.4m), les conditions de flux résultantes sont telles que les pressions maximales sont atteintes et que les prix aux différents nœuds du réseau divergent. Au contraire, lorsque celui-ci est plus important (0.7m) les pressions n'atteignent jamais leurs limites supérieures et les prix aux différents nœuds du réseau sont identiques pour une même période. Ceci permet notamment d'évaluer l'opportunité d'introduire aux côtés des produits de flexibilité des produits d'« ajustement » permettant à l'opérateur de réseau d'augmenter l'offre de capacité en obtenant un certain contrôle sur les injections et soutirages de certains consommateurs. Leurs mécanismes précis ne sont pas décrits en détail dans ce résumé.

Ces travaux montrent qu'offrir la flexibilité du réseau de transport de gaz de manière explicite peut mener à une gestion plus efficace du réseau et à une augmentation du bien-être social lié à l'activité gazière. Cependant de tels gains ne sont possibles que dans le cas où les acteurs de marché feraient face à une incertitude temporelle. Enfin, la complexité d'un tel système pourrait réduire sa lisibilité et sa transparence, et compromettre sa mise en œuvre. Il est en particulier nécessaire de résoudre au préalable le défi que représentent les calculs techniques requis pour la validation de telles enchères par l'opérateur de réseau.

Ces travaux ont été présentés aux conférences INFORMS 2015 à Philadelphie et EURO 2016 à Poznan.

VII.7 Conclusion

Les trois principaux chapitres de cette thèse montrent l'importance d'évaluer les interactions entre la régulation économique et les phénomènes physiques à l'œuvre au sein des réseaux de transport de gaz. Ils soulignent également la nécessité d'effectuer cette analyse à l'aide d'outils quantitatifs permettant de juger de l'ampleur de ces interactions et des inefficacités qu'elles peuvent créer, ou de l'opportunité d'éventuels ajustements régulatoires. Face à la complexité de ces modèles, combinant des représentations économiques et techniques du réseau de transport de gaz, de futurs efforts de recherche seront essentiels pour permettre l'analyse de systèmes et marchés de taille réelle.

VII.8 Références

Abada I, Ehrenmann A, Smeers Y (2017) Modeling Gas Markets with Endogenous Long-Term Contracts. *Operations Research* 65(4):856–877.

Abada I, Massol O (2011) Security of supply and retail competition in the European gas market. *Energy Policy* 39(7):4077–4088.

Averch H, Johnson LL (1962) Behavior of the firm under regulatory constraint. *The American Economic Review*:1052–1069.

Babonneau F, Nesterov Y, Vial JP (2012) Design and Operations of Gas Transmission Networks. *Operations Research* 60(1):34–47.

Boiteux M, Stasi P (1952) Sur la détermination des prix de revient de développement dans un système interconnecté de production-distribution. *Reprinted in G. Morlat et F.Bessière, 1971, "Vingt-cinq ans d'économie électrique," Paris Dunod, 361-400.*

Brouwer J, Gasser I, Herty M (2011) Gas Pipeline Models Revisited: Model Hierarchies, Nonisothermal Models, and Simulations of Networks. *Multiscale Model. Simul.* 9(2):601–623.

Callen J, Mathewson GF, Mohring H (1976) The Benefits and Costs of Rate of Return Regulation. *The American Economic Review* 66(3):290–297.

Caramanis MC, Bohn RE, Schweppe FC (1982) Optimal Spot Pricing: Practice and Theory. *IEEE Transactions on Power Apparatus and Systems* PAS-101(9):3234–3245.

Carter RG, Rachford HHJ (2003) Optimizing Line-Pack Management to Hedge Against Future Load Uncertainty. *PSIG Annual Meeting*.

Chaton C, Creti A, Villeneuve B (2009) Storage and security of supply in the medium run. *Resource and Energy Economics* 31(1):24–38.

Chenery HB (1949) Engineering Production Functions. *The Quarterly Journal of Economics* 63(4):507–531.

Chenery HB (1952) Overcapacity and the Acceleration Principle. *Econometrica* 20(1):1–28.

Creti A, Villeneuve B (2004) Longterm Contracts and Take-or-Pay Clauses in Natural Gas Markets. *Energy Studies Review* 13(1).

De Wolf D, Smeers Y (1993) Optimal Dimensioning of Pipe Networks with Application to Gas Transmission Networks.

Egging R, Gabriel SA, Holz F, Zhuang J (2008) A complementarity model for the European natural gas market. *Energy Policy* 36(7):2385–2414.

European Commission (2017) *Commission Regulation (EU) 2017/460 of 16 March 2017 establishing a network code on harmonised transmission tariff structure for gas* (European Commission).

Fodstad M, Midthun KT, Tomasgard A (2015) Adding flexibility in a natural gas transportation network using interruptible transportation services. *European Journal of Operational Research* 243(2):647–657.

Gabriel SA, Zhuang J, Kiet S (2005) A large-scale linear complementarity model of the North American natural gas market. *Energy Economics* 27(4):639–665.

Grimm V, Martin A, Schmidt M, Weibelzahl M, Zöttl G (2016) Transmission and generation investment in electricity markets: The effects of market splitting and network fee regimes. *European Journal of Operational Research* 254(2):493–509.

Hallack M, Vazquez M (2013) European Union regulation of gas transmission services: Challenges in the allocation of network resources through entry/exit schemes. *Utilities Policy* 25:23–32.

Hunt P (2008) *Entry–Exit Transmission Pricing with Notional Hubs: Can it Deliver a Pan-European Wholesale Market in Gas?* (Oxford Institute for Energy Studies).

ICF International (2012) *The future of natural gas in Mozambique: towards a gas master plan - executive summary* (ICF International, Fairfax, Virginia, USA).

Joskow PL (1999) Regulatory Priorities for Reforming Infrastructure Sectors in Developing Countries. *Annual World Bank Conference on Development Economics 1998*. (The World Bank, Washington, D.C.), 191–223.

Keyaerts N, Hallack M, Glachant JM, D'haeseleer W (2011) Gas market distorting effects of imbalanced gas balancing rules: Inefficient regulation of pipeline flexibility. *Energy Policy* 39(2):865–876.

Klevorick AK (1971) The "Optimal" Fair Rate of Return. *The Bell Journal of Economics and Management Science* 2(1):122–153.

Koch T, Hiller B, Pfetsch ME, Schewe L eds. (2015) *Evaluating Gas Network Capacities* (Society for Industrial and Applied Mathematics, Philadelphia, PA).

Mahlke D, Martin A, Moritz S (2007) A simulated annealing algorithm for transient optimization in gas networks. *Math Meth Oper Res* 66(1):99–115.

Menon ES (2005) *Gas Pipeline Hydraulics* (CRC Press).

Midthun KT, Bjorndal M, Tomasgard A (2009) Modeling optimal economic dispatch and system effects in natural gas networks. *The Energy Journal* 30(4):155–180.

Miriello C, Polo M (2015) The development of gas hubs in Europe. *Energy Policy* 84:177–190.

Moritz S (2007) *A mixed integer approach for the transient case of gas network optimization*. (TU Darmstadt).

Qadrdan M, Chaudry M, Wu J, Jenkins N, Ekanayake J (2010) Impact of a large penetration of wind generation on the GB gas network. *Energy Policy* 38(10):5684–5695.

Read EG, Ring BJ, Starkey SR, Pepper W (2012) An LP Based Market Design for Natural Gas. Sorokin A, Rebennack S, Pardalos PM, Iliadis NA, Pereira MVF, eds. *Handbook of Networks in Power Systems II*. Energy Systems. (Springer Berlin Heidelberg), 77–113.

Rious V, Hallack M (2009) *Tariffs for European Gas Transmission Networks* (Florence School of Regulation).

Schweppe FC, Caramanis MC, Tabors RD, Bohn RE (1988) *Spot Pricing of Electricity* (Springer US, Boston, MA).

Vazquez M, Hallack M (2013) Need and design of short-term auctions in the EU gas markets. *Energy Policy* 63:484–493.

Yépez RA (2008) A Cost Function for the Natural Gas Transmission Industry. *The Engineering Economist* 53(1):68–83.

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